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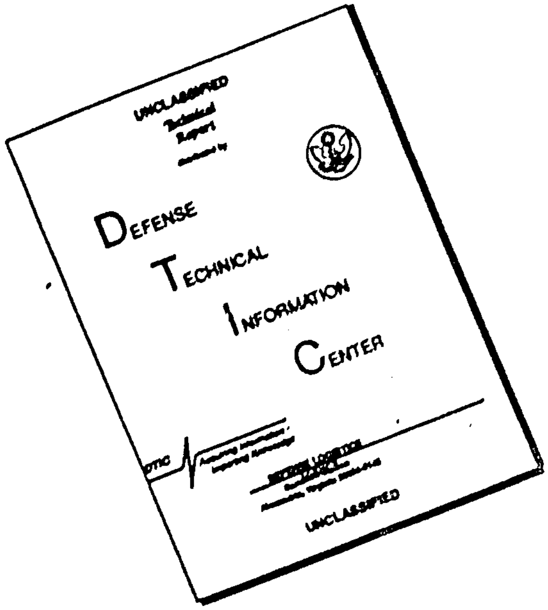
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AFSC-TR-65-2 Vol III

WEAPON SYSTEM EFFECTIVENESS INDUSTRY ADVISORY COMMITTEE (WSEIAC)

FINAL REPORT

of

TASK GROUP II

PREDICTION - MEASUREMENT (TECHNICAL SUPPLEMENT)

January 1965

Systems Effectiveness Division
Systems Policy Directorate
Headquarters, Air Force Systems Command
Andrews Air Force Base, Maryland

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ERRATA

8 March 65

FINAL REPORTS OF THE
WEAPON SYSTEM EFFECTIVENESS INDUSTRY ADVISORY COMMITTEE (WSEIAC)

AFSC-TR-65-1

AFSC-TR-65-2 (Vols I, II & III)

AFSC-TR-65-3

AFSC-TR-65-4 (Vols I & III)

(Note: This Errata sheet applies to each of the above reports.)

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WEAPON SYSTEM EFFECTIVENESS

INDUSTRY ADVISORY COMMITTEE (WSEIAC)

FINAL REPORT

of

TASK GROUP II

PREDICTION - MEASUREMENT
(TECHNICAL SUPPLEMENT)

FOREWORD

This is Volume III of the final report of Task Group II of the Weapon System Effectiveness Industry Advisory Committee (WSEIAC). It is submitted to the Commander, AFSC, in partial fulfillment of Task Group II objectives cited in the committee Charter. The final report is contained in three separate volumes:

> Volume I contains an overview of Task Group II findings, including a summary of Volumes II and III, conclusions, and recommendations.
>
> Volume II contains a discussion of effectiveness concepts, a description of specific tasks required to evaluate effectiveness, and a detailed example illustrating the method.
>
> Volume III contains descriptions of effectiveness analysis methods applied to four typical Air Force systems using the techniques described in Volume II.

The membership of Task Group II was as follows:

Mr. D. F. Barber (Chairman)	RADC (EMER)
Mr. I. Bosinoff	Sylvania Electronics System Division
Mr. I. Doshay	Space General Corporation
Dr. B. J. Flehinger	IBM - Thomas J. Watson Research Laboratories
Mr. W. Haigler	Rocketdyne - Division of North American Aviation, Inc.
Mr. H. J. Kennedy	ARINC Research Corporation
Mr. C. R. Knight (Technical Director)	ARINC Research Corporation
Mr. A. J. Monroe	TRW Space Technology Laboratories
Mr. M. H. Saunders	OOAMA (OONEW)
Mr. M. M. Tall	Radio Corporation of America
Mr. H. D. Voegtlen	Hughes Aircraft Company

Other task group reports submitted in fulfillment of the committee's objectives are:

AFSC-TR-65-1	Final Report of Task Group I "Requirements Methodology"
AFSC-TR-65-3	Final Report of Task Group III "Data Collection and Management Reports"

AFSC-TR-65-4	Final Report of Task Group IV "Cost Effectiveness Optimization"
AFSC-TR-65-5	Final Report of Task Group V "Management Systems"
AFSC-TR-65-6	Final Summary Report "Chairman's Final Report"

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

APPROVED

William F. Stevens

William F. Stevens, Colonel, USAF
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DCS Systems

WSEIAC CHARTER

In order that this report of Task Group II may be studied in context with the entire committee effort, the purpose and task group objectives as stated in the WSEIAC Charter are listed below:

Purpose

The purpose of the Weapon System Effectiveness Industry Advisory Committee is to provide technical guidance and assistance to AFSC in the development of a technique to apprise management of current and predicted weapon system effectiveness at all phases of weapon system life.

Task Group Objectives

Task Group I - Review present procedures being used to establish system effectiveness requirements and recommend a method for arriving at requirements that are mission responsive.

Task Group II - Review existing documents and recommend uniform methods and procedures to be applied in predicting and measuring systems effectiveness during all phases of a weapon system program.

Task Group III - Review format and engineering data content of existing system effectiveness reports and recommend uniform procedures for periodically reporting weapon system status to assist all levels of management in arriving at program decisions.

Task Group IV - Develop a basic set of instructions and procedures for conducting an analysis for system optimization considering effectiveness, time schedules, and funding.

Task Group V - Review current policies and procedures of other Air Force commands and develop a framework for standardizing management visibility procedures throughout all Air Force commands.

ABSTRACT

This Technical Supplement is concerned primarily with four examples of effectiveness evaluations. The systems involved are: The avionics system in a tactical fighter-bomber (Example A); a squadron of intercontinental ballistic missiles (Example B); a fixed radar surveillance and threat evaluation system (Example C); and, a spacecraft system (Example D). In addition to the variety of system types included, an attempt has been made to illustrate procedures employed at different phases of development. The evaluation of the Avionics system takes place during Program Definition; the ICBM squadron, during Operation; the Radar system, during Definition and Operation; and, the Spacecraft, during Acquisition. Since evaluation during the Conceptual phase will generally be based on a gross comparison with existing, similar systems, it was not felt that an example of such an analysis was necessary. Further, each example is intended to illustrate to a different level of detail, various aspects of tl · evaluation. The avionics system example, for instance, shows the possibility of combining independent evaluations of several subsystems. The radar example shows simplifications which can be made in order to minimize the number of system states to be considered. In the ICBM example, illustrations of many of the detailed procedures required to evaluate components of the vectors and matrices are shown. Finally, the spacecraft example addresses itself to techniques for determining elements of the Dependability matrix. It is stressed, however, that these examples do not purport to illustrate <u>all</u> possible methods of application and use of the evaluation procedures. Rather they are intended to show some methods for applying the concepts, areas of flexibility in their application, and some uses which might be made of the evaluations.

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SECTION I

INTRODUCTION

Resumé of Task Group II Effort

In Volume II of the final report of Task Group II, Weapon System Effectiveness Industry Advisory Committee, a discussion of the concept of System Effectiveness was presented. In addition, a mathematical model to facilitate the evaluation of Effectiveness was proposed; the tasks to be accomplished in using the model were delineated and discussed; and a tutorial example showing the application of the procedures to a radar system was presented.

It was appreciated by members of Task Group II that Effectiveness evaluation for large weapon systems is a complex task, subject to many variations in detailed procedures depending upon the system type, available information/data, and the stage of development. In order to provide a preliminary analysis of the utility of the methods proposed, Task Group II applied the procedures in evaluating the Effectiveness of several hypothetical systems. While these exercises cannot be considered to have raised and answered all questions that will occur during actual evaluations, they do suggest some areas of difficulty and types of solutions applicable.

Aside from providing a preliminary evaluation of the proposed procedures, it was felt that presentation of these examples would provide the reader with additional comment on the application of the techniques. For this reason, they are discussed at some length in SECTION II of this Technical Supplement.

In addition to the examples, this supplement also contains two technical papers not generally available, yet of interest to personnel concerned with Effectiveness evaluation. These papers comprise the two appendixes to this supplement.

Finally, a tabulation of data sources which may be employed in the analysis of System Effectiveness is included as a BIBLIOGRAPHY.

The following paragraphs present an analytical framework common to the treatment of the four examples.

Mathematical Framework

The specific, basic, analytical model proposed by Task Group II in its symbolic form is

$$E = \overline{A}' \left[D\right] \overline{C}$$

where

E = System Effectiveness, is a measure of the extent to which a system may be expected to achieve a set of specific mission requirements and is a function of availability, dependability, and capability.

$\overline{A}'$ = Availability, is a measure of the system condition at the start of a mission and is a function of the relationships among hardware, personnel, and procedures.

$\left[D\right]$ = Dependability, is a quantitative measure of the system condition at one or more points during the mission, given the system condition(s) at the start of the mission, and may be stated as the probability (or probabilities or other suitable mission oriented measure) that the system will enter and/or occupy any one of its significant states during a specified mission.

$\overline{C}$ = Capability, is a measure of the ability of a system to achieve the mission objectives, given the system condition(s) during the mission, and specifically accounts for the performance spectrum of a system.

This basic framework is not intended to be restrictive. This point is illustrated in the radar, detection and tracking example of Volume II where the following variations on the basic model are illustrated:

$$E_1 = \overline{A}' C(0)$$

$$E_2 = \overline{A}' \left[C(0)\right] \left[D(30)\right] \overline{C}(30)$$

$$E_3 = \frac{E_2}{E_1}$$

In the first variation, the system effectiveness (E_1) is defined to be the probability that the radar will adequately perform initial detection of the target. In this case the dependability matrix reduces to unity since "mission duration" is measured from the point of initial detection, and $\overline{C}$ applies to detection capability only (denoted by $\overline{C}(0)$. In the second variation, the system effectiveness (E_2) is defined to be the probability of initial detection and track for a period of thirty minutes. In this case the elements of the detection capability vector $\overline{C}(0)$ become the elements of a capability matrix $C\left[C(0)\right]$ The original availability vector $\overline{A}$ and this new capability matrix $C\left[C(0)\right]$ are now multiplicatively combined with a dependability matrix $\left[D(30)\right]$ and a new capability vector $\overline{C}(0)$ which express the tracking capability of the radar for a period of thirty minutes. In the final variation; the system effectiveness (E_3) is defined to be the probability of successful track; given initial detection. This conditional measure is the ratio of the two previously treated variations.

The intended flexibility of approach is further illustrated in the avionics example, which is Example A of this Technical Supplement, where the following series of effectiveness measures are illustrated,

$$E_j^{(i)} = \overline{A}_j \left[D\right]_j \overline{C}_j^{(i)}$$

$$E^{(i)} = \prod_{j=i}^{k} E_j^{(i)}$$

$$E = \sum_{i=1}^{m} P_i E^{(i)}$$

The first measure $E_j^{(i)}$ treats the effectiveness of the j^{th} system function or subsystem in the i^{th} mode of operation in terms of the basic analytical model. The system effectiveness in the i^{th} mode of operation ($E^{(i)}$) is then treated as the continued product of the $E_j^{(i)}$ over the k subsystems (or functions) that collectively define the avionics system. Finally, the net effectiveness of the entire avionics system (E) is the sum of the effectiveness of the system in each of its modes of operation $E^{(i)}$ multiplied by the probability P_i of utilizing that mode of system operation, where m is the number of modes of operation.

The common elements in these variations are availability, dependability, and capability. The precise manner in which they combine depends wholly upon the specific definition of system effectiveness which is to be considered.

SECTION II

EXAMPLES OF SYSTEM EFFECTIVENESS EVALUATION

This section consists of four examples of effectiveness evaluation. The examples relate to the following systems:

Example A - the avionics system in a tactical fighter-bomber
Example B - a squadron of intercontinental ballistic missiles
Example C - a fixed radar surveillance and threat evaluation system
Example D - a spacecraft system

As stated previously in the Abstract, each example illustrates, to a different level of detail, various aspects of the evaluation.

The examples do not presume to illustrate all possible methods of application and use of the evaluation procedures. It is the intent of the examples, however, to show some methods for applying the concepts, areas of flexibility in their application, and some uses which could be made of the evaluations.

EXAMPLE A

AIRBORNE AVIONICS SYSTEM

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ILLUSTRATIONS

TABLES

I. INTRODUCTION AND SUMMARY

This example shows the application of the expression

$$E = \overline{A}'\lfloor D \rfloor \overline{C}$$

to the avionics system of a tactical fighter-bomber aircraft. The evaluation proceeds by performing the analyses outlined in the eight-step task analysis , VOLUME II.

The evaluation is made in this example by determining the Effectiveness of each of several functions of the avionics system for each of three mission types. These figures are then combined to provide an indication of the overall Effectiveness of the system.

A computer program was written for the model so that parameter variation was feasible. Curves showing the influences on Effectiveness of variations in basic reliability and maintainability characteristics of the several equipments are shown. A relationship is also shown between the required number of systems to provide assurance of mission accomplishment and the effectiveness of the system.

II. EFFECTIVENESS ESTIMATION

1.0 Mission Definition

At any random time when an execution order is received, the aircraft shall take off immediately, receive a target assignment, proceed to target area, deliver weapon within 500 feet of target, and return to assigned operating base.

2.0 System Description

2.1 General Configuration

The system being considered consists of three major subsystems which are, where appropriate, sub-divided into equipments.

a. Fire Control Subsystem

   1. Radar (Search and Terrain Avoidance functions)
   2. Toss-bomb Computer
   3. Sight System

b. Doppler Navigator

   The Doppler Navigator in this example is considered to be a single equipment.

c. Communication-Identification-Navigation (CIN)

1. UHF direction finder
2. Tacan
3. Instrument Landing System (ILS)
4. UHF transmitter-receiver
5. Identification equipment
6. Audio amplifier equipment

The equipments itemized are independent of each other, i.e., the condition of any equipment does not influence the condition of any other.

2.1.1 Functions of Equipments

The Fire Control Subsystem is employed in actual weapon delivery. It provides a radar display of the target and computation of weapon release point in the toss-bombing mode. It also provides, through the Sight System, the aiming point for "lay-down" delivery.

In addition, the "terrain avoidance" feature of the radar provides automatic control of the aircraft so that high speed, low level target approaches are possible. To simplify the example, it will be assumed that the equipment required for the terrain avoidance function is separate from that required for the bombing function.

The Doppler Navigator provides the prime navigation function by computing and displaying information on both present position and distance/heading to target. Alternate navigation procedures are provided by the Tacan and the UHF Direction Finder. Each of these, however, requires ground station facilities. If ground station transmitters are available, operating and within range, the Tacan provides distance and bearing information, while the Direction Finder provides bearing data only.

The Instrument Landing System (ILS) provides the ability to land the aircraft under ceiling and visibility conditions which would otherwise prevent landing.

The UHF transmitter-receiver is the only radio communication device, and is employed for all in-flight radio communication. For the mission being considered, the essential communication function is that of receiving and acknowledging target assignment information. The Audio Amplifier equipment is employed with the UHF transmitter-receiver only, and may be considered as a part of that equipment.

The Identification equipment (IFF) provides a coded identification signal in response to an interrogation by friendly forces. Failure to provide the proper response can result in attack by friendly forces.

2.2 Block Diagram

A general block diagram of the system is shown in Figure 1. The essential functions to be performed are indicated in the upper diagram, while the equipment(s) capable of performing the functions are shown in the lower diagram.

2.3 Mission Profile

A time-line analysis of the mission being considered is shown in Figure 2. The upper section shows the function(s) being performed during various phases of the mission. The lower section shows the times during the mission when the functioning of each equipment is desired. Because the demands upon the equipments vary with the type of bomb delivery, the requirements are shown for each of the three bomb delivery modes, viz., visual lay-down (VL), visual toss (VT), and blind toss (BT).

2.4 Delineation of Mission Outcomes

(A) Mission accomplished exactly as noted in (1.0)

(B) Mission not accomplished exactly as noted in (1.0)

  (1) Aircraft does not proceed without delay.

    (a) One or more subsystems known or thought to be in such state that aircraft is not launched.

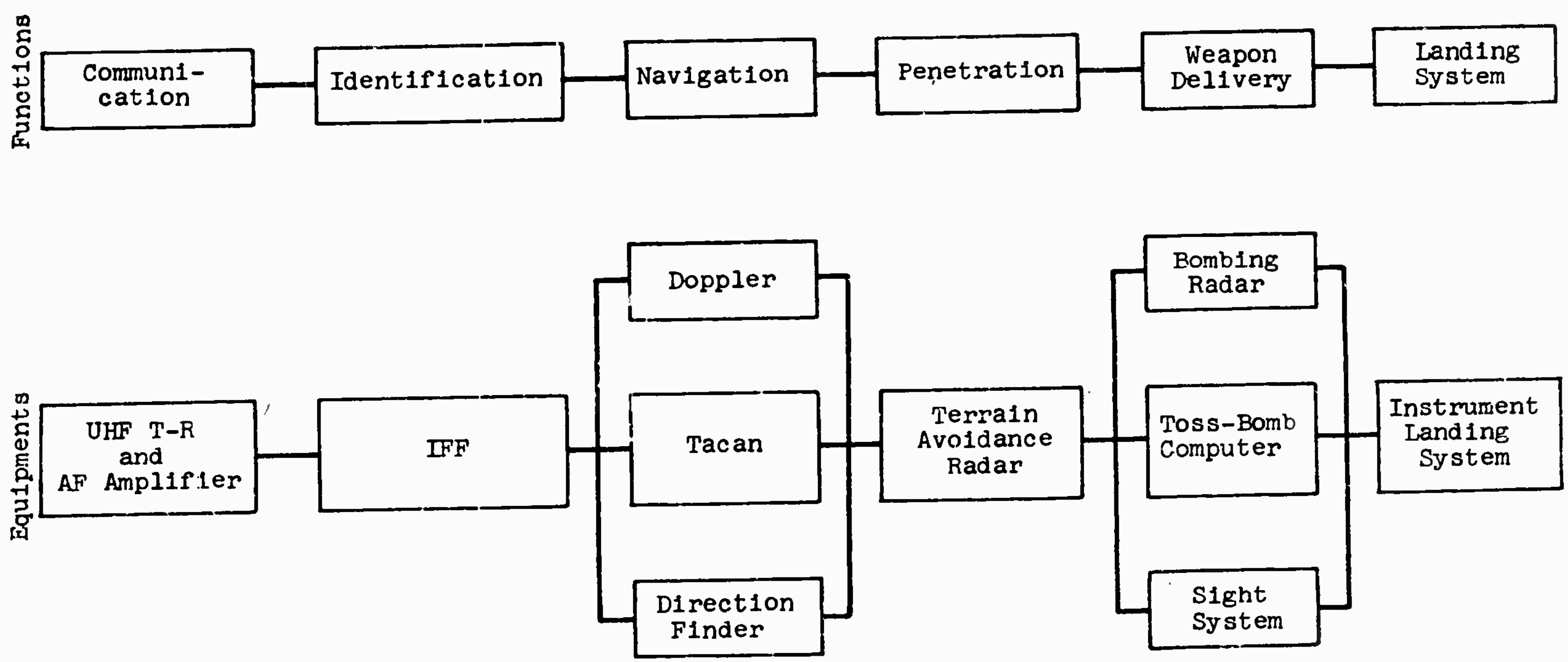

FIGURE 1

SYSTEM BLOCK DIAGRAM

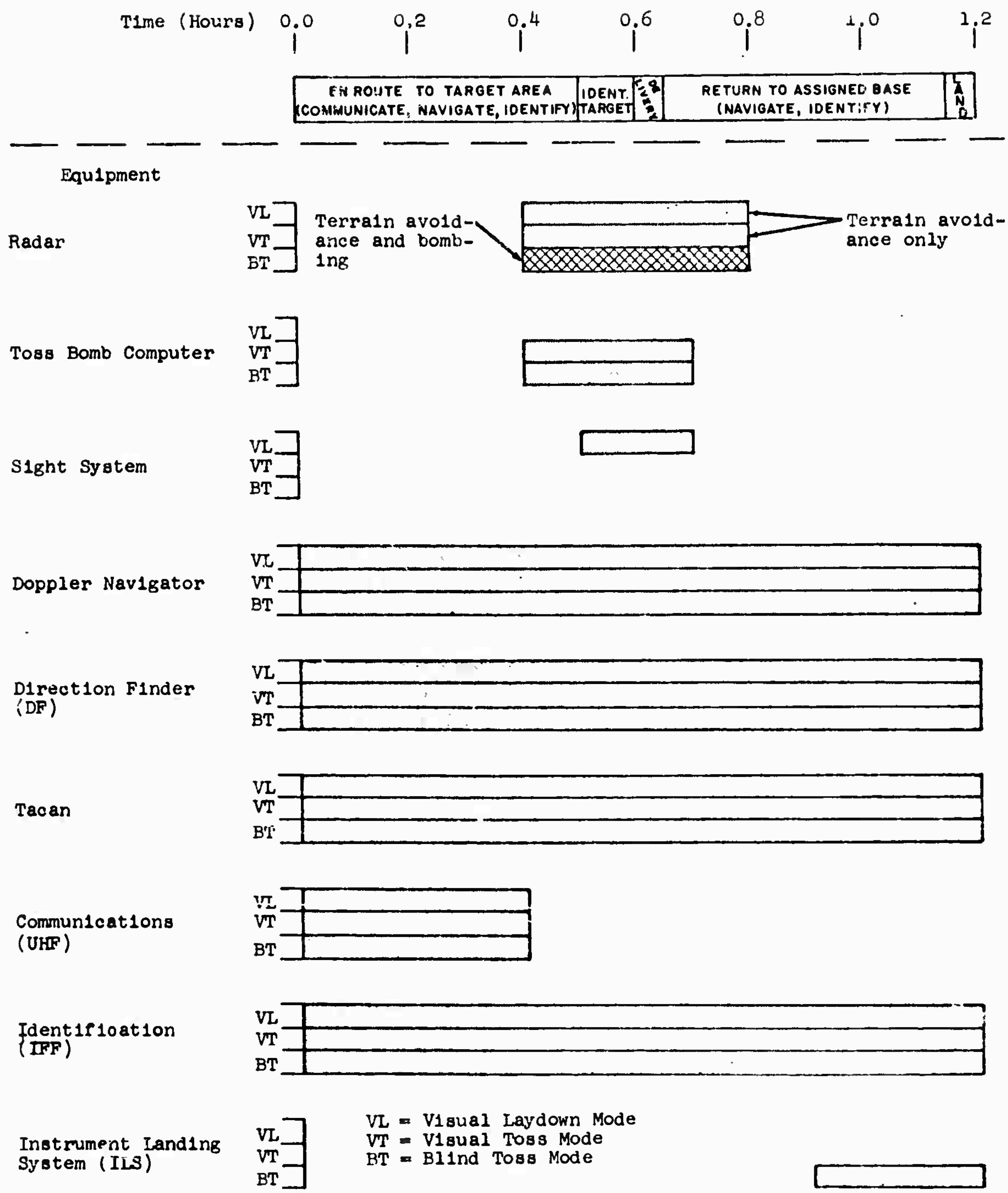

FIGURE 2

MISSION PROFILE AND PERIODS
DURING WHICH USE OF EACH EQUIPMENT IS DESIRED

(2) Aircraft does not receive target assignment.

(a) Failure or inadequacy of one or more subsystems prevents receipt of target assignment

(3) Aircraft does not deliver weapon within 500 feet of target.

(a) Aircraft does not reach target area. (No weapon release.)

(a-1) Failure or inadequacy of one or more subsystems prevents reaching target area.

(b) Aircraft does not identify target. (No weapon release.)

(b-1) Failure or inadequacy of one or more subsystems prevents identification of target.

(c) Aircraft does not place weapon within 500 feet of target. (Release)

(c-1) Failure or inadequacy of one or more subsystems results in inaccurate delivery.

(4) Aircraft does not return to assigned operating base.

(a) Aircraft lost.

(a-1) Failure or inadequacy of one or more subsystems results in aircraft loss.

(b) Aircraft returns to wrong base.

(b-1) Failure or inadequacy of one or more subsystems prevents return to assigned base.

3.0 Specification of Figures-of-Merit

For this specific mission requirement, the major figure-of-merit is the probability that the mission, as defined, will be accomplished.

Accomplishment of the mission, however, depends upon the successful performance of several individual functions. Following take-off, the required functions are:

a. Receipt and acknowledgement of target assignment.

b. Navigation to a point not more than five miles from target.

c. Proper identification when interrogated.

d. Penetration of enemy defenses.

e. Identification of target and weapon delivery within 500 feet of target.

f. Navigation to within 10 miles of assigned operating base.

g. Landing.

The probability of accomplishing each of these functions may also be regarded as an appropriate figure of merit of interest to particular levels of management. For this reason, each will be evaluated.

4.0 Identification of Accountable Factors

4.1 Tabulation of Factors

a. Operational conditions

- Physical environment (climate)
- Day vs. night conditions
- Good (VFR) vs. bad (IFR) weather
- Modes of weapon delivery
- Enemy counteractions
- Actions by friendly forces

b. Support situation

- Ground operating equipment
- Ground support equipment
  - Availability and adequacy
    - Test equipment
    - Repair facilities
- Maintenance personnel
  - Number and skill levels
  - Number of shifts

Spare parts and units

Availability
Repair philosophy

Module vs. part replacement

4.2 Discussion of Factors

Climate: The evaluation is to be conducted for a semi-tropical environment. The ground temperatures range from 70°-105°F, humidity between 60-100%. Atmospheric conditions which result in improper radar function are anticipated 1% of the time.

Visibility: Daylight conditions exist for 14 of the 24 hours per day, or for 58% of the time.

Bad weather (IFR) conditions exist, on the average, 20% of the time, night or day.

Visibility conditions of such a nature that the Instrument Landing System is essential to safe landing exist 5% of the time.

Influence of visibility conditions on mode of weapon delivery: The weapon delivery mode depends upon both the visibility conditions and the tactical requirement. Visual modes can be used only under daylight VFR conditions. The tactical requirements are such that the lay-down mode will be preferred 80% of the time. (The decision concerning lay-down or toss must be made prior to take-off, since a different type weapon is required for each.) If toss-bombing is preferred, the visual method will be selected whenever possible, i.e., weather and daylight permitting.

Enemy Action: Enemy defensive action, i.e., the enemy's ability to destroy intruding aircraft, is such that

1. A 30% loss of aircraft is anticipated for aircraft approaching at altitudes in excess of 1000 feet at normal attack speed.
2. A 5% loss of aircraft is anticipated for aircraft approaching at altitudes of less than 1000 feet at normal attack speed.

Friendly Action: Friendly defenses in the area are such that 90% of the aircraft entering the defense area are challenged. If electronic identification equipment in friendly aircraft does not respond properly to a challenge, a 0.10 probability of destruction of the aircraft by friendly defense exists. (This figure reflects the occasions when secondary methods of identification, e.g., visual, prevent attack on friendly aircraft.)

Availability of Ground Station Equipments:

Tacan: It is expected that a Tacan ground station will be available, operating, and within range 50% of the time.

UHF Ground Station: It is expected that a UHF ground station will be available, operating, and within range 40% of the time.

Ground Support Equipment: Sufficient ground equipment will be provided so that no delays in repair due to this factor will occur. Further, test equipment and repair facilities will be available and adequate to the degree that the mean-down-times presented in a later section are anticipated.

Maintenance Personnel: The quantity of maintenance personnel of various skill levels is such that the down-times referred to above represent also the influence of this factor.

Spare Parts/Units: All repairs to the avionics system are to be made through replacement of "flight-line replaceable units". No in-shop maintenance is anticipated at this echelon. Sufficient spare units will be provided to prevent logistic delays.

5.0 Model Construction

5.1 Delineation of System States

Only two states of each equipment, i.e., operative and failed, are to be considered. It will be observed that if all combinations of two states of each of ten equipments are considered, more than 1000 system states are defined. This situation would obviously complicate the system evaluation.

In this case, however--and in many actual cases--simplifications can be developed. It was noted in Paragraph 2.0 that all equipments are independent. For this reason, the Effectiveness of each equipment could be determined individually and the resulting figures combined to determine the system effectiveness. Because of an interest in the effectiveness of each major function, however, this procedure will be applied at the "function" level rather than at the equipment level. Therefore, the three navigation equipments will be treated collectively, so that the eight possible combinations of the three equipment states will be considered. Also, the four combinations of the Radar and Toss-bomb Computer states will be considered.

5.2 Operational Considerations and Equipment Usage

In this analysis, two methods of weapon delivery (Toss and Lay-down), and two basic environmental conditions (Daylight or VFR, and Night or IFR) will be considered. However, the Lay-down type delivery is only attempted during daylight (VFR).

NOTE No especially serious attempt has been made to make the example completely realistic. Conditions and requirements have generally been selected to demonstrate procedures to be employed.

In Figure 2 were shown the several mission components on a time scale. In addition, for each of three combinations of delivery mode and environmental conditions, the equipments required during various portions of the missions are indicated.

The three situations are:

VL - Visual conditions, lay-down-type delivery

VT - Visual conditions, toss-bomb delivery

BT - Blind conditions, toss-bomb delivery.

The probabilities of accomplishing the mission in each of the three situations will be evaluated. The overall Effectiveness will then be determined by combining the three figures, weighted by the probabilities of occurrence of each situation.

5.3 System Model

The system model must express the probability of successfully completing a mission as a function of (1) the effectiveness of the system for each of the three delivery modes, and (2) the probability of employing each delivery mode. This can be represented by the following simple model:

$$E = \sum_{i=1}^{3} E_i P_i$$

where

E = System effectiveness

E_i = System effectiveness in Mode i

P_i = Probability of using Mode i.

The three values of P_i will be determined from consideration of tactical requirements and operational conditions. The values of E_i will be derived by combining the Effectiveness figures for each mission function, e.g., navigation, communication, in accordance with the requirement for each function in the particular mission type. The individual function effectiveness figures will be computed from the proposed basic model:

$$E = \bar{A}'[D]\bar{C}$$

Further description of the individual models will be presented in Section 7.0.

6.0 Data Acquisition

Because this evaluation is being made during the Program Definition phase, predictions of the several components of

Effectiveness will be required. Suitable prediction techniques must, therefore, be specified.

While several methods for predicting reliability and maintainability are available, the procedures developed for the Aeronautical Systems Division, AFSC, by ARINC Research[1/] are appropriate for this evaluation.

It is assumed that estimates of the basic capabilities of the various equipments have been made by individuals who are expert in regard to specific equipment types. This is a reasonable assumption, since it generally cannot be expected that one individual will be sufficiently experienced in all areas to make such estimates independently.

7.0 Parameter Estimation

7.1 Basic Equipment Characteristics

The prime purpose of this example is to illustrate a procedure for evaluation of Effectiveness. While the prediction of the basic components of Effectiveness for any

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1/ H. Balaban & A. Drummond, "Prediction of Field Reliability for Airborne Electronic Systems", ARINC Research Publication No. 203-1-344, 31 December 1962.

G. Harrison, H. Leuba, & E. Schneider, "Maintainability Prediction - Theoretical Basis and Practical Approach" (Revised), ARINC Research Publication No. 267-02-6-420, 31 December 1963.

equipment is certainly basic to the evaluation, a detailed description of the application of reliability and maintainability prediction techniques will not enhance this example. For further discussion of these procedures, the reader is referred to the list of references.

For the purposes of this example, assume that reliability and maintainability predictions made in accordance with the procedures specified resulted in the individual mean-times-between-failures (t_f) and the mean-down-time (t_d) shown in Table I. Further, the State Readiness figure, V_1,[2/] is calculated from

$$V_1 = \frac{t_f}{t_f + t_d}$$

The probability that the equipment is not ready, i.e., is in State 0, is

$$V_0 = 1 - V_1$$

The basic Capability indices will be discussed in Section 7.4.

2/ The subscript notations "1" and "0" will be employed throughout this example to indicate respectively, operative state and failed state. Where the individual states of several equipments determine functional states, e.g., Navigation, an alphabetic and numeric subscript will be employed. For example, the situation in which the Doppler and the Direction Finder are each in State 1 and the Tacan is in State 0 is identified as N_3.

TABLE I

Reliability, Maintainability, and State Readiness Indices

Equipment	Mean-time-between-failure--t_f (hours)	Mean-down-time--t_d (hours)	V_1	V_0
Radar				
Bombing	32	6	0.842	0.158
Terrain Avoidance	40	8	0.833	0.167
Toss-bomb Computer (TBC)	20	4	0.833	0.167
Sight System	200	2	0.990	0.010
Doppler	20	15	0.571	0.429
Direction Finder (DF)	100	2	0.980	0.020
Tacan	50	4	0.926	0.074
Instrument Landing System (ILS)	150	3	0.980	0.020
Communication Equipment (UHF & Amplifier)	70	2	0.972	0.028
Identification Equipment (IFF)	100	3	0.971	0.029

7.2 Determination of Availability

In this example, two factors will be considered in establishing the Availability vector.

V = The probability that an equipment (or group of equipments) is in a particular state of readiness, and

W = The probability that an aircraft will be launched with the equipments in a particular state of readiness.

These two factors will be discussed in the following sections.

7.2.1 State Readiness

Except for the Navigation function and the Blind-Toss Bomb function, the state readiness for each function is defined by the state readiness of the equipment performing that function. Therefore, with the two exceptions noted, the state readiness figures, V_1, are as shown in Table I. The exceptions are discussed below.

(a) Navigation Equipment

Considering two possible states of each of three navigational equipments results in eight (8) different states of the overall navigational system. These are defined in Table II.

TABLE II Navigation System States			
Navigational State Designation	Doppler State	Tacan State	Direction Finder State
N_1	1	1	1
N_2	1	1	0
N_3	1	0	1
N_4	0	1	1
N_5	1	0	0
N_6	0	1	0
N_7	0	0	1
N_8	0	0	0

The Navigational state readiness figure may be determined by multiplying the probabilities that each of the three equipments will be in the prescribed state. For example,

$$
\begin{aligned}
V_{N_4} &= V_0(\text{Doppler}) \cdot V_1(\text{Tacan}) \cdot V_1(\text{DF}) \\
&= 0.429 \times 0.926 \times 0.980 \\
&= 0.389.
\end{aligned}
$$

The probability that the combined Doppler-Tacan-Direction Finder group will be in each of the eight defined states is:

State Number		
V_{N_1}	=	$(0.571)(0.926)(0.980) = 0.518$
V_{N_2}	=	$(0.571)(0.926)(0.020) = 0.011$
V_{N_3}	=	$(0.571)(0.074)(0.980) = 0.041$
V_{N_4}	=	$(0.429)(0.926)(0.980) = 0.389$
V_{N_5}	=	$(0.571)(0.074)(0.020) = 0.001$
V_{N_6}	=	$(0.429)(0.926)(0.020) = 0.008$
V_{N_7}	=	$(0.429)(0.074)(0.980) = 0.031$
V_{N_8}	=	$(0.429)(0.074)(0.020) = 0.001$

(b) Blind-Toss Bombing Equipment

As in the case of the navigational system, multiple states exist for the Blind-Toss Bombing function. The four possible states are defined in Table III.

TABLE III

Blind-Toss System States

Blind-Toss State Designation	Radar State	Toss-Bomb Computer State
B_1	1	1
B_2	1	0
B_3	0	1
B_4	0	0

The probability that the combined Bombing Radar-Toss Bomb Computer group will be in one of the four states is

State Number

$$V_{B_1} = (0.842)(0.833) = 0.701$$

$$V_{B_2} = (0.842)(0.167) = 0.141$$

$$V_{B_3} = (0.158)(0.833) = 0.132$$

$$V_{B_4} = (0.158)(0.167) = 0.026$$

7.2.2 Probability of Launch

We shall now consider the fact that launch will not always be precluded because a particular equipment is not ready. Since in many cases, some bombing capability exists even with inoperative equipments, the possibility of launching aircraft in degraded states should be considered. Estimates of the probabilities of launch for various equipment states are assumed to be as shown in Table IV.

7.3 Determination of Dependability

The next step in the evaluation procedure is to determine the state transition probabilities for each equipment during the mission. Because no in-flight repair is possible,

TABLE IV Probabilities of Launch		
Equipment	State	Probability of Launch (W)
For All Mission Type		
Radar (Terrain Avoidance)	1 0	1.0 0.0
Communications	1 0	1.0 0.0
Identification	1 0	1.0 0.2
Landing System	1 0	1.0 0.95
Navigation	N_1 N_2 N_3 N_4 N_5 N_6 N_7 N_8	1.0 1.0 1.0 0.1 0.8 0.0 0.0 0.0
For Lay-down Delivery 3/		
Sight System	1 0	1.0 0.8
For Visual Toss 4/		
Toss Bomb Computer	1 0	1.0 0.7
For Blind Toss 5/	B_1 B_2 B_3 B_4	1.0 0.5 0.0 0.0

3/ Condition of Bombing Radar and Toss Bomb Computer not significant.

4/ Condition of Bombing Radar and Sight System not significant.

5/ Condition of Sight System not significant.

no transition from State 0 to State 1 is possible ($R_{01}=0$). For the same reason, an equipment which starts in State 0 is certain to remain in that state during the flight ($R_{00}=1.0$). The remaining transition probabilities may be determined from:

(a) $R_{11} = e^{-t_m/t_f}$

where

t_m = mission time during which equipment will be in operation, and

t_f = mean-time-between-failures.

(b) $R_{10} = 1 - R_{11}$

These probabilities are shown in Table V.

7.4 Determination of Capability

The remaining parameter to be determined is the Capability for each of the functional equipment groupings. The capability figures will be discussed in the following for each of these groupings.

a. Navigation Equipment

The aircraft must be able to navigate to within 5 miles of the target by use of the Navigation equipment; from this point, target identification can be accomplished by other

TABLE V

Equipment Transition Probabilities

Equipment	Mean-time-between-failures--t_f (hours)	Mean-down-time--t_d (hours)	R_{11}	R_{10}
Radar				
Bombing	32	0.4	0.9876	0.0124
Terrain Avoidance	40	0.4	0.9900	0.0100
Toss-bomb Computer (TBC)	20	0.3	0.9851	0.0149
Sight System	200	0.2	0.999	0.001
Doppler	20	1.2	0.9418	0.0582
Direction Finder (DF)	100	1.2	0.9881	0.0119
Tacan	50	1.2	0.9763	0.0237
Instrument Landing System (ILS)	150	0.3	0.998	0.002
Communication Equipment (UHF & Amplifier)	70	0.4	0.9943	0.0057
Identification Equipment (IFF)	100	1.2	0.9881	0.0119

methods. On its return flight, it must be able to navigate to within 10 miles of its assigned base. While the navigation function can be supplied by three different equipments, the capability of each is different. The Doppler has a basic capability (C) of 0.95; the Tacan, 0.9; and the DF, 0.8. That is, the Doppler navigator can provide the required accuracy with a probability of 0.95; the Tacan, with 0.9 probability; and the DF with 0.8 probability.

However, because the Tacan and DF depend upon external signals from associated ground equipment, the probabilities that these signals will be available must also be considered. This can be most easily accomplished by modifying the equipment Capability figures. While the Doppler can be used at any time that it is operating properly, a Tacan ground station will be available only 50% of the time, and a DF ground station, only 40% of the time.

The actual capabilities for each equipment, then, are:

$$C_{Doppler} = 0.95$$
$$C_{Tacan} = 0.9(0.5) = 0.45$$
$$C_{DF} = 0.8(0.4) = 0.32.$$

Consideration must now be given to the overall Navigation Capability in each of the eight (8) states of the navigation system. It is significant that the aircraft is not committed to any particular state situation. That is, if a state transition occurs, navigation in the resultant state will be undertaken. The capabilities are shown in Table VI.

TABLE VI

Navigation Equipment Capabilities

Navigation State	Doppler State	Tacan State	DF State	State Capability
N_1	1	1	1	0.95
N_2	1	1	0	0.95
N_3	1	0	1	0.95
N_4	0	1	1	0.61
N_5	1	0	0	0.95
N_6	0	1	0	0.45
N_7	0	0	1	0.32
N_8	0	0	0	0

The capability of each state is usually the capability of the operating equipment whose individual capability is highest. In the case of State 4, however, the probabilities that the ground stations for Tacan and DF will be available must also be considered. The capability of State 4, then, is:

$$\begin{aligned} C_{N_4} &= \{\text{Probability that Tacan can be used}\}\{\text{Tacan capability}\} + \\ &\quad \{\text{Probability that only DF can be used}\}\{\text{DF capability}\} \\ &= (0.5)(0.9)+(1 - 0.5)(0.4)(0.8) \\ &= 0.45 + 0.16 \\ &= 0.61. \end{aligned}$$

b. Communication Equipment

For this particular mission, the communication function is only required so that specific target assignment can be made or changed after the aircraft has taken off. It will be assumed for this example that specific assignments are always made when the aircraft is in flight.

The communication function is supplied by the UHF Transmitter-Receiver. A necessary accessory equipment is the audio amplifier. Assuming a properly operating ground station at the base, contact between the aircraft and the base can be maintained, under average environmental conditions, for the first 1/3 and for the last 1/3 of the mission. (During the remaining 1/3 of the mission the aircraft is not within communication range of the ground station.) It is estimated that in 90% of the cases specific target assignments and changes will be made before the aircraft is out of range. In the remaining 10%, an unsuccessful mission will result.

It is estimated that environmental conditions and difficulties with the ground station equipment will prevent required communication 5% of the time when the aircraft is within range of the base. These effects will be reflected in the capability figure for the airborne system.

The capability of the Communication System, then, is expressed as the probability that target designation and/or change is received and acknowledged by the aircraft.

$$C_U = \text{(probability of successful communication, given the aircraft is within range)} \times \text{(probability of being within range when message is transmitted)}$$

In State 1 (subsystem operative),

$$C_{U_1} = (0.95)(0.90) = 0.855.$$

In State 0 (subsystem failed),

$$C_{U_0} = 0.$$

c. Identification Equipment

During the mission, the aircraft--if not able to identify itself properly--is in danger of being attacked and destroyed by friendly forces. The Identification Equipment (IFF) provides the identification function. It has a State 1 capability of 1.0. That is, in all cases, a properly operating subsystem will respond properly to a friendly challenge and the aircraft has a probability of 1.0 of surviving friendly defense.

Destruction of the aircraft is not certain, however, even when this subsystem is in State 0. This fact can be conveniently accounted for in the State 0 capability figure.

The aircraft will survive if:

(a) it is not challenged, or

(b) it is challenged, but not destroyed.

$$\begin{aligned} C_{I_0} &= \text{Probability}\{\text{no challenge}\} + \\ &\quad \text{Probability}\{\text{challenge}\} \times \\ &\quad \text{Probability}\{\text{not destroyed}\} \\ &= (0.1) + (0.9)(0.9) \\ &= 0.1 + 0.81 \\ &= 0.91. \end{aligned}$$

d. Terrain Avoidance Equipment

The Terrain Avoidance function of the radar is the only avionics equipment that contributes to the penetration ability of the aircraft. This equipment permits flying the aircraft at normal attack speeds at low altitudes, i.e., below 1000 feet. Without this equipment, such low-level approaches are not possible. It will be recalled that the anticipated loss due to enemy action was 5% for low altitude approaches and 30% for high altitude approaches. This might also be stated as 0.95 probability of survival for low altitude approach, and 0.7, for high altitude approach.

Atmospheric conditions which result in improper radar returns are anticipated 1% of the time. This condition is reflected in the Terrain Avoidance radar basic capability of 0.99.

The penetration capabilities (the probability of penetrating enemy defenses), when the effectiveness of enemy action is considered, are:

State 1 - Terrain Avoidance function operable

$$
\begin{aligned}
C_{P_1} &= \text{(Probability that radar permits low approach) x} \\
&\quad \text{(Probability of survival, given low approach) +} \\
&\quad \text{(Probability radar does \underline{not} permit low approach) x} \\
&\quad \text{(Probability of survival, given high approach)} \\
&= (0.99)(0.05)+(0.01)(0.70) \\
&= 0.9405 + 0.007 \\
&= 0.9475.
\end{aligned}
$$

State 0 - Terrain Avoidance function inoperable

$$
\begin{aligned}
C_{P_0} &= \text{Probability of survival, given high approach} \\
&= 0.70.
\end{aligned}
$$

e. <u>Target Identification and Weapon Delivery Equipment</u>

The target can be identified either visually or by means of the radar equipment. The method of identifying the target will be visual if the delivery method is "visual", and by radar, if the delivery method is "blind".

The ability to deliver a weapon within 500 feet of an identified target is dependent upon the mode of delivery and the equipment states. For this example, it is assumed that the probabilities of delivery within the prescribed 500 feet have been estimated for the indicated states and delivery modes. These probabilities are shown in Table VII.

TABLE VII

Delivery Capabilities by Mode and State

Delivery Mode	Mode State	Radar State	Toss-Bomb Computer State	Sight System State	Capabilities
Lay-down	L_1	n.a.	n.a.	1	0.90
Lay-down	L_0	n.a.	n.a.	0	0.70
Visual Toss	V_1	n.a.	1	n.a.	0.80
Visual Toss	V_0	n.a.	0	n.a.	0.60
Blind Toss	B_1	1	1	n.a.	0.75
	B_2	1	0	n.a.	0.40
	B_3	0	1	n.a.	0.0
	B_4	0	0	n.a.	0.0

n.a. = not applicable

f. <u>Instrument Landing Equipment</u>

The instrument landing system (ILS) when functioning properly has a capability of 0.99. That is, a landing without damage to the aircraft or injury to the pilot can be made 99% of the time. In weather during which this equipment is not required, however, the probability of successful landing is 1.0.

Recalling that visual landing procedures are possible 95% of the time, the probability of successful landing if the ILS is operable is:

$$
\begin{aligned}
C_{T_1} &= \text{(Probability of visual landing)} \times \\
&\quad \text{(Probability of successful landing under visual conditions)} + \\
&\quad \text{(Probability of ILS landing)} \times \\
&\quad \text{(Probability of successful landing under ILS conditions)} \\
&= (0.95)(1.0)+(0.05)(0.99) \\
&= 0.95 + 0.0495 \\
&= 0.9995.
\end{aligned}
$$

If the ILS is not operable, no capability under ILS conditions exist, and the overall landing capability is

$$
\begin{aligned}
C_{T_0} &= (0.95)(1.0)+(0.05)(0) \\
&= 0.95.
\end{aligned}
$$

8.0 Model Exercise

8.1 Effectiveness of Individual Functions

With all of the basic parameters now available, the individual Effectiveness figures for each mission function can now be determined. The probability of performing each required mission function will first be determined. These probabilities, since they are independent, will then be combined to establish the mission effectiveness.

a. <u>Communication</u>

$$E_C = \overline{A}_C'\left[D_C\right]\overline{C}_C$$

$$\overline{A}_C' = \left[V_1 V_0\right]\begin{bmatrix} W_1 & 0 \\ 0 & W_0 \end{bmatrix}$$

$$= \left[0.972 \quad 0.028\right]\begin{bmatrix} 1.0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\left[D_C\right] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} 0.9943 & 0.0057 \\ 0 & 1.0 \end{bmatrix}$$

$$\overline{C}_C = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} = \begin{bmatrix} 0.855 \\ 0 \end{bmatrix}$$

$$E_C = 0.8265$$

b. Navigation

$$E_N = \bar{A}'_N [D_N] \bar{C}_N$$

$$\bar{A}'_N = [V_1 V_2 V_3 V_4 V_5 V_6 V_7 V_8] \begin{bmatrix} W_1 & & & & & & & \\ & W_2 & & & & & & \\ & & W_3 & & & & & \\ & & & W_4 & & & & \\ & & & & W_5 & & & \\ & & & & & W_6 & & \\ & & & & & & W_7 & \\ & & & & & & & W_8 \end{bmatrix}$$

$$= [.518\ .011\ .041\ .389\ .001\ .008\ .031\ .001] \begin{bmatrix} 1.0 & & & & & & & \\ & 1.0 & & & & & & \\ & & 1.0 & & & & & \\ & & & 0.1 & & & & \\ & & & & 0.8 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{bmatrix}$$

$$[D_N] = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{18} \\ D_{21} & D_{22} & \cdots & D_{28} \\ \vdots & \vdots & & \vdots \\ D_{81} & D_{82} & \cdots & D_{88} \end{bmatrix}$$

In this matrix, the following elements, for example, are computed from:

$$D_{11} = D_{D_{11}}\ D_{T_{11}}\ D_{DF_{11}}$$

$$D_{12} = D_{D_{11}}\ D_{T_{11}}\ D_{DF_{10}}$$

$$D_{35} = D_{D_{11}}\ D_{DF_{10}}$$

$D_{36} = 0$ (Transition from State 3 to State 6 is not possible.)

where subscripts D, T, and DF represent respectively, Doppler, Tacan, and Direction Finder.

$$\left[D_N\right] = \begin{bmatrix} .9085 & .0109 & .0220 & .0561 & .0003 & .0007 & .0014 & .0000 \\ 0 & .9195 & 0 & 0 & .0223 & .0568 & 0 & .0014 \\ 0 & 0 & .9306 & 0 & .0112 & 0 & .0575 & .0007 \\ 0 & 0 & 0 & .9647 & 0 & .0116 & .0234 & .0003 \\ 0 & 0 & 0 & 0 & .9418 & 0 & 0 & .0582 \\ 0 & 0 & 0 & 0 & 0 & .9763 & 0 & .0237 \\ 0 & 0 & 0 & 0 & 0 & 0 & .9881 & .0019 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

$$\bar{C}_N = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0.95 \\ 0.95 \\ 0.61 \\ 0.95 \\ 0.45 \\ 0.32 \\ 0 \end{bmatrix}$$

$E_N = 0.5537.$

c. Identification

$$E_I = \overline{A}_I'[D_I]\overline{C}_I$$

$$\overline{A}_I' = [V_1 V_0]\begin{bmatrix} W_1 & 0 \\ 0 & W_0 \end{bmatrix} = [.971 \quad .029]\begin{bmatrix} 1.0 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$[D_I] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} .9881 & .0119 \\ 0 & 1.0 \end{bmatrix}$$

$$\overline{C}_I = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ .91 \end{bmatrix}$$

$$E_I = 0.9751$$

d. Penetration

$$E_P = \overline{A}_P'[D_P]\overline{C}_P$$

$$\overline{A}_P' = [V_1 V_0]\begin{bmatrix} W_1 & 0 \\ 0 & W_0 \end{bmatrix} = [.833 \quad .167]\begin{bmatrix} 1.0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[D_P] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} .990 & .010 \\ 0 & 1.0 \end{bmatrix}$$

$$\overline{C}_P = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} = \begin{bmatrix} .9475 \\ .70 \end{bmatrix}$$

$$E_P = 0.7875$$

e. Landing

$$E_T = \bar{A}_T [D_T] \bar{C}_T$$

$$\bar{A}'_T = [V_1 V_0] \begin{bmatrix} W_1 & 0 \\ 0 & W_0 \end{bmatrix} = [.980 \quad .020] \begin{bmatrix} 1.0 & 0 \\ 0 & .95 \end{bmatrix}$$

$$[D_T] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} .998 & .002 \\ 0 & 1.0 \end{bmatrix}$$

$$\bar{C}_T = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} = \begin{bmatrix} .9995 \\ .95 \end{bmatrix}$$

$$E_T = 0.9975$$

f. Weapon Delivery

Lay-down Mode

$$E_L = \bar{A}'_L [D_L] \bar{C}_L$$

$$\bar{A}'_L = [V_1 V_0] \begin{bmatrix} W_1 & 0 \\ 0 & W_0 \end{bmatrix} = [.990 \quad .010] \begin{bmatrix} 1.0 & 0 \\ 0 & 0.8 \end{bmatrix}$$

$$[D_L] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} .999 & .001 \\ 0 & 1.0 \end{bmatrix}$$

$$\bar{C}_L = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} = \begin{bmatrix} .90 \\ .70 \end{bmatrix}$$

$$E_L = 0.8964$$

Visual Toss Mode

$$E_V = \bar{A}_V' \left[D_V\right] \bar{C}_V$$

$$\bar{A}_V' = \left[V_1 V_0\right] \begin{bmatrix} W_1 & 0 \\ 0 & W_0 \end{bmatrix} = \begin{bmatrix} .833 & .167 \end{bmatrix} \begin{bmatrix} 1.0 & 0 \\ 0 & 0.7 \end{bmatrix}$$

$$\left[D_V\right] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} .9851 & .0149 \\ 0 & 1.0 \end{bmatrix}$$

$$\bar{C}_V = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} = \begin{bmatrix} .80 \\ .60 \end{bmatrix}$$

$$E_V = 0.7342$$

Blind Toss Mode 6/

$$E_B = \bar{A}'_B [D_B] \bar{C}_B$$

$$\bar{A}'_B = \begin{bmatrix} V_1 V_0 \end{bmatrix} \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} = \begin{bmatrix} (.842)(.833) & (.842)(.167) \end{bmatrix} \begin{bmatrix} 1.0 & 0 \\ 0 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} .701 & .141 \end{bmatrix} \begin{bmatrix} 1.0 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$[D_B] = \begin{bmatrix} D_{11} & D_{10} \\ D_{01} & D_{00} \end{bmatrix} = \begin{bmatrix} (.9876)(.9851) & (.9876)(.0149) \\ 0 & (.9876) \end{bmatrix}$$

$$= \begin{bmatrix} .9729 & .0147 \\ 0 & .9876 \end{bmatrix}$$

$$\bar{C}_B = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} .75 \\ .40 \end{bmatrix}$$

$$E_B = 0.5439$$

6/ Because the Capability in States 3 and 4 is zero, these states need not be treated explicitly in the computation.

8.2 Effectiveness for Individual Mission Types

The individual functional effectiveness figures may now be combined to evaluate the system effectiveness for each mission type.

Lay-down Delivery Mission (E_1)

$$E_1 = (E_C\ E_I\ E_N\ E_T\ E_P)\ E_L$$

$$= \left[(.8265)(.9751)(.5537)(.9975)(.7875)\right]\ .8964$$

$$= (.3500)(.8964)$$

$$= 0.3142$$

Visual Toss Delivery Mission (E_2)

$$E_2 = (E_C\ E_I\ E_N\ E_T\ E_P)\ E_V$$

$$= (.3500)(.7342)$$

$$= 0.2574$$

Blind Toss Delivery Mission (E_3)

$$E_3 = (E_C\ E_I\ E_N\ E_T\ E_P)\ E_B$$

$$= (.3500)(.5439)$$

$$= 0.1907$$

8.3 Overall System Effectiveness

The single, overall system effectiveness figure is now obtained from

$$E = E_1P_1 + E_2P_2 + E_3P_3$$

where P_1, P_2, and P_3 are the probabilities that each mission type will be flown.

P_1(probability of Lay-down Delivery) = (Probability of daytime mission) x
(Probability of VFR conditions) x
(Probability that Lay-down Delivery is preferred)
= (.58)(.8)(.8)
= 0.3712

P_2(probability of Visual Toss Delivery) = (Probability of daytime mission) x
(Probability of VFR conditions) x
(Probability that Toss Bombing is preferred)
= (.58)(.8)(.2)
= 0.0928

P_3(probability of Blind Toss Delivery) = (Probability of night mission) +
(Probability of IFR conditions) -
(Probability of night mission and IFR conditions)
= .42 + .2 - (.42)(.2)
= 0.536

$$E = (.3142)(.3712)+(.2574)(.0928)+(.1907)(.536)$$
$$= 0.2427.$$

8.4 Application of Model Results

It was stated in the introduction that this evaluation was being performed during the Program Definition phase, and that Force Structure, i.e., the number of systems required to accomplish a specific mission, was of prime concern.

It can be shown that if one system has a probability, E, of accomplishing a mission, the the probability that at least one of N systems will accomplish the mission (S) is:

$$S = 1 - (1-E)^N$$

In order to determine the number of systems required to attain a fixed value of S for a particular value of E the equation may be written:

$$N = \frac{\ln(1-S)}{\ln(1-E)}$$

Figure 3 shows this relationship for S values of 0.95 and 0.90. That is, any point on the 95% curve shows the number of systems of effectiveness E that would be required to provide 0.95 assurance of successful mission completion.

Considering the upper curve, note that for the System Effectiveness of 0.24 computed in the previous section, eleven (11) systems would be required to provide a 0.95

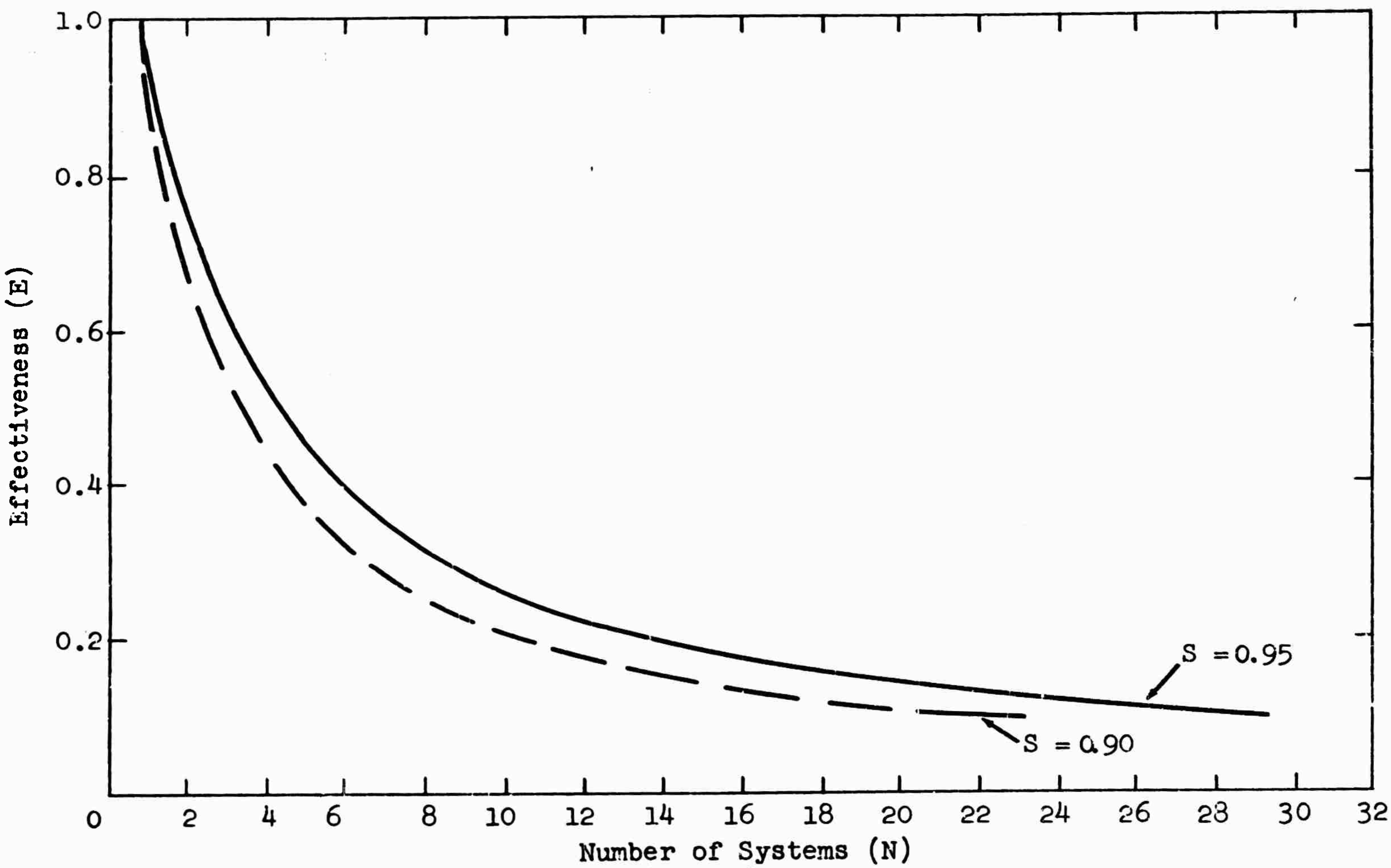


FIGURE 3
NUMBER OF SYSTEMS (N) REQUIRED TO PROVIDE ASSURANCE (S) OF MISSION ACCOMPLISHMENT AS A FUNCTION OF SYSTEM EFFECTIVENESS (E)

assurance of a successful mission. If the Effectiveness could be raised to 0.4, six (6) systems could provide the same assurance. A question that might be asked, then, is "What is the optimum method for attaining the required assurance of mission success?" Should the expected Effectiveness be accepted and the required quantity of aircraft be obtained; or should efforts be made to increase the Effectiveness so that fewer aircraft would be required?

No effort will be made here to treat optimization procedures in general. The reader is referred to the report of Task Group IV for this purpose. However, an elementary procedure that might be employed in the initial trade-off analyses is described in the following.

While the many inputs to the model represent the effects of a wide range of influencing factors, assume that the analysis being performed during this particular phase of Program Definition is concerned only with those factors over which the hardware designer has some degree of control. These are essentially the capability, the reliability, and the maintainability of each equipment. If each of these factors is varied over some pre-determined range and the resultant Effectiveness figures computed, an indication of the areas of high potential pay-off will be available. This procedure was followed in this example for the reliability and maintainability characteristics. The calculations described in the preceding sections were repeated for six values of mean time between failures and five values of mean down time for each equipment. Utilization of even modest computing equipment makes this procedure completely feasible. Figures 4 and 5 show the results of these analyses.

An initial examination of these figures shows that the influence on Effectiveness of a given percentage change in either t_f or t_d will be greatest for the Doppler, followed by the Terrain Avoidance Radar, the Bombing Radar, the Toss Bomb Computer, etc.[1/]

These results would initiate a re-examination of the reliability and maintainability predictions for the equipments in the order listed. Some criteria against which

---

1/ In this relatively simple example, these results might seem to point out the obvious, e.g., that the Doppler could have been recognized from Table I as the major problem area. Note, however, that the mean-time-between-failures (t_f) for the Computer is equal to that for the Doppler. Had corrective actions been based only upon the t_f figures and equal efforts accorded these two equipments, the improvement in Effectiveness per unit of effort would have been considerably less than had the major effort been applied to the Doppler.

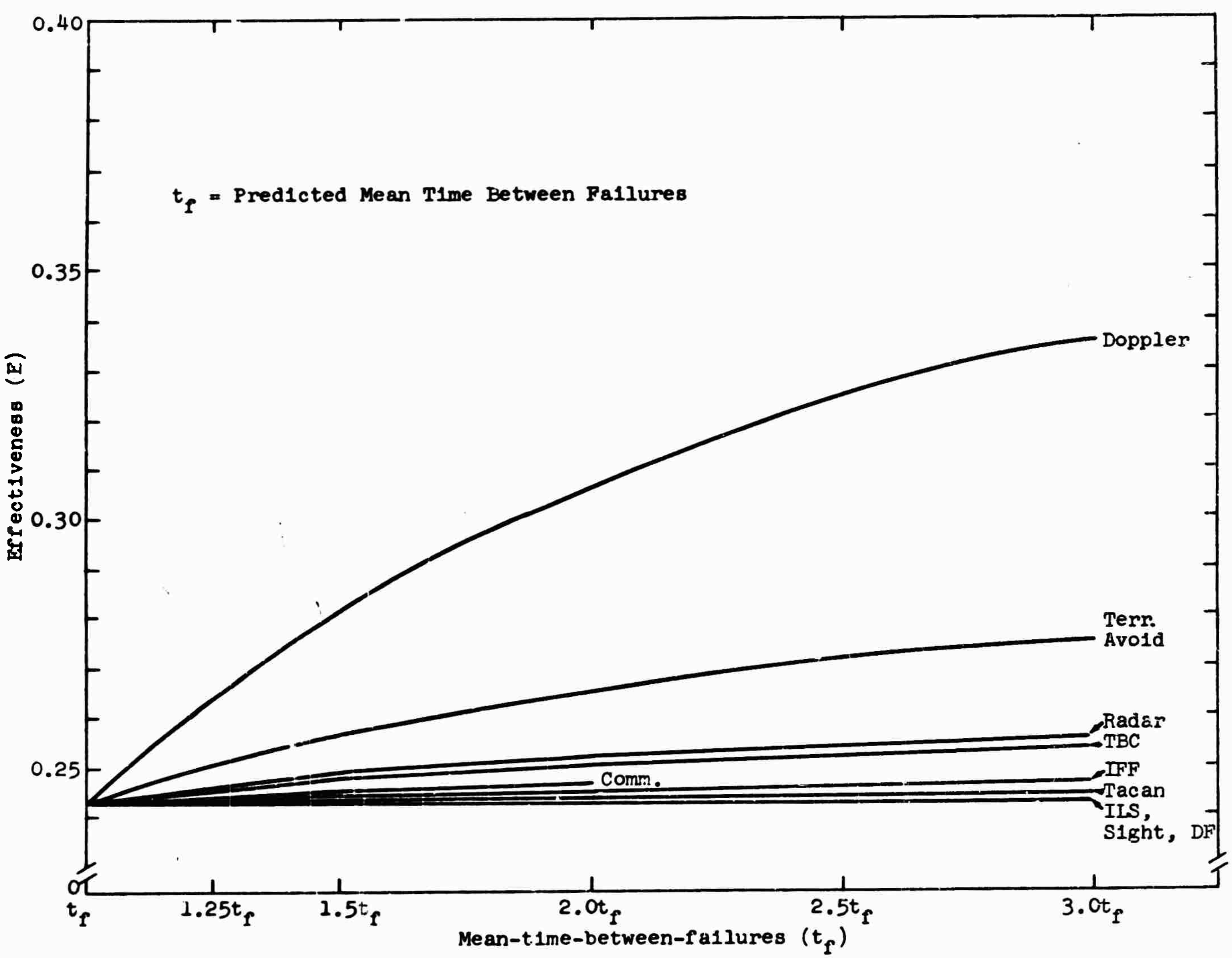


FIGURE 4

INFLUENCE ON SYSTEM EFFECTIVENESS OF VARIATIONS
IN MEAN-TIMES-BETWEEN-FAILURE FOR EACH EQUIPMENT

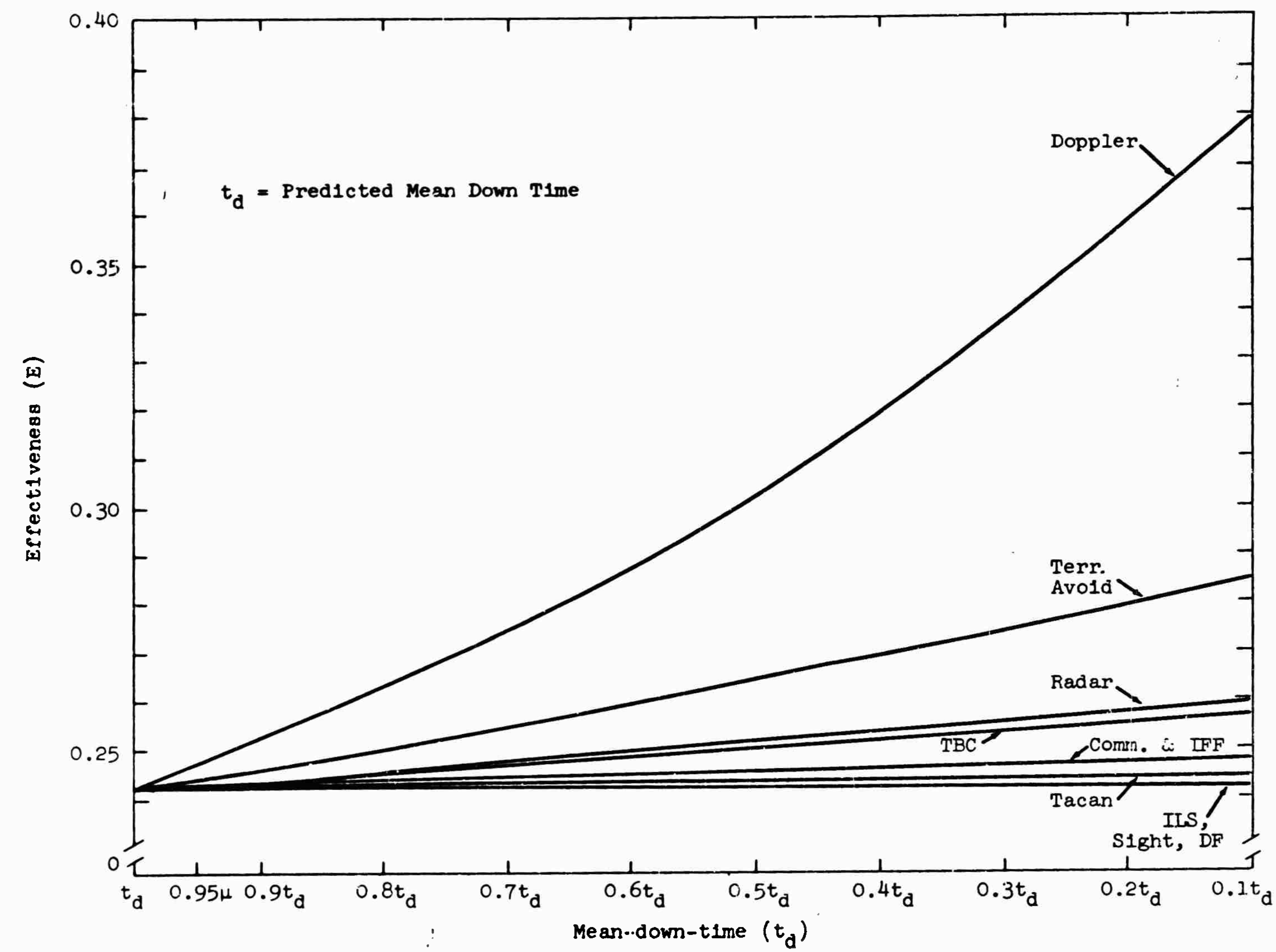


FIGURE 5

INFLUENCE ON SYSTEM EFFECTIVENESS OF VARIATIONS IN MEAN-DOWN-TIMES FOR EACH EQUIPMENT possible changes in the equipments might be weighed are now available. For example, a 50% reduction in mean-down-time for the Doppler would be equivalent to reducing the number of aircraft required for a successful mission from 11 to 9, or a force reduction of about 18%. An approximation of the projected savings to be realized by such a reduction can then be weighed against the costs to be incurred in decreasing the down-time by 50%.

EXAMPLE B

INTERCONTINENTAL BALLISTIC MISSILE SQUADRON

CONTENTS

CONTENTS (Continued)

CONTENTS (Continued)

CONTENTS (Continued)

CONTENTS (Continued)

ILLUSTRATIONS

ILLUSTRATIONS (Continued)

TABLES

I. INTRODUCTION AND SUMMARY [1/]

It is the specific object of this document to provide an example of the analysis of an ICBM fleet which will illustrate the formal mathematical structure adopted by Task Group II of the WSEIAC. Symbolically, this structure is given by

$$E = \overline{A}' \, [D] \, \overline{C} \tag{1}$$

where

E is system effectiveness

$\overline{A}$ is the readiness vector and $\overline{A}'$ is its transpose.

$[D]$ is the dependability matrix.

$\overline{C}$ is the design capability (performance) vector.

The point of view which is adopted here is that the evaluation and/or prediction of system effectiveness is the result of the interaction of

- Weapon system criteria
- Mission description
- Weapon system description

Because the ICBM fleets have reached the acquisition and operational phases of system life, this memorandum does not reflect the application of models in the conceptual and program definition phases of system development. This is perhaps unfortunate since hindsight frequently has the quality of 20-20 vision. Nevertheless, it is felt that this document will prove most useful if it is concentrated on methods and techniques for current and future weapon system evaluation and improvements. Therefore, we shall limit the discussion on the role of models in the various phases of system life to the following brief remarks.

1/ The material presented in this example is an abstraction from "A Compendium of Atlas-Sponsored Developments in Reliability and Availability." Vol. I, AD 420882; Vol. II, AD 420883; and Vol. III, AD 420884.

A system evolves through four relatively distinct phases, namely;

- conceptual phase
- program definition phase
- acquisition phase
- operational phase

In the conceptual phase, feasibility studies are conducted to test the ability of the current state of the art to support the proposed system development.

Out of this phase a set of specific operational requirements emerges.

The program definition phase continues the feasibility studies, pinpoints potential problem areas, and results in a firm system description to the major subsystem level. This phase terminates with a set of firm system specifications which initiates the acquisition phase.

In the acquisition phase system hardware is designed, developed, and tested.

System production initiates the operational phase.

The precise manner in which a model is implemented in any of these phases depends upon the point in time at which the evaluation is made. Consider, for example, the problem of designing a launch vehicle for an information retrieval spacecraft in 1965. Specifically, let the problem be to determine the feasibility of achieving a certain reliability of countdown and a certain reaction time consistent with a narrow launch window.

A countdown may be regarded as an event during which the vehicle and its launch complex act as a single unit. There are two properties of a countdown of particular interest here.

- the probability of completing a countdown.
- the duration of a countdown in excess of scheduled time.

We may express these two properties as

$$P_{cd}[t] = P_{cd}[\infty] \; \widetilde{P}_{cd}[t/cd]$$

where

$P_{cd}[\infty]$ = probability of completing a countdown without regard for its duration (no abort).

$\widetilde{P}_{cd}[t/cd]$ = probability that a countdown will exceed the scheduled countdown duration by t or less; given that the countdown is completed.

During the feasibility studies of the conceptual phase, gross generic data would be utilized from all available sources. For example, AMR data on Atlas D development launches might be used without much regard for the finer differences in hardware or procedures between the Atlas D and the proposed system.

Once the feasibility has been established and the program definition phase is well along, a second look at the system is taken. The system is now fairly well defined to the subsystem level, but there is still no actual hardware from which to obtain data, so the Atlas D data would again be used; except that now that data would be examined at the subsystem level and all non-relevant data rejected.

During the acquisition phase, the scope of modeling would be extended to the piece part level using generic failure rate data. The system data on the Atlas D launches would no longer be useful since the structure of the model is now far more detailed than in the preceding phase.

Toward the end of the acquisition phase and in the early part of the operational phase, a considerable body of subsystem test data tends to accumulate. During this time period the model structure will tend to simplify again in a direction which can accept subsystem data rather than piece part data. Finally, after a sufficiently large number of operational units are in existence, the model structure tends to simplify to the gross system level, although the detailed subsystem and piece part models will still play a part in assessing proposed system alterations at those levels of detail. Thus, broadly speaking, there are three levels of model structure

- gross system model
- subsystem model
- piece part model.

In the present document we shall illustrate these three levels in some detail for an ICBM, with the understanding that their degree of applicability depends upon which phase of system life is under consideration.

The system which has been chosen for illustration in this example is a squadron of ICBM's consisting of nine launch sites with one missile per site. The squadron is treated as an entity without reference to its interface with other strategic weapons or possible enemy counter measures. The lowest level of consideration is a subsystem, as opposed to a lesser aggregate of equipment, except in the case of the re-entry vehicle for which a piece part reliability model is developed. Redundancy is illustrated in this latter model.

The maintenance policy is a combination of scheduled maintenance, continuous monitoring, and a fortuitous implementation of TCTO's performed at the subsystem level. The tests are not assumed to be either accurate or complete. Repair, if it is required, is accomplished by remove and replace at the subsystem level. It is assumed that one maintenance crew tends all nine missile sites so that queuing can occur, but transportation lag time is not accounted for. Spares provisioning is assumed to be adequate and no administrative down time occurs.

Several figures of merit are illustrated commencing with the highest level figure defined as "the expected number of targets destroyed per squadron when an execution directive is received at a random point in time." Among lesser figures of merit considered are:

- relative subsystem rank by reliability indices and mode of operation.
- relative subsystem ranking by availability.

Both true and apparent availability are considered at a random point in time and as a function of warning time. Countdown reliability is considered in terms of reaction time and success ratio.

Repair of aborts during a tactical situation is treated accounting for a limited spares provisioning. System capability is defined in terms of guidance accuracy, warhead lethality, and a given targeting policy.

An S.O.R. for the squadron is postulated. Requirements are placed on readiness, launch reliability and reaction time, flight reliability, and unit kill probability. Analytical models reflecting the figures of merit defined above are developed for the squadron by site and subsystem. It is assumed that during the system acquisition and early operational phases, a body of data has been obtained as a result of system and subsystem tests. Equations are developed for processing this data into numerical estimates of the model parameters. The model is exercised using these estimates to produce estimates of availability, dependability, and capability and the product of these factors.

The model outputs are compared to the S.O.R. This comparison indicates that the minimum acceptable values for system reliability in countdown and flight are met, although the reliability of the re-entry vehicle is clearly susceptible of improvement. The <u>true</u> availability of the system is woefully lower than the acceptable minimum, although the <u>apparent</u> availability is relatively high. The per unit kill probability is also in drastic need of improvement.

Parameter variation studies are initiated on the availability and capability factors to assess the potential for system improvement. It is shown that

- improved monitoring and increased reliability of the power generation and distribution subsystem in conjunction

with

- a drastic shortening of the times between scheduled checkouts on several subsystems

and

- an increase in guidance accuracy by a factor of two

will be required to achieve minimum acceptable system performance.

The questions of costs, schedules, confidence factors, relative strategic value of the system, and technical feasibility of accomplishing the required system alterations are not considered.

A more serious shortcoming of this document is the lack of an illustrative decision algorithm (for aiding management) that accounts for cost, schedules, expected product life, and the host of other factors which (conceivably) influence decisions in a real situation. The current example limits itself to a trade off study based strictly on the technical factors which enter into decisions. Thus, there is a certain flavor of real life missing from this example.

Although the example developed here illustrates the formal mathematical framework referred to above, it was found that this framework can be too restrictive under certain circumstances. The difficulty is implicit in the structure of the product $\overline{A}' \overline{D} \, \overline{C}$ which assumes that readiness, dependability, and design capability may be expressed as mutually exclusive, independent factors. This assumption can break down for availability and dependability in the case of ICBM's when several launch attempts are permitted (with repair from the preceding aborts).

It is shown in Appendix II that this situation cannot be formulated within the present formal framework adopted by Task Group II.

II. EFFECTIVENESS EVALUATION BY TASK ANALYSIS DESIGNATOR NUMBERS

1.0 MISSION DEFINITION

1.1 Functional Definition of Mission

Any missile of an ICBM fleet should be ready to accept a launch directive at a random point in time, or at an arbitrary time after an initial warning has been received at a random point in time. It should then launch successfully within a prescribed reaction time,[2/] fly a ballistic trajectory, penetrate, arm, fuse, impact within the prescribed target area, detonate and yield as planned with a prescribed probability of target kill.

1.2 System Requirements

The basic numerical criteria used in guiding the design of ICBM fleets is given in a document called "Specific Operational Requirements." For example, the Atlas and Titan I fleet requirements are given in SOR-104.

For our example analysis we shall assume that the SOR requires:

	Minimum accept. value	Objective value
Countdown reliability	0.8 *	0.95*
Flight reliability	0.7	0.90
Fleet in commission rate	0.5	0.90
Per unit probability of kill	0.8	0.9

* Assumed reaction time of 2 1/2 hours.

We shall also assume that the SOR specifies one or more objectives of the following nature:

2/ Multiple launch attempts with repair of aborts is permissible.

- Crisis criterion:

  Maximize the number of missiles available for launch.
- Cold war criteria:
  - Fixed budget criterion:

    Maximize target coverage within a fixed allocation of resources.
  - Per unit cost criterion:

    Maximize target coverage per dollar consumed.
  - System efficiency criterion:

    Minimize the dollars required to obtain a specified target kill probability

It should be noted that these criteria define acceptance and objective levels and a course of action. They do not necessarily specify Figures of Merit.

However, the SOR probably should specify one or more Figure of Merit to be used in assessing the developed system. We shall, therefore, assume that our hypothetical SOR requirements are based upon the expected number of objectives destroyed per squadron when an execution directive is given at a random point in time and three missiles are targeted per objective.

2.0 SYSTEM DESCRIPTION

2.1 General Configuration

The system is a squadron. A squadron consists of nine launch sites, each containing one missile.

Each missile contains the following launch critical subsystems

Subsystem	Subsystem Designator
Re-entry vehicle	A
Guidance	B
Autopilot	C
Propulsion	D
Structure	E

. Each launch facility contains the following launch critical subsystems.

Subsystem	Subsystem Designator
Overhead door	F
Air conditioning	G
Power generation and distribution	H

2.2 Block Diagram

Block diagrams of a system are useful in showing the organization of a system. In particular, they are a useful reference in establishing the interfaces between equipments and settling the question of redundancy. The functional flow diagram of Figure 1 illustrates the degree of complexity and amount of detail normally available from such diagrams.

2.3 Engineering Drawings

The engineering drawings define the details of the hardware of the system. From these drawings information is extracted to support the integrated task index, unit manning document, data handbook, provisioning requirements document, equipment running time line analysis, and the reliability functional block (RFB) diagram.

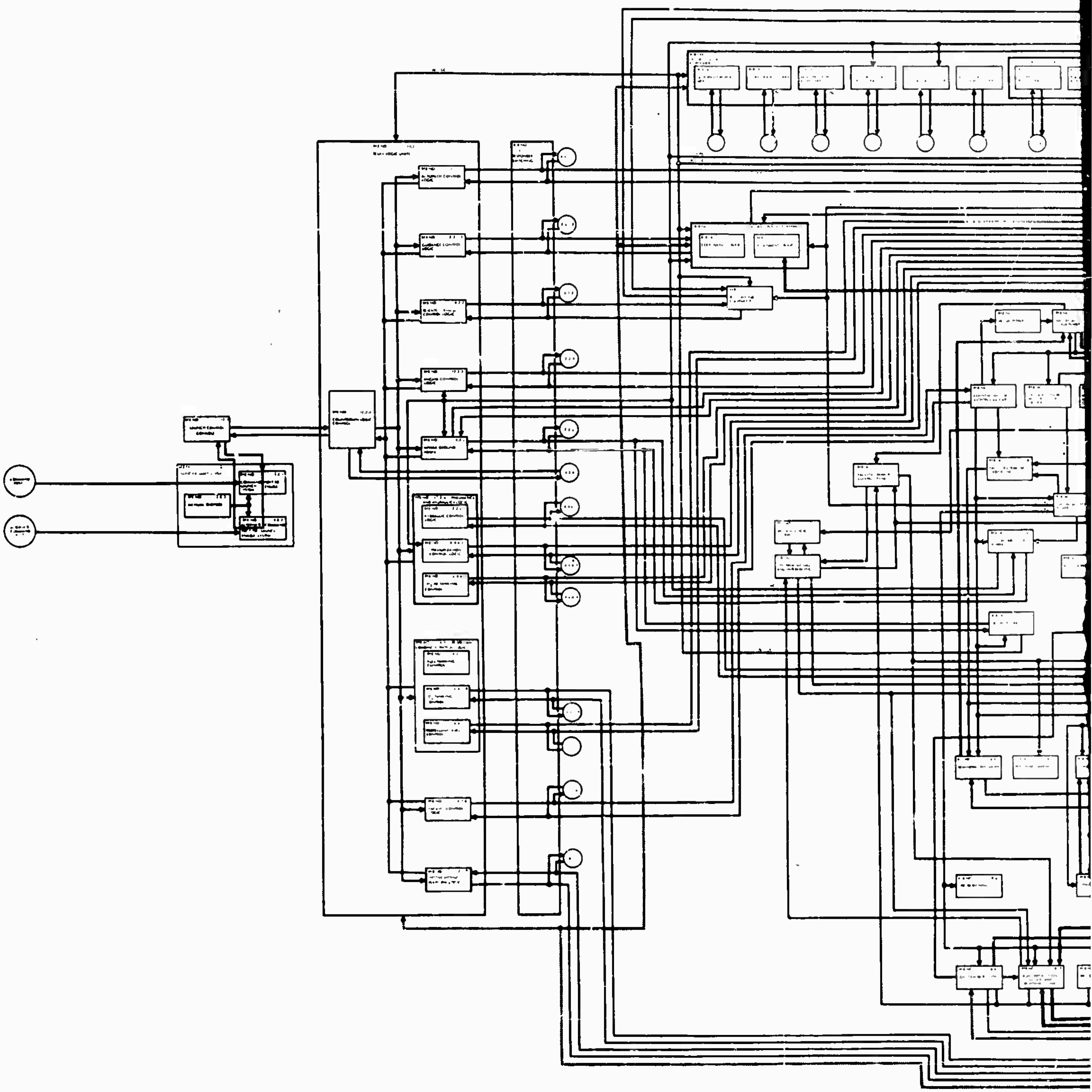


FIGURE 1. FUNCT
OPERA

1

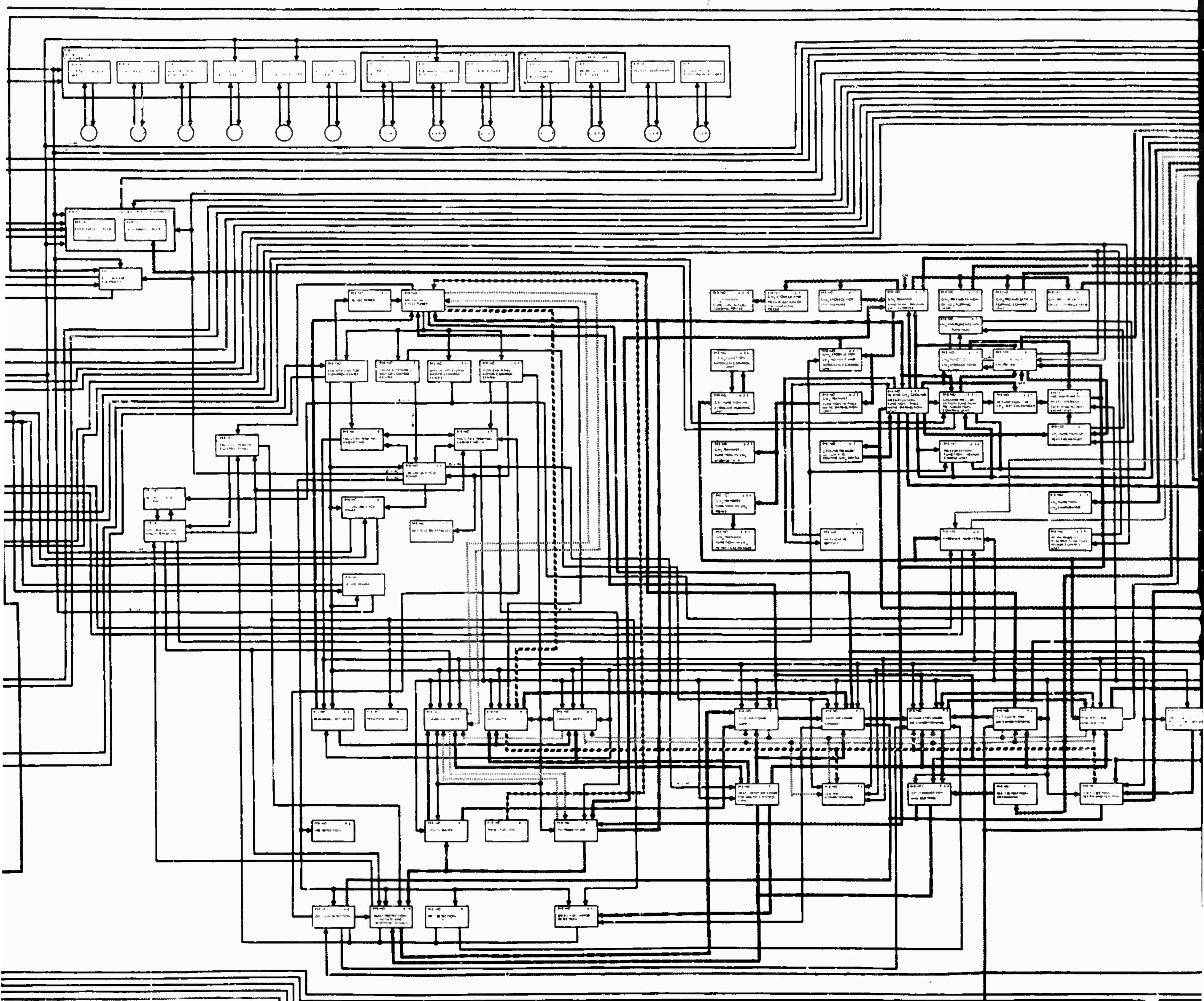
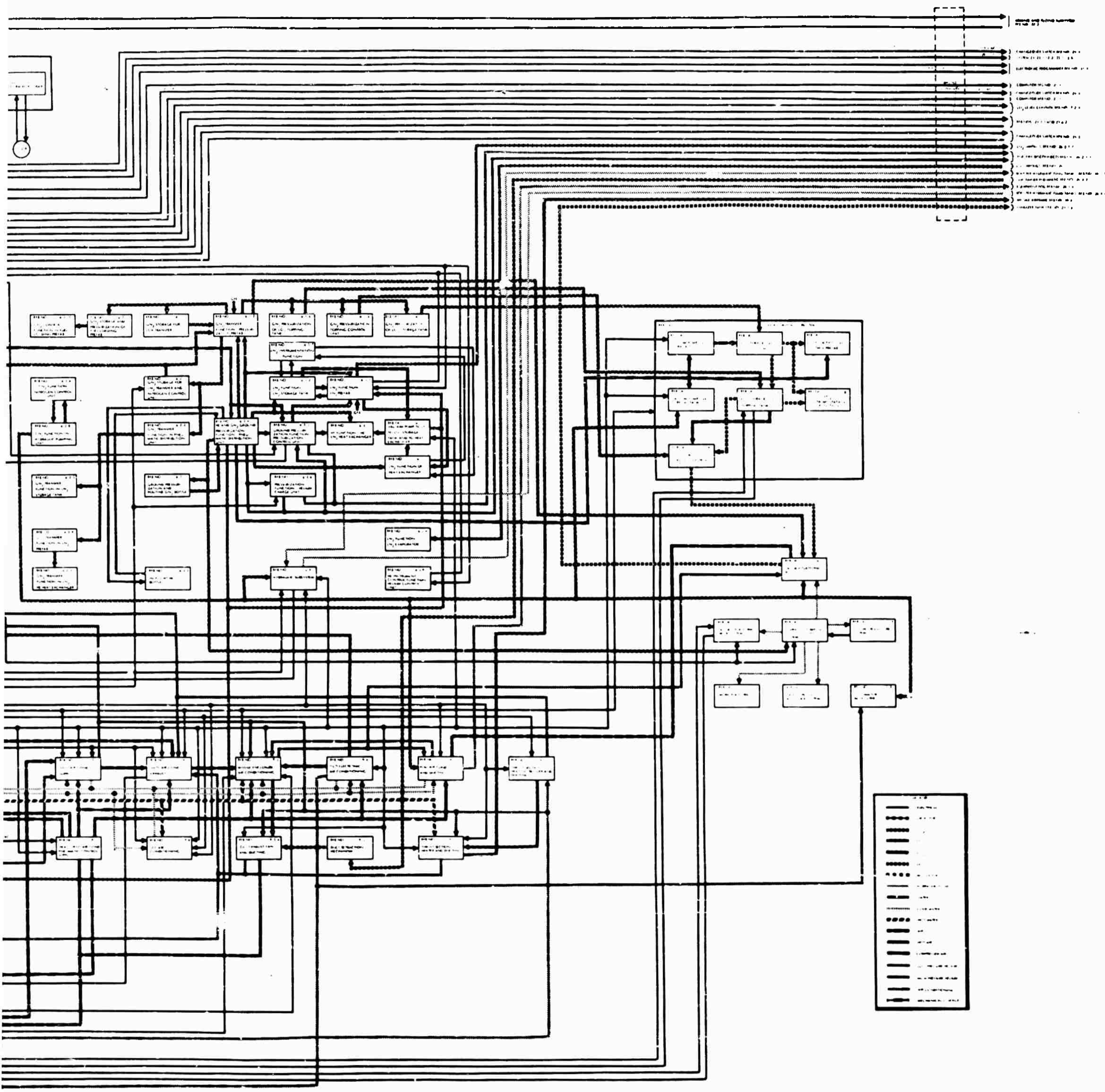


FIGURE 1. FUNCTIONAL FLOW DIAGRAM OF ATLAS WEAPON SYSTEM OPERATIONAL GROUND EQUIPMENT

2



M OF ATLAS WEAPON SYSTEM
IPMENT

3

2.4 System Function Analysis

A system function analysis (F/A) is a task oriented analysis of the time and sequence of the events necessary to support and utilize a weapon system. It provides the base line from which the equipment running time line analysis, integrated task index, and unit manning document are prepared. A function analysis is documented as a set of configuration control engineering drawings.

The method is illustrated by the two Atlas F series drawings of Figures 2a and 2b.

2.5 Physical Factors Summary Documents

These documents are usually a series of design reports of all system factors.

2.6 Equipment Running Time Line Analysis

A running time line analysis of each equipment group is performed for each standard tactical operating condition (STOC) implied by the mission description. For example, consider Figure 3 which illustrates the time line analysis of a hypothetical re-entry vehicle during countdown.

2.7 Integrated Task Index

The integrated task index of the system uniquely identifies all of the tasks which must be accomplished to maintain and operate the weapon system. It lists the required skill level (AFSC), number of people required, sequence and duration of the tasks. This is illustrated in Figure 4 for an Atlas E Series periodic inspection.

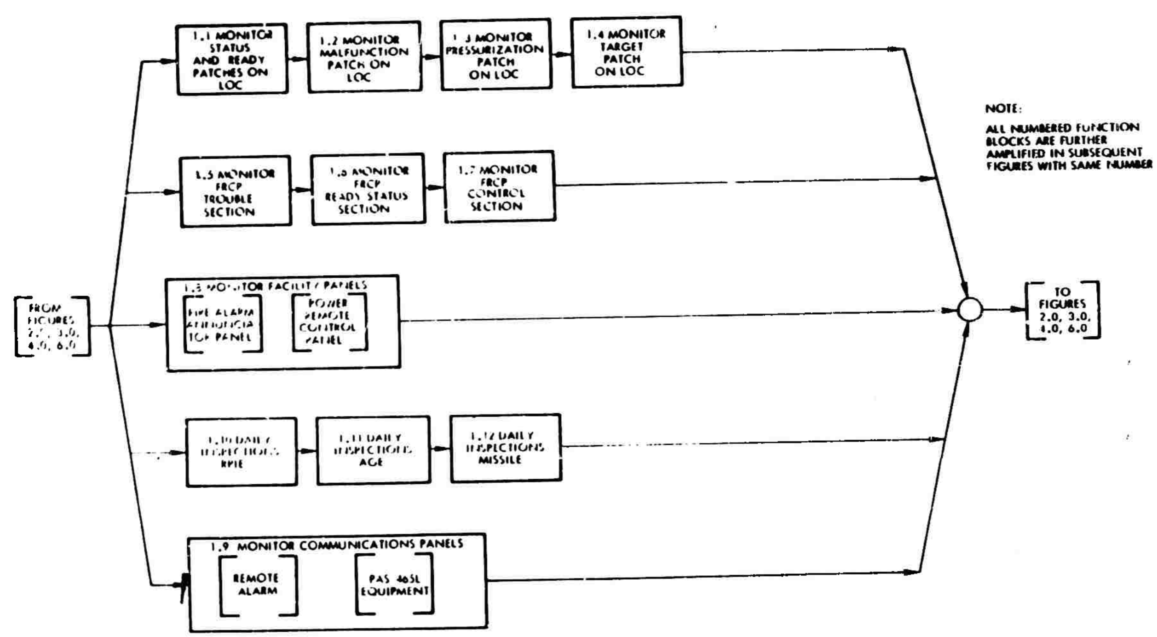


FIGURE 2a. MONITOR SITE EWO READINESS (FIGURE 1.0-1)

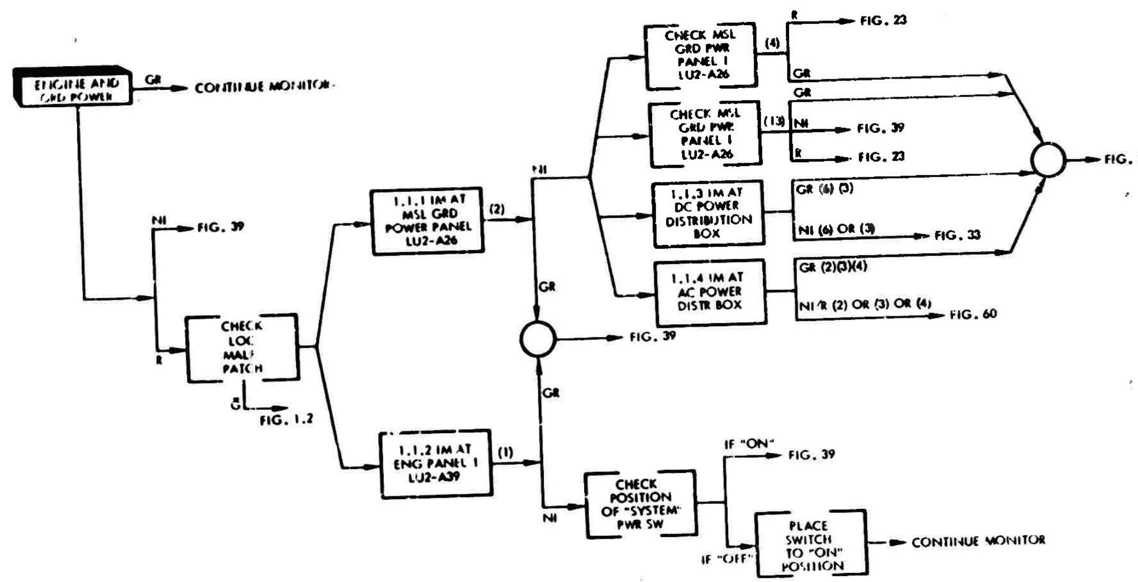


FIGURE 2b. MONITOR STATUS AND READY PATCHES DURING STANDBY (FIGURE 1.1-1)

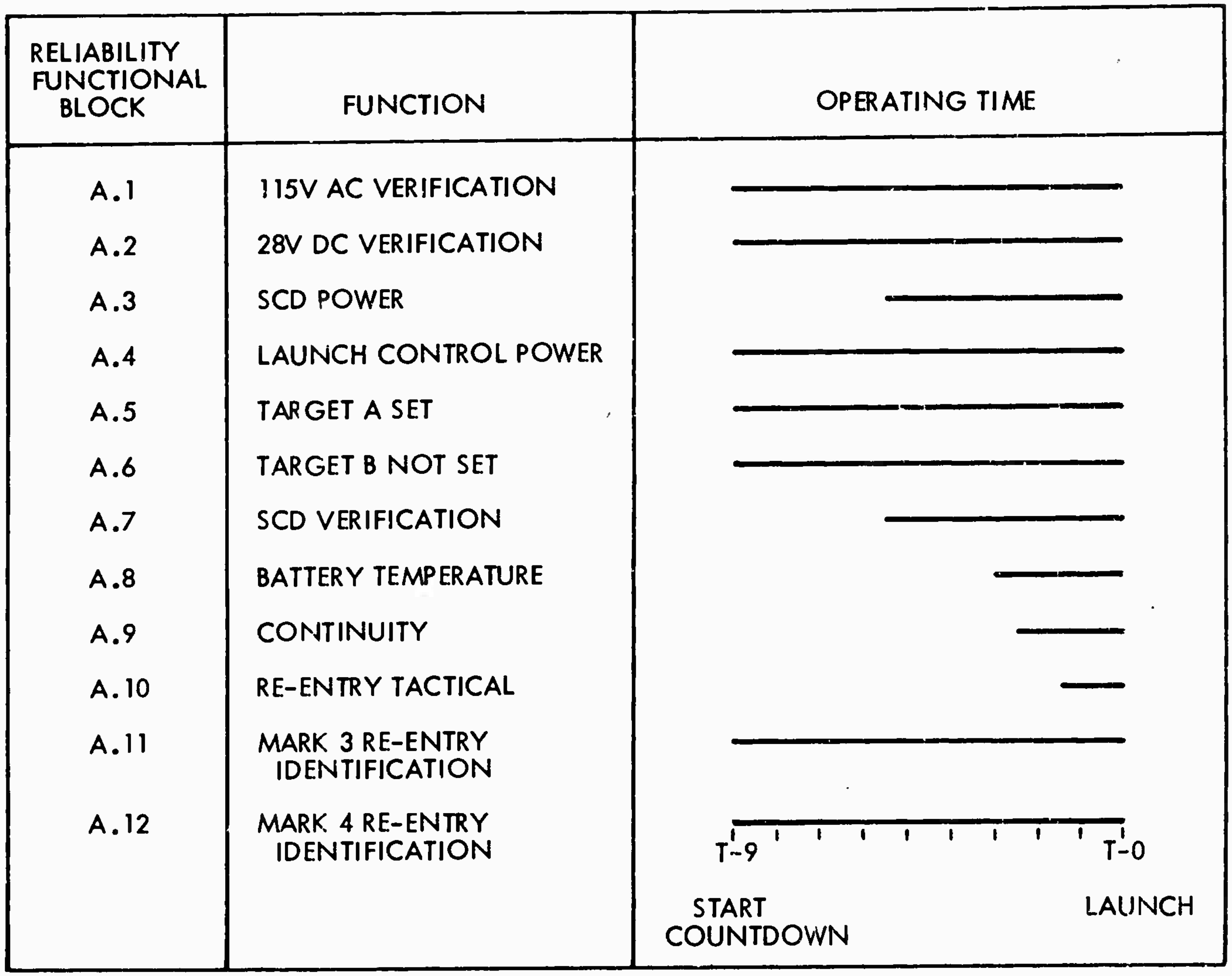


FIGURE 3. TIME LINE FOR REENTRY VEHICLE IN S.T.O.C. (COUNTDOWN)

BLOCK NO.	TITLE	AFSC	NO.	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH	11TH	12TH	13TH	14TH	15TH	16TH	17TH	18TH	19TH
				30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
120.5	PERFORM INITIAL INSPECTION AND DOCUMENT INSPECTION	433X0 683X0	1 1			120																
121.1	POSITION MISSILE FOR MAINTENANCE	433X0 433X1 603X0	1 2 3			120																
121.1A	PREPARE FOR INSPECTION	433X0 433X1 603X0	2 2 3							235												
121.1A 121.2	(IF NEW MISSILE) PREPARE FOR INSPECTION AND INSTALL LOOSE EQUIPMENT	433X0 433X1 421X3 603X0	2 2 1 3							275												
121.2	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	433X0	2								90											
122.1	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	314X0F	1							10												
123.1	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	423X0								10												
124.1	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	421X2									45											
125.1	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	421X2									15											
127.46	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	314X0P	1							15												
161.1	PERFORM VISUAL INSPECTION AND NECESSARY REPLACEMENT	433X1	2							5												
162.8	PREPARE PROPELLANT SYSTEM MGE FOR MISSILE	433X1	1							30												
133.11	PREPARE ELECTRICAL MGE FOR MISSILE INSPECTION	561X0	1								70											
134.5	PREPARE HYDRAULIC MGE FOR MISSILE INSPECTION	421X2 568X0B	1 1								50											
135.14	PREPARE PNEUMATIC MGE FOR MISSILE INSPECTION	568X0B	1								15											
132.1	PREPARE FOR APCHE SELF-CHECK	314X0P	1								35											
132.1A	PERFORM APCHE SELF-CHECK	314X0P	2									90										
135.11A	PREPARE MDU FOR MISSILE	421X2	1										35									
120.6	READY SYSTEM FOR MISSILE CHECKOUT	314X0P 421X2	1 1											120								
124.4	PERFORM HYDRAULIC SYSTEM CHECKOUT (FILL AND BLEED DECK 233)	314X0P 421X2	2 2													125						
161.9	PERFORM PROPULSION SYSTEM CHECKOUT	433X1 433X0 421X2	2 1 1																		270	
161.8	PERFORM ENGINE RELAY BOX CHECKOUT	314X0P 421X2 433X0	2 1 1																		30	
161.7	PERFORM APCHE PROPULSION IGNITTERS AND HEATERS CONTINUITY TEST	314X0P 433X1	2 2																			60
127.47	PERFORM PU SYSTEM CHECKOUT (DECK 240)	314X0P	2																			3
127.49	PERFORM PU SYSTEM TEST (DECK 219)	314X0P	2																			
123.5	PERFORM AIRBORNE ELECTRICAL SYSTEM CHECKOUT	314X0P	2																			
125.4	PERFORM AIRBORNE PNEUMATIC SYSTEM CHECKOUT	433X0 421X2	1 2																			
152.1		311X0P	1																			
252.1	PREPARE COUNTDOWN GROUP FOR CHECKOUT	311X0P	1																			
122.6	PERFORM AUTOPILOT SYSTEM CHECKOUT	314X0P 421X2 433X0 433X1	2 1 2 2																			
151.4	PREPARE MGS FOR CHECKOUT	311X0P	1																			
151.5	PERFORM MGS CHECKOUT	311X0P	1																			
252.50	DEENERGIZE IGS	311X0P	1																			
252.2	SECURE COUNTDOWN GROUP FROM CHECKOUT	311X0P	1																			
152.2	SECURE ALINGMENT GROUP FROM CHECKOUT	311X0P	1																			
122.7	PERFORM APCHE AUTOPILOT FREQUENCY RESPONSE TEST (DESK 256)	314X0P	2																			
122.8	PERFORM INTEGRATED A/P IGS TESTS	311X0P	1																			
	PERFORM PNEUMATIC LEAK TEST	421X2	1																			
121.3	SECURE FROM INSPECTION	433X0 433X1	2 2																			
122.12	SECURE FROM INSPECTION	314X0P	1																			
123.10	SECURE FROM INSPECTION	423X0	1																			
	SECURE FROM INSPECTION	421X2 314X0P	2 1																			
125.6	SECURE FROM INSPECTION	421X2	1																			
127.50	SECURE FROM INSPECTION	314X0P 433X0	1 1																			
161.10	SECURE FROM INPSECTION	433X1 421X2	2 1																			
151.13	SECURE MGS FROM CHECKOUT	311X0P 433X0	1 1																			
152.2	SECURE ALIGNMENT GROUPFROM CHECKOUT	311X0P	1																			
252.2	SECURE COUNTDOWN GROUP FROM CHECKOUT	311X0P	2																			
120.2	PREPARE MISSILE FOR TRANSPORT	433X0 603X0	2 3																			
120.3	STORE MISSILE	433X0 433X1 603X0	2 2 3																			
120.4	REMOVE MISSILE FROM STORAGE	433X0 603X0	2 3																			

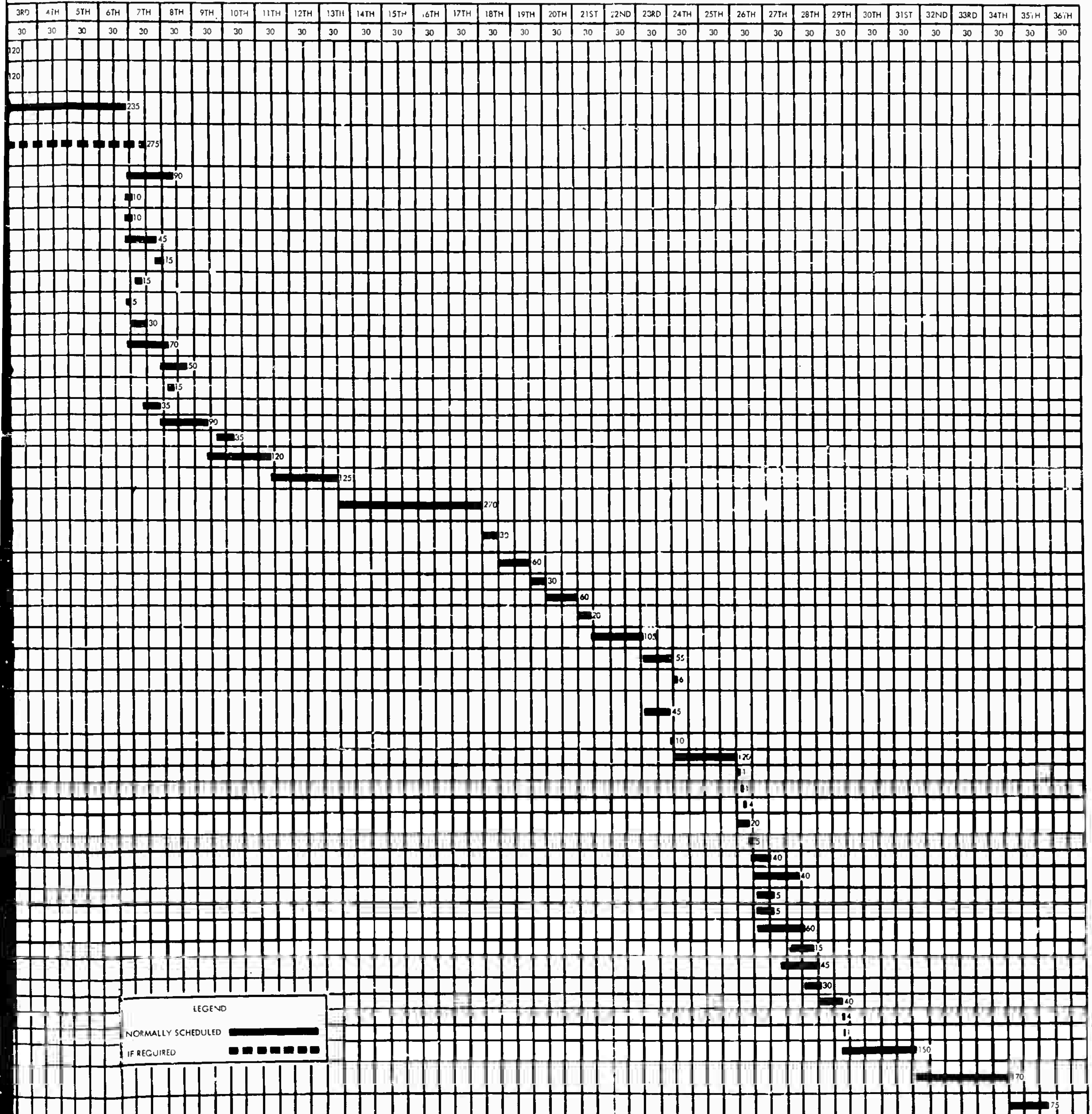
LEGEND

NORMALLY SCHEDULED

IF REQUIRED

1

FIGURE 4. SMA MISSILE PERIODIC INSPECTION - TASK DURATIONS AND MANNING REQUIRE



E 4. SMA MISSILE PERIODIC INSPECTION -
TASK DURATIONS AND MANNING REQUIREMENTS

2

2.8 Unit Manning Document

The unit manning document describes the skill levels required and the number of people allocated to the weapon system.

2.9 Reliability Indices Reports

The reliability indices reports list each reliability functional block, identify its function, and give its failure rate. The raw data from which the failure rate is estimated is also listed in the document. A typical excerpt is shown in Figure 5.

2.10 The Data Handbook

As the system is developed, a running estimate of each of the pertinent system parameters is maintained.

2.11 Provisioning Requirements Document

The number of items of support equipment and the allocation of spares is documented.

2.12 Cost Indices Document

(Not pertinent to this technical document.)

2.13 RFB Diagram

Each equipment group is subjected to a detailed reliability analysis. The resultant reliability functional block (RFB) diagram shows the inputs to each equipment block, the outputs of each equipment block, and the internal relations of each equipment block.

Figure 6 illustrates such a diagram for a hypothetical re-entry vehicle.

RELIABILITY FUNCTIONAL BLOCK	FUNCTION	FAILURES PER UNIT TIME λ
A.1	115 VAC VERIFICATION	1.0×10^{-6}
A.2	28V DC VERIFICATION	0.8×10^{-6}
A.3	SCD POWER	0.8×10^{-6}
A.4	LAUNCH CONTROL POWER	0.8×10^{-6}
A.5	TARGET A SET	0.4×10^{-6}
A.6	TARGET B SET	0.4×10^{-6}
A.7	SCD VERIFICATION	0.8×10^{-6}
A.8	BATTERY TEMPERATURE	15.0×10^{-6}
A.9	CONTINUITY	1.0×10^{-6}
A.10.1 A.10.2	RE-ENTRY VEHICLE TACTICAL	20.0×10^{-6}
A.11	MARK 3 RE-ENTRY VEHICLE	0.8×10^{-6}
A.12	MARK 4 RE-ENTRY VEHICLE	0.8×10^{-6}

FIGURE 5. FAILURE RATE ESTIMATES BASED ON GENERIC DATA AND LIMITED SUBSYSTEM TESTS

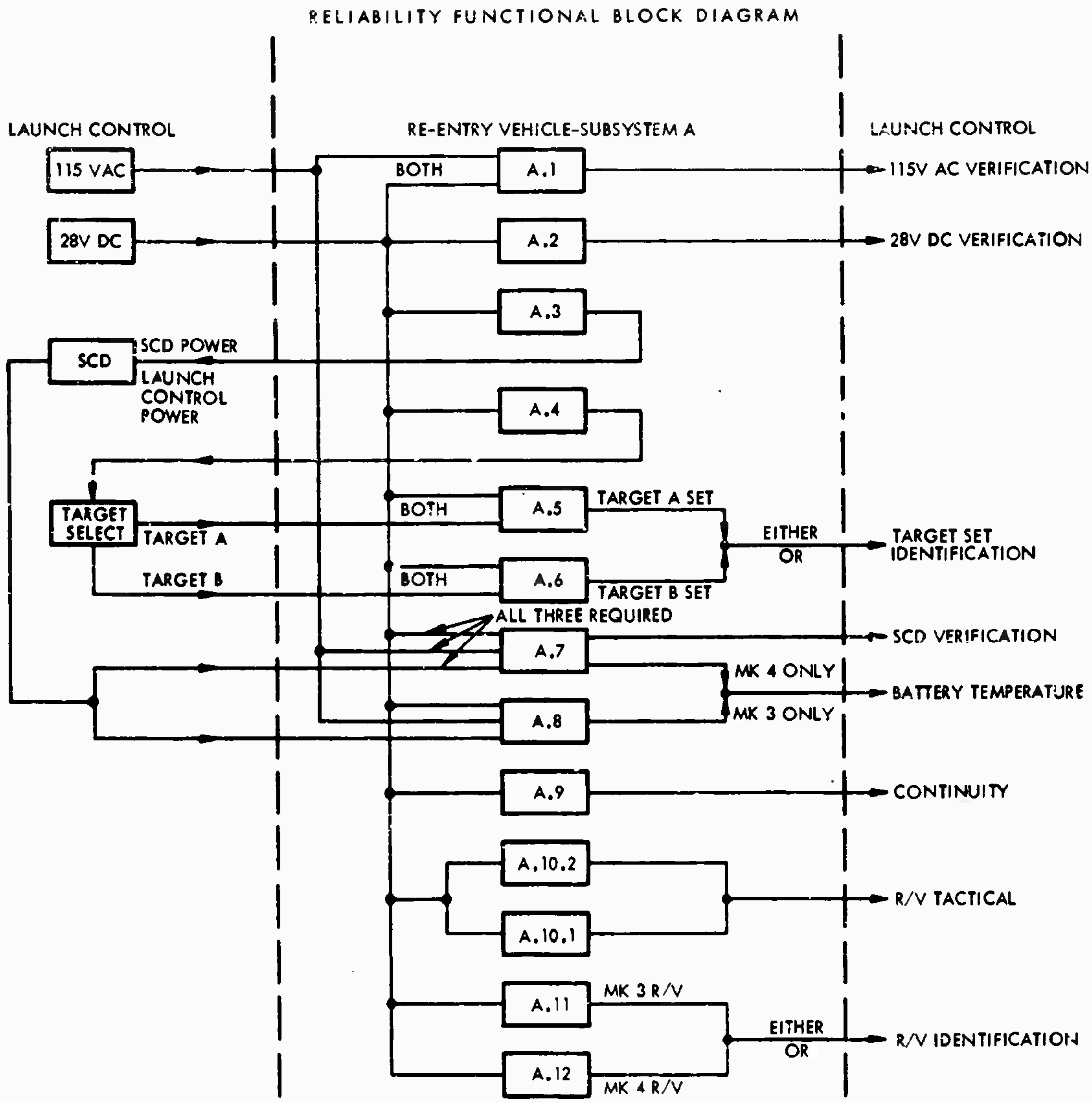


FIGURE 6. TYPICAL RFB DIAGRAM (SUBSYSTEM A, REENTRY VEHICLE DURING COUNTDOWN)

2.14 Weapon System Summary

It is assumed that the eleven tasks indicated above have been satisfactorily completed to produce a weapon system summary document. For the present example, we assume the following summary.

2.14.1 Delineate the STOC[3/] and Their Time Lines by Subsystem

The STOC for a launch site are

- EWO[4/] readiness
- Guidance checkout
- Re-entry vehicle recycle
- Periodic checkout
- Countdown
- Return to standby
- Flight

Typical time lines are illustrated by Figures 3 and 4.

2.14.2 Delineate Targeting Policy

A squadron is targeted on three objectives, three missiles to an objective.

2.14.3 Delineate Physical Factors

The launch site may be regarded to be impervious to countermeasures except when the overhead door is open. (Consider ground invulnerability to be unity.)

For the class of target considered, the warhead exhibits a unity damage function.

The cross range and down range miss distances arising from errors of the guidance system are normally distributed and independent.

3/ Standard Tactical Operating Conditions

4/ Emergency War Order

The probability of propellant depletion is zero for the target ranges used.

Under tactical launch conditions two launch attempts may be made, since each site stocks sufficient spares to repair one countdown abort. No retargeting capability exists.

The reliability and performance capability of the communication system is unity.

Penetration probability is unity.

2.14.4 Delineate Personnel Composition

Each squadron is supported by four maintenance crews. A crew works an eight hour shift with every fourth day off. During emergency conditions not lasting longer than one week all crews may be put on twelve hour duty, two crews operating simultaneously. Maintenance equipment is redundant to this extent. It requires a full crew to maintain, checkout, and/or repair a failed missile or launch facility. Scheduled maintenance does not create queuing problems.

Each launch site is fully manned twenty-four hours a day.

2.14.5 Delineate Maintenance Policy Types and Time Lines

Each launch site is maintained using a hybrid maintenance policy. Subsystems G and H are continuously monitored and enter unscheduled maintenance when a failure is indicated by malfunction lights. Repair is by remove and replace and requires a mean time of one day. Subsystem A is an unmonitored system which is replaced once a year. The time for replacement is constant and takes one day.

Subsystem B is periodically checked after standing on alert for ten days. The duration of the checkout, when it is all-go, is one hour. The system can be returned to alert in ten minutes from any point in all-go checkout. The mean time for repair, which is by remove and replace, is [illegible] hours.

Subsystems C, D, E, and F stand on alert for thirty days. At the end of thirty days, a checkout requiring 0.6 day is performed. The system is off alert during this time. Repairs are by remove and replace.

- Spares are unlimited.
- Deployment of the squadron is such that travel time for unscheduled maintenance is negligible compared to the duration of maintenance activity.
- At irregular intervals TCTO must be accomplished (off alert).[5/]
- Scheduled maintenance does not create queuing problems.

Figures 7 through 13 illustrate the time lines for each subsystem maintenance policy and the values of the parameters.

3.0 SPECIFICATION OF FIGURES OF MERIT (F.O.M.)

The various figures of merit useful in making decisions for or against system alterations and for use in targeting are, in order of increasing detail;

3.1 $E \triangleq \overline{A}' \, [D] \, \overline{C}$ = expected targets destroyed per squadron

3.2 $[D]$ = dependability matrix per squadron

3.3 $\overline{C}$ = System capability vector

3.4 $\overline{A}$ = Squadron availability vector, and $\overline{A}'$ is its transpose

3.5 Relative subsystem, site, squadron rank by reliability indices by mode of operation

3.6 Relative subsystem, site, squadron rank by availability indices

3.7 Relative subsystem, site, squadron rank by consumption rate by mode of operation

3.8 Relative subsystem, site, squadron rank by repair time

3.9 Relative subsystem, site, squadron rank by lag time

3.10 Relative subsystem, site, squadron rank by duration of go checkout

3.11 Relative subsystem rank by test quality and coverage by mode of operation.

5/ Time Compliance Technical Order.

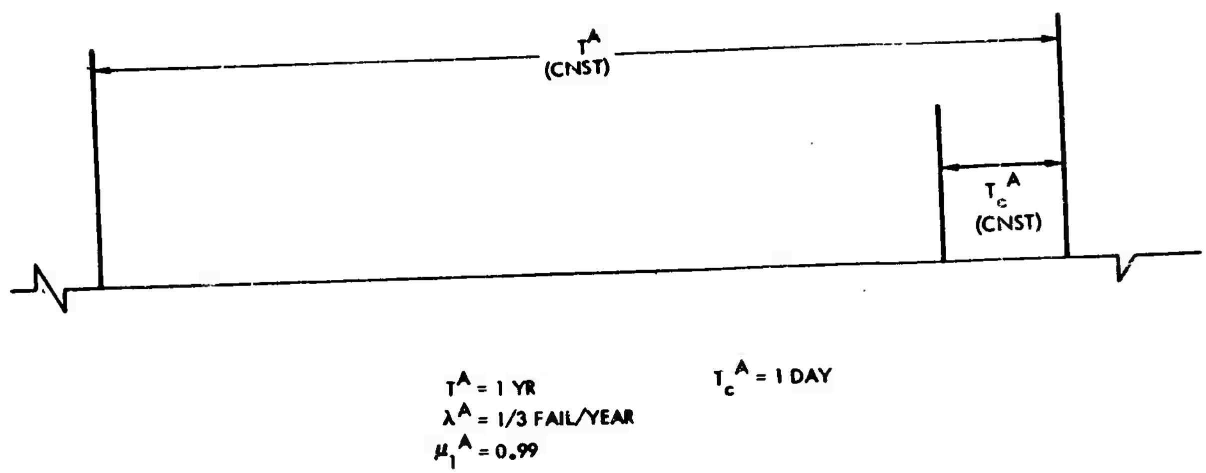
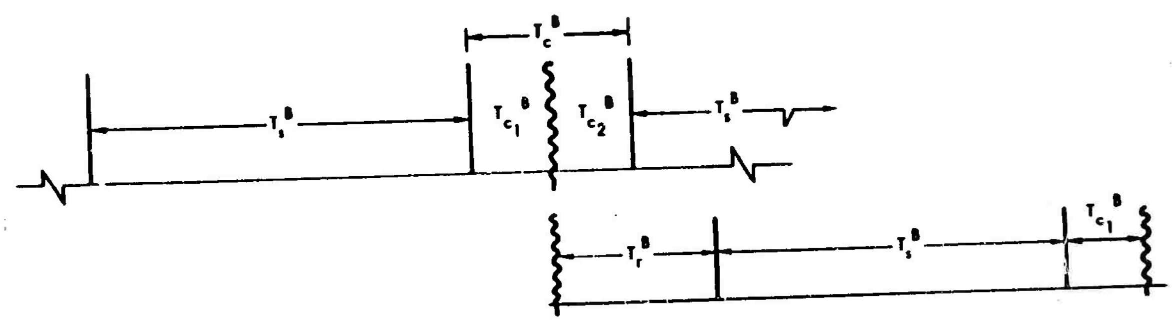


FIGURE 7. TIME LINE ANALYSIS OF CALENDAR REPLACEMENT POLICY FOR SUBSYSTEM A

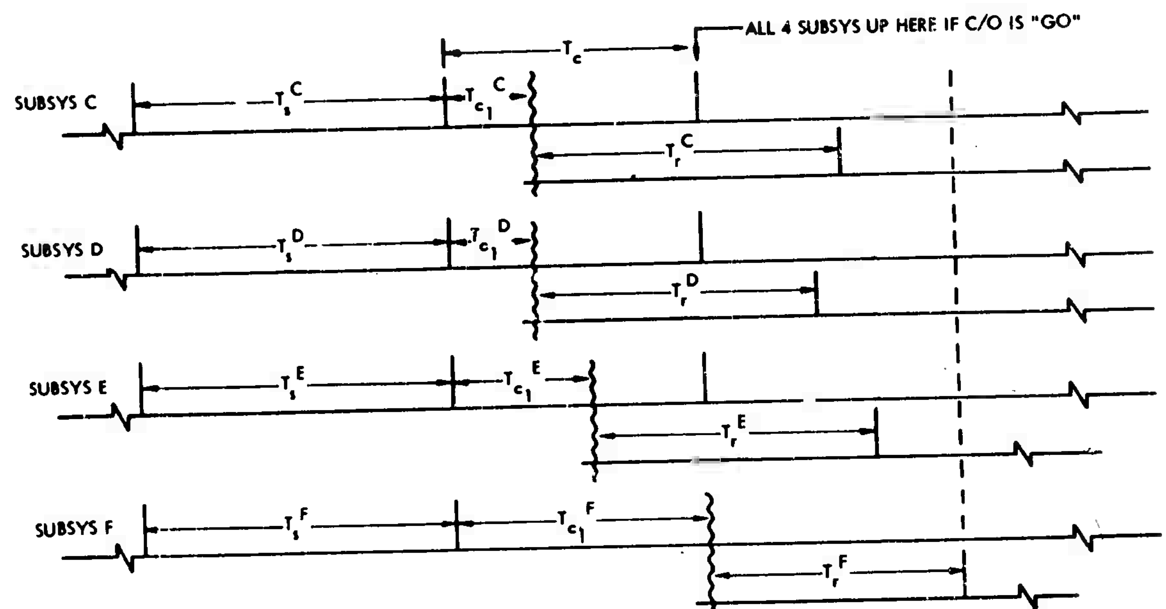


$T_s^B = 10$ DAYS
$T_{c_1}^B = 1/25$ day
$(T_{c_1}^B)' = 1/144$ day
$T_r^B = 1/3$ DAY
$T_{c_2}^B = 0$

$\lambda_{d_s}^B = 0.002$ FAIL/DAY
$\lambda_{u_s}^B = 0$
$P_{d_{c_2}}^B = 1$
$\lambda_{d_c}^B = 0.2$ FAIL/DAY

$\alpha^B = 0.1$
$\mu_1^B = 0.9$
$\mu_2^B = 0.1$
$\mu_3^B = 0$
$\beta^B = 0.01$

FIGURE 8. TIME LINE ANALYSIS OF SEQUENTIAL CHECKOUT POLICY FOR SUBSYSTEM B



NOTE:

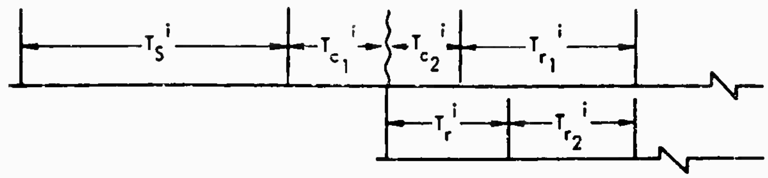
$T_s^C = T_s^D = T_s^E = T_s^F = \text{CNST}$,

$T_{c_1}^C = T_{c_1}^D = \text{CNST}$

$T_r^D < T_r^E = \text{CNST}$

$T_{c_1}^F = T_c^F = \text{CNST}$

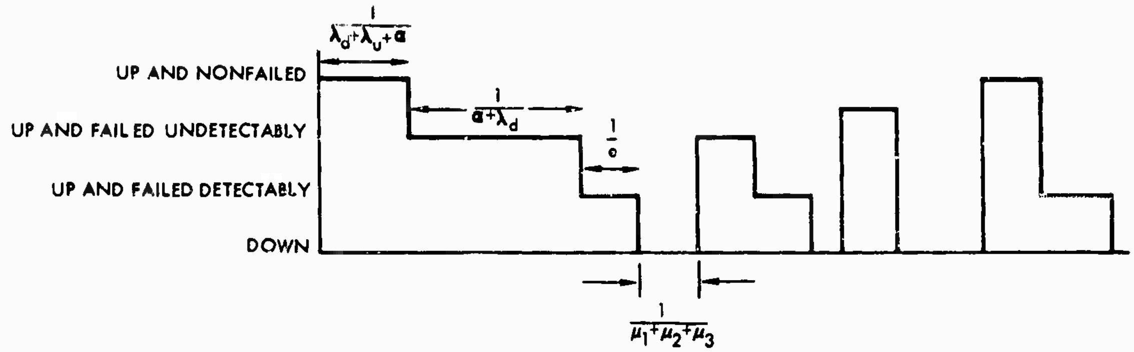
FIGURE 9. TYPICAL TIME LINE ANALYSES OF JOINT MAINTENANCE POLICY OF SUBSYSTEMS C, D, E and F



PARAMETER*	SUBSYSTEM: INDIVIDUAL MODELS C	D	E	F	COMPOSITE MODEL C	D	E	F
T_S	(VARIABLE)				30	30	30	30
T_c	0.1	0.1	0.2	0.05	0.6	0.6	0.6	0.6
T_{c_1}	0.05	0.05	0.1	0.05	0.1	0.1	0.3	0.6
T_r	0.1	3	10	1				
α	0.05	0.01	0	0.01				
β	0.01	0.01	0	0				
$P_{d_{c_2}}$	0.9	1.0	1.0	1.0				
$1/\lambda_{d_s}$	50	200	2000	500				
$1/\lambda_{u_s}$	500	2000	∞	∞				
μ_1	0.99	0.985	1.0	1.0				
μ_2	0.005	0.01	0	0				
μ_3	0.005	0.005	0	0				
T_{r_1}					(SEE TEXT)			
T_{r_2}					(SEE TEXT)			
$1/\lambda_c$	5	20	2000	50				

*UNITS ARE DAYS OR PER DAY

FIGURE 10. EQUIVALENT TIME LINE FOR THE Ith SUBSYSTEM OF THE JOINT MAINTENANCE POLICY FOR SUBSYSTEMS C, D, E, AND F



PARAMETER	SUBSYSTEM	
	H	G
$\frac{1}{\mu_1}$	1 DAY *	1 DAY *
$\frac{1}{\mu_2}$	∞	∞
$\frac{1}{\mu_3}$	∞	∞
$1/\alpha$	10 DAYS	500 DAYS
$1/e$	$\frac{1}{24}$ DAYS	$\frac{1}{24}$ DAYS
$1/\lambda_d$	5 DAYS	50 DAYS
$1/\lambda_u$	100 DAYS	∞
$\frac{1}{\mu_1+\mu_2+\mu_3}$	1 DAY	1 DAY

* INCLUDES APPROXIMATELY
1/2 DAY LAG TIME
DUE TO QUEUING

FIGURE 11. TYPICAL TIME LINE OF A CONTINUOUSLY MONITORED SYSTEM SHOWING DWELL TIME IN VARIOUS STATES (SUBSYSTEMS G AND H)

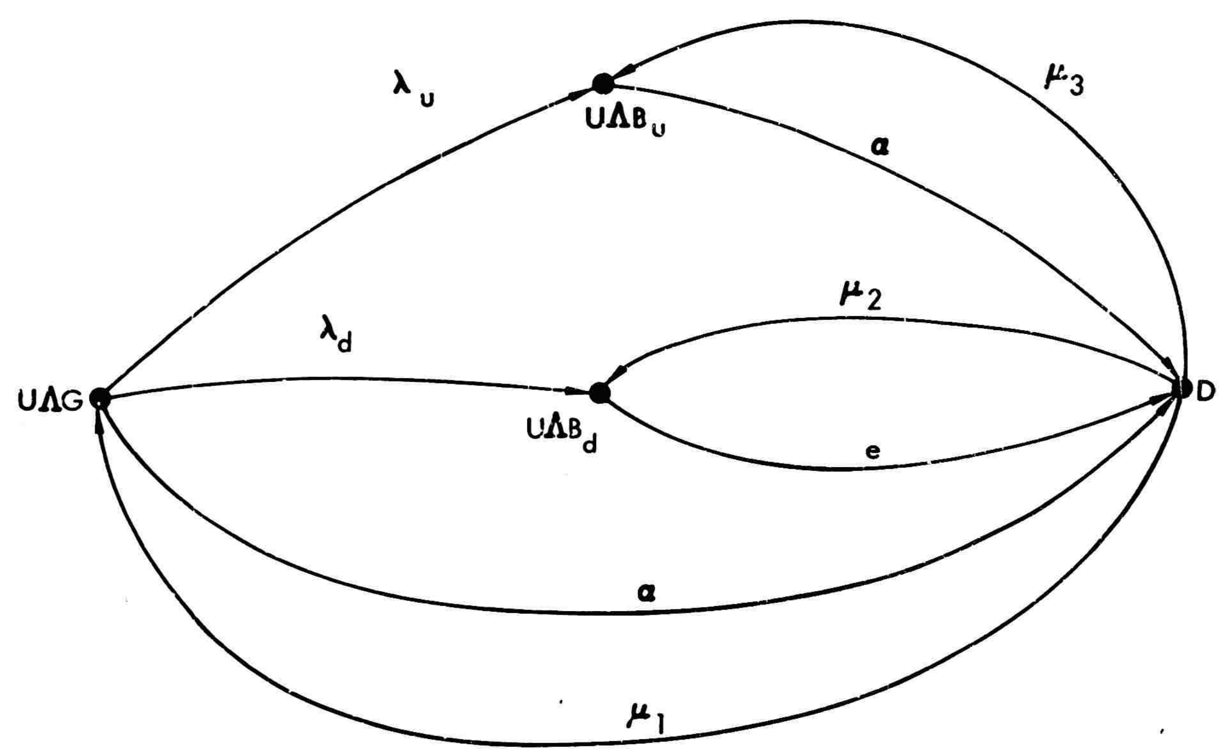
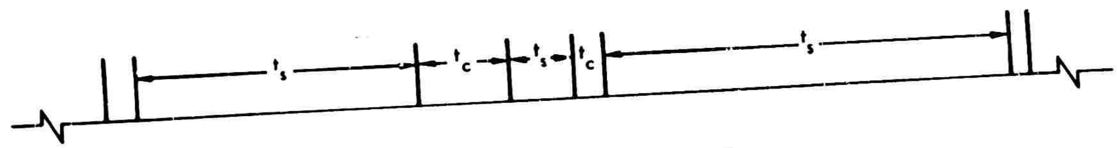


FIGURE 12. POSSIBLE STATE TRANSITIONS FOR CONTINUOUSLY MONITORED SYSTEMS SHOWING RATES OF TRANSITION (SUBSYSTEMS G AND H)



t_s AND t_c ARE RANDOM VARIABLES:

$$p(t_s) = \lambda_o e^{-\lambda_o t_s}$$

$$p(t_c) = \mu_o e^{-\mu_o t_c}$$

PARAMETER	TCTO
$1/\lambda_o$	60 DAYS
$1/\mu_o$	1 DAY

FIGURE 13. TIME LINE ANALYSIS OF TCTO

The calculation of 3.1 shall reflect the conditions of 1.0 and 2.0 In the present memorandum only the highest level F.O.M. (No. 3.1) will be considered since all other F.O.M. are obtained as intermediate by products of proper data processing.

4.0 IDENTIFICATION OF ACCOUNTABLE FACTORS

The total number of factors which must be accounted for are determined by the system complexity and the nature and detail of the questions which it is expected must be answered by the modeling effort.

It is convenient to group the areas of consideration under four headings:

Personnel
Procedures
Hardware
Logistics

4.1 Define Level of Accountability

The degree of accountability (in this example) places the least accountable level at a subsystem, and the highest accountable level at a squadron.

The depth of detail to be accounted for is specified in the following four sections. Each factor is to be explicitly accounted for in the structure of the model by subsystem, by site, and by squadron.

4.2 Hardware

The models shall reflect the possibility of four failure stress levels for periodically checked subsystems depending upon the modes of operation:

- Alert
- Checkout and/or countdown
- Flight
- Demating

The model shall also reflect the possiblity of inherently undetectable failures.

4.3 Procedures

The model shall specifically account for at least the following properties of a test

- Test coverage
- Test error
  - false alarm
  - Oversight
- Test duration
  - On alert
  - Off alert

4.4 Personnel

The model shall reflect the possibility of queuing in unscheduled maintenance due to insufficient personnel.

The model shall not explicitly differentiate between inherent failures and human induced failures.

The model shall not explicitly differentiate procedural errors from human errors.

4.5 Logistics

The model shall specifically account for lag time due to transportation delays and the deployment of the launch sites.[6/]

The model shall specifically account for spares provisioning.[7/]

6/ Zero by assumption since time did not permit an analysis.

7/ Accounted for in launch probability only.

4.6 Specify Data Constraints

Data shall be obtained as a result of the normal routine of system checkouts, maintenance actions, and repairs. Data from existing data systems shall be utilized to the maximum degree possible.

Special field exercises shall be kept to the minimum consistent with obtaining accurate estimates of crucial parameters.[8/]

Field data shall be supplemented by depot and qualification testing results wherever possible.

8/ The question of confidence levels and intervals is not treated herein.

5.0 MATHEMATICAL MODEL CONSTRUCTION [9]

5.1 Assumptions

- The failure distribution which holds during standby is an exponential distribution.
- The failure distributions in checkout, countdown, and flight may be binomial, exponential, or both.
- The means of all distributions are finite.
- Subsystems fail independently.
- Test errors are binomially distributed.
- The launch sites/missiles are a homogeneous population.

5.2 Definitions and Symbols

The following definitions and symbols hold throughout the analysis.

$\overline{A}$ is the availability vector. $\overline{A}'$ is its transpose.

A_i is the ith element of $\overline{A}$.

$A_s[\tau]$ total system readiness expressed as a function of time τ.

$A_s[\infty]$ total system readiness, limiting value as $\tau \longrightarrow \infty$.

$A_u[\tau]$ apparent readiness expressed as a function of time τ.

$\overline{C}$ is the design capability (performance) vector.

C_i is an element of $\overline{C}$.

C_k^n is the combination of n things taken k at a time.

$[D]$ system dependability matrix.

d_{ij} element of $[D]$.

e the rate of detection of failures of the inherently detectable in principle class.

---

[9] The principles used in this section are discussed in Appendix I.

e_k — The expected kill as a function of the number of missiles targeted per objective, other parameters held constant.

E — The expected kill per squadron (system effectiveness, defined to be a function of readiness, reliability, and design capability).

$f_g[R_O]$ — Conditional delivery probability. The probability of successful flight and penetration to the target area; given a successful launch and no gross malfunction of any part of the system. A measure of system performance excluding reliability and/or readiness.

i; $\bar{i}$ — i is the logical indication of the entrance of subsystems C, D, E, or F into repair. $\bar{i}$ is the logical negation of i; i.e. checkout is go.

L — Rate of termination of launch attempts irrespective of manner of termination, but excluding enemy counter measures.

$\overline{P_i[F]}$ — Is the mean likelihood that the ith subsystem will fail to pass the test during checkout. (periodically tested subsystem)

$P_{CD}[\infty]$ — The probability of successful launch on the first attempt without regard for duration.

$P_{CD}[t/CD]$ — The probability of successful launch on the first attempt in time t or less; given that the launch is successfully completed.

P_{d_c} — The probability that all of the equipment characteristics which are monitored during a periodic checkout will survive the checkout; given that they were unfailed at entrance to checkout. Failure of any such characteristic is termed to be "inherently detectable in principle."

$P_{d_{c_1}}$ — The probability that the inherently detectable equipment characteristics survive checkout up to the point of test decision; given that they are nonfailed at entrance to checkout.

$P_{d_{c_2}}$ — The probability that the inherently detectable equipment characteristics survive the demating process post checkout test decision; given that they were passed and were actually nonfailed at the test decision point.

$P_{d_{r_1}}$; $P_{d_{r_2}}$	The probability that the inherently detectable equipment characteristics survive the expected waiting times T_{r_1} and T_{r_2} ; given that they were unfailed at completion of checkout and repair respectively.
P_{d_s}	The probability that the inherently detectable equipment characteristics survive the standby period; given that they were unfailed at the time of assignment to standby.
$P_D[R_O]$	The weapons effect damage function; expressed as a function of the radial miss distance R.
P_f	System flight reliability.
P_g	Guidance accuracy dispersion.
$\overline{P_{g/u}^{i}}$	The mean likelihood that the ith unit is nonfailed; given that it is assigned "UP" .
$P_i[G; t_{s_k}]$	The probability that the ith unit is nonfailed at entrance to standby.
P_k	Unit probability of kill.
$P_k[\tau]$	The probability that exactly k units are "down" τ units of the time after initiation of an alarm condition. Down means in repair or awaiting repair.
$P_L[\tau]$	The probability of launch for one or more successive attempts. τ is measured from the initiation of first attempt.
$P_L[\infty]$	The limiting value of $P_L[\tau]$ as $\tau \rightarrow \infty$.
$P_{NPD}[r_O]$	The probability of no propellant depletion expressed as a function of target range r_O.
P_p	Penetration probability.
$P_{ub_d}[t]$	Probability of being up and bad, (failed) but detectable in principle at time t.
$P_{ub_u}[t]$	Probability of being up and bad, and not detectable in principle at time t.
$P_{dg}[t]$	Probability of being "down" (assigned to repair), but "good" (nonfailed) at time t.
$P_{db_d}[t]$	Probability of being down with a detectable class of failure at time t.

$P_{db_u}[t]$	Probability of being down with an undetectable class of failure at time t.
$P_{ug}[t]$	Probability of being "up" (assigned to service) and "good" (nonfailed) at time t.
P_{u_c}	The probability that all of those equipment characteristics which are not monitored during a periodic checkout will survive the checkout; given that they were unfailed at entrance to checkout. Failure of any such characteristic is termed to be "inherently undetectable in principle."
$P_{u_{c_1}}$	The probability that the inherently undetectable equipment characteristics survive checkout up to the point of test decision; given that they were unfailed at entrance to checkout.
$P_{u_{c_2}}$	The probability that the inherently undetectable equipment characteristics survive the demating process post checkout test decision; given that they were passed and unfailed at the point of test decision.
$P_{u_{r_1}}$; $P_{u_{r_2}}$	The probability that the inherently undetectable equipment characteristics survive the expected waiting times T_{r_1} and T_{r_2}; given that they were unfailed at the completion of checkout and repair respectively.
P_{u_s}	The probability that the inherently undetectable equipment characteristics survive the standby period; given that they were unfailed at the time of assignment to standby.
P_{WH}	Warhead yield function.
r_0	Target range.
R	Target miss distance measured radially from the target to the point of impact
$R_{CD}^{\,i}$	Reliability of the ith subsystem in countdown.
$R_f^{\,i}$	Reliability of the ith subsystem in flight.
R_L	Lethal radius of warhead.
t	Time
t_c; $\bar{t}_c$	Duration and mean duration of checkout, respectively.
t_{CD}	Duration of countdown.

t_d; $\bar{t}_d$	Duration and mean duration of down time, respectively.
t_s; $\bar{t}_s$	Duration and mean duration of standby time, respectively.
t_u; $\bar{t}_u$	Duration and mean duration of up time, respectively.
T_c	Duration of a constant duration checkout.
T_{c_1}, T_{c_2}	Duration of the first and second half of checkout when constant.
$T_r^{\,i}$	Remove/replace time for ith subsystem repair.
$T_{r_1}^{\,i}$; $T_{r_2}^{\,i}$	Expected time awaiting reassignment to standby for the ith subsystem after successful completion of checkout and repair, respectively.
T_s	Constant standby duration.
$U[x]$	Unit step at $t = x$.
α	<u>Periodic maintenance</u> - the probability of false alarm. <u>Continuous monitoring</u> - the rate of false alarms.
β	<u>Periodic maintenance</u> - the probability of passing a failed characteristic of the inherently detectable in principle class.
$\delta(x)$	Delta dirac at $t = x$.
λ	System failure rate of the continuously monitored subsystems during standby.
λ_0	Rate of occurrance of TCTO actions.
λ_L	System failure rate during countdown.
$\lambda_{d_s}^{\,i}$	Failure rate of the inherently detectable characteristics of the ith subsystem during standby.
$\lambda_{u_s}^{\,i}$	Failure rate of the inherently undetectable characteristics of the ith subsystem during standby.
μ	Equivalent system repair rate for aborted countdowns.
μ_0	Rate of completion of TCTO actions

μ_1 Periodic maintenance - probability of successful repair.

Continuous monitoring - the rate of successful repair.

μ_2 Periodic maintenance - the probability of leaving repair with a failure of the inherently detectable in principle class.

Continuous Monitoring - The rate of leaving repair with a failure of the inherently detectable in principle class.

μ_3 Periodic maintenance - The probability of leaving repair with a failure of the inherently undetectable in principle class.

Continuous monitoring - The rate of leaving repair with a failure of the inherently undetectable in principle class.

$\prod$ Indicates continued product.

σ Standard deviation.

τ Time duration.

5.3 Delineation of Possible Outcomes

- Total failure (full target survival)
  - Not ready to enter countdown.
  - Aborts countdown.
  - Catastrophic failure in flight.
  - Destroyed by counter measures.
  - No yield.
  - Falls outside target area.
- Partial failure (or success): (incomplete target destruction.)
  - Falls wide of target with proper yield.
  - Falls on target with low yield.
- Total success (target destroyed).

5.4 Delineation of System States

During the prealarm condition of system readiness, availability is calculated under the assumption that each subsystem of each site can occupy any one of six basic states, namely:

- up and nonfailed
- up and failed detectably
- up and failed undetectably
- down and nonfailed
- down and failed detectably
- down and failed undetectably

In addition, there is an overall system administrative state, namely:

- down in TCTO

Since there are five launch critical subsystems, there are 7^5 possible launch sites states (16,807 states). Since there are nine launch sites to be considered, the squadron can theoretically occupy any one of

$$C^{m+r-1}_{r-1}\Bigg|_{\substack{r = 16,807 \\ m = 9}} = \frac{16,815!}{16,806!9!} = \text{on the order of } 10^{31}$$

basic states. The permissible state transitions are shown in Figure 12.

In the post alarm environment one additional state is accounted for, namely;

- down and in queue

Brief attention is given to multiple tactical launch attempts. For this calculation the 16,807 basic states of a launch site are subsumed into seven gross states:

- on alert and nonfailed at the time of receipt of the launch directive.
- on alert, but failed, at the time of receipt of the launch directive.
- in repair out of countdown entered upon receipt of launch directive.
- in repair at time of receipt of launch directive.
- counting down after first abort or after repair that was being completed at time launch directive was received, called final countdown.
- launched.
- aborted out of final countdown.

These states and the permissible state transitions are shown in Figure B-1 of Appendix II.

The dependability matrix identifies 81 system states, each of which corresponds to the probability that if i missiles are available when the execution directive is received at a random point in time, j of them will successfully launch, fly, and impact within the specified target area.

5.5 Availability

5.5.1 System Models

5.5.1.1 The Availability Vector

If we denote the availability of any member of the squadron by $A_s[\infty]$ or $A_s[\tau]$ where the first symbol refers to steady state availability and the second symbol refers to transient (augmented) availability then the availability vector is given by

$$\bar{A} = \begin{bmatrix} A_9 \\ A_8 \\ \cdot \\ \cdot \\ \cdot \\ A_1 \\ A_0 \end{bmatrix} \tag{2}$$

where

$$A_k = C_k^9 \; (A_s[\infty])^k \; (1 - A_s[\infty])^{9-k} \tag{3}$$

or

$$A_k = C_k^9 \; (A_s[\tau])^k \; (1 - A_s[\tau])^{9-k} \tag{4}$$

The components A_i of $\bar{A}$ are read "The probability that exactly i missiles of the squadron are available."

5.5.1.2 Composite Steady State Model

Total missile/launch site availability may be expressed in the steady state by;

$$A_s[\infty] = A_0[\infty] \; A_A[\infty] \; A_G[\infty] \; A_H[\infty] \; A_B[\infty] \; A_{CDEF}[\infty] \tag{5}$$

where,

$A_O[\infty]$ = Impact of TCTO on availability

$A_A[\infty]$ = Availability of re-entry vehicle

$A_B[\infty]$ = Availability of guidance

$A_{CDEF}[\infty]$ = Joint availability of autopilot, propulsion, structure, and overhead door

$A_G[\infty]$ = Availability of air conditioning

$A_H[\infty]$ = Availability of power generation and distribution

<u>Alert Degradation due to TCTO</u>

It is assumed that the only effect of a TCTO action is to remove the launch site from alert. The time between TCTO actions (t_s) is distributed with density function;

$$p_O[t_s] = \lambda_O e^{-\lambda_O t_s} \tag{6}$$

The durations (t_c) of TCTO actions are distributed with density function;

$$p_O[t_c] = \mu_O e^{-\mu_O t_c} \tag{7}$$

The system availability due to TCTO actions is therefore given by,

$$A_O[\infty] = \frac{\bar{t}_s}{\bar{t}_s + \bar{t}_c} \tag{8}$$

$$\bar{t}_s = \int_0^\infty e^{-\lambda_O t_s} dt_s = \frac{1}{\lambda_O} \tag{9}$$

$$\bar{t}_c = \int_0^\infty e^{-\mu_O t_c} dt_c = \frac{1}{\mu_O} \tag{10}$$

where

$\bar{t}_s$ = Mean time between TCTO actions.

$\bar{t}_c$ = Mean duration of TCTO actions.

The Joint Availability of Subsystems, C, D, E, and F

The weapon system summary (2.14) indicated that subsystems C, D, E, and F stand in readiness for the same time interval T_s, at the end of which time they enter checkout. If the checkout is "go" for all subsystems, the checkout duration is T_c. Checkout and repair of the subsystems is conducted in parallel. Each subsystem is assigned up when its repair or checkout is complete. The last system up defines the point of entry in T_s for all four subsystems.

A typical time line of this joint maintenance policy is shown in Figure 9. The equivalent time line for any given one of the subsystems is shown in Figure 10. It will be noted that, in general, there will be an expected waiting time $T_{r_1}{}^i$ or $T_{r_2}{}^i$ on each maintenance cycle during which the ith subsystem is on alert, but one or more of the remainder of the subsystems is down. This waiting time must be accounted for in the structure of the model.

Accordingly, we have,

$$A_{CDEF}[\infty] = \frac{\prod_{i=C}^{F} \overline{P_i[G;t_{s_k}]} \left\{ 1 - e^{-\sum_{i=C}^{F} (\lambda_s{}^i + \lambda_u{}^i) T_s} \right\} \Big/ \sum_{i=C}^{F} (\lambda_s{}^i + \lambda_u{}^i)}{T_s + \bar{t}_d} \qquad (11)$$

where,

$$\overline{P_i[G;\, t_{s_k}]} = \frac{\mu_1{}^i(1-\beta^i)\, P_{d_{r_2}}{}^i\, P_{u_{r_2}}{}^i \left\{1 - (1-\alpha^i)\, P_{d_s}{}^i\, P_{d_c}{}^i\, P_{d_{r_1}}{}^i\right\}}{\left\{1 - P_{d_s}{}^i\, P_{d_c}{}^i\, P_{u_s}{}^i\, P_{u_c}{}^i\, P_{u_{r_1}}{}^i\, P_{d_{r_1}}{}^i\,(1-\alpha^i)\right\} A} \quad (12)$$

where

$$A = 1 + \left\{(1-\alpha^i-\beta^i)(1-\mu_2{}^i)\, P_{d_{r_2}}{}^i - (1-\alpha^i)\, P_{d_{c_2}}{}^i\, P_{d_{r_1}}{}^i\right\} P_{d_s}{}^i\, P_{d_{c_1}}{}^i \quad (13)$$

Denote

$$i = \overline{P_i[F]} \quad (14)$$

$$\bar{i} = 1 - \overline{P_i[F]} \quad (15)$$

Then,

$$\overline{P_i[F]} = \frac{1}{A}(1-\beta^i)\left\{1 - (1-\alpha^i)\, P_{d_s}{}^i\, P_{d_c}{}^i\, P_{d_{r_1}}{}^i\right\} \quad (16)$$

and for the inequalities given in the weapon system summary (2.14).[10]

$$\bar{t}_d = \bar{D}\,\bar{E}\,T_c + E(T_{c_1}{}^E + T_r{}^E) + D\,\bar{E}\,(T_{c_1}{}^D + T_r{}^D) + \bar{D}\,\bar{E}\,F\,T_r{}^F \quad (17)$$

and

$$T_{r_1}{}^C = D\,\bar{E}\,(T_{c_1}{}^D + T_r{}^D - T_c) + E\,(T_{c_1}{}^E + T_r{}^E - T_c) + \bar{D}\,\bar{E}\,F\,T_r{}^F + \bar{D}\,\bar{E}\,\bar{F}\,(0) \quad (18)$$

$$T_{r_2}{}^C = \bar{D}\,\bar{E}\,\bar{F}\,(T_c - T_{c_1}{}^C - T_r{}^C) + E\,(T_{c_1}{}^E + T_r{}^E - T_{c_1}{}^C - T_r{}^C) + \bar{E}\,D\,(T_{c_1}{}^D + T_r{}^D - T_{c_1}{}^C - T_r{}^C) + \bar{D}\,\bar{E}\,F\,(T_c + T_r{}^F - T_{c_1}{}^C - T_r{}^C) \quad (19)$$

$$T_{r_1}{}^D = E\,(T_{c_1}{}^E + T_r{}^E - T_c) + \bar{E}\,F\,T_r{}^F + \bar{E}\,\bar{F}\,(0) \quad (20)$$

$$T_{r_2}{}^D = E\,(T_{c_1}{}^E + T_r{}^E - T_{c_1}{}^D - T_r{}^D) + \bar{E}\,(0) \quad (21)$$

$$T_{r_1}{}^E = D\,(T_{c_1}{}^D + T_r{}^D - T_c) + \bar{D}\,F\,T_r{}^F + \bar{D}\,\bar{F}\,(0) \quad (22)$$

$$T_{r_2}{}^E = 0 \quad (23)$$

$$T_{r_1}{}^F = E\,(T_{c_1}{}^E + T_r{}^E - T_c) + \bar{E}\,D\,(T_{c_1}{}^D + T_r{}^D - T_c) + \bar{D}\,\bar{E}\,(0) \quad (24)$$

$$T_{r_2}{}^F = E\,(T_{c_1}{}^E + T_r{}^E - T_c - T_r{}^F) + \bar{E}\,D\,(T_{c_1}{}^D + T_r{}^D - T_c - T_r{}^F) + \bar{D}\,\bar{E}\,(0) \quad (25)$$

10/ See Appendix III for derivations

where, as a typical example,

$$P_{d_{r_1}}{}^{C} = D\,\overline{E}\,e^{-\lambda_s^C(T_{c_1}{}^{D} + T_r{}^{D} - T_c)} + E\,e^{-\lambda_s^C(T_{c_1}{}^{E} + T_r{}^{E} - T_c)} + \overline{D}\,\overline{E}\,\overline{F}\,e^{-\lambda_s^C T_r{}^{F}} + \overline{D}\,\overline{E}\,\overline{F} \qquad (26)$$

$$\begin{aligned} P_{d_{r_2}}{}^{C} = {} & \overline{D}\,\overline{E}\,\overline{F}\,e^{-\lambda_s^C(T_c - T_{c_1}{}^{C} - T_r{}^{C})} \\ & + E\,e^{-\lambda_s^C(T_{c_1}{}^{E} + T_r{}^{E} - T_{c_1}{}^{C} - T_r{}^{C})} \\ & + \overline{E}\,D\,e^{-\lambda_s^C(T_{c_1}{}^{D} + T_r{}^{D} - T_{c_1}{}^{C} - T_r{}^{C})} \\ & + D\,\overline{E}\,F\,e^{-\lambda_s^C(T_c + T_r{}^{F} - T_{c_1}{}^{C} - T_r{}^{C})} \end{aligned} \qquad (27)$$

The Availability of Subsystems A, B, G, and H

The system models for the availability of subsystems A, B, G and H do not differ from the respective subsystem models for these subsystems. Accordingly, discussion of these availability models is delayed till Section 5.5.2.

5.5.1.3 Transient (Augmented) Availability

In the event that prior warning is received, steps may be taken to augment the availability of a squadron. Specifically, all scheduled maintenance may be deferred and the maintenance crews may be put on twelve hour shifts, two crews working in parallel to take care of unscheduled maintenance.

As noted earlier there are of the order of 10^{31} basic system states. An equivalent number of state transition equations is required to express the possible interactions between the nine launch sites and the two maintenance crews.

It is evident that the state equation approach to augmented availability is not a feasible approach, even for the simple illustrative system used here. On the other hand, machine simulation methods using Monte Carlo techniques are quite satisfactory for this and considerably more complex systems. Since Monte Carlo methods are beyond the intended scope of the present document, we shall use approximations that will permit a solution to be obtained by pencil and paper methods.

Divide the system into two equipment groups

- Continuously monitored (Subsystems G and H)
- Periodically checked (all Subsystems except G and H)

Let it be assumed that the system is returned to alert from scheduled activities in essentially zero time. Assume that unscheduled maintenance on Subsystems G and H is the only activity which can now remove the system from alert. Further assume that

- Repair is perfect at the equivalent repair rate μ,[11]

$$\mu = \frac{(\lambda_d^G + \alpha_c^G)\,\mu_1^G}{\lambda_d^G + \lambda_d^H + \alpha^G + \alpha^H} + \frac{(\lambda_d^H + \alpha_c^H)\,\mu_1^H}{\lambda_d^G + \lambda_d^H + \alpha^G + \alpha^H} \tag{28}$$

- The net launch site observable failure rate λ is

$$\lambda = \lambda_d^G + \alpha^G + \lambda_d^H + \alpha^H \tag{29}$$

- There is no delay in detecting failures, i.e.,

$$e^G = e^H \longrightarrow \infty \tag{30}$$

[11] Implies that only one subsystem can fail at a time.

The probability that the periodically checked portion of a site will be good τ units of time after the warning is received is given to an excellent degree of approximation by;

$$P_p[\tau] = \prod_{i=A}^{F} \overline{P_i[G;t_{s_k}]} \; \frac{1 - e^{-\lambda_s{}^i T_s{}^i}}{\lambda_s{}^i \; T_s{}^i} \; e^{-\left(\sum_{i=A}^{F} \lambda_s{}^i\right) \tau} \tag{31}$$

The queuing equations which express the probability $P_k[\tau]$ that exactly k sites will be down in the post warning environment are given by;

(See page 113)

$$
\begin{bmatrix} \dot{P}_9[\tau] \\ \dot{P}_8[\tau] \\ \dot{P}_7[\tau] \\ \dot{P}_6[\tau] \\ \dot{P}_5[\tau] \\ \dot{P}_4[\tau] \\ \dot{P}_3[\tau] \\ \dot{P}_2[\tau] \\ \dot{P}_1[\tau] \\ \dot{P}_0[\tau] \end{bmatrix}
=
\begin{bmatrix}
-2\mu & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2\mu & -(2\mu+\lambda) & 2\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2\mu & -(2\mu+2\lambda) & 3\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2\mu & -(2\mu+3\lambda) & 4\lambda & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2\mu & -(2\mu+4\lambda) & 5\lambda & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\mu & -(2\mu+5\lambda) & 6\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu & -(2\mu+6\lambda) & 7\lambda & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\mu & -(2\mu+7\lambda) & 8\lambda & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu & -(\mu+8\lambda) & 9\lambda \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & -9\lambda
\end{bmatrix}
\begin{bmatrix} P_9[\tau] \\ P_8[\tau] \\ P_7[\tau] \\ P_6[\tau] \\ P_5[\tau] \\ P_4[\tau] \\ P_3[\tau] \\ P_2[\tau] \\ P_1[\tau] \\ P_0[\tau] \end{bmatrix}
\qquad (32)
$$

The (average) probability of being up for any one member of the squadron is given by

$$P_u[\tau] = 1 - \frac{1}{9} \sum_{i=1}^{9} i \, P_i[\tau] \tag{33}$$

The total expression for augmented availability is then given by the approximate expression,

$$A_s[\tau] \cong P_p[\tau] \, P_u[\tau] \; P_{g/u}{}^{G} \, P_{g/u}{}^{H} \tag{34}$$

where $P_{g/u}{}^{i}$ is given by;

$$P_{g/u}{}^{i} = \frac{\mu_1{}^{i}}{(\lambda_d{}^{i} + \lambda_u{}^{i} + \alpha^{i}) \left\{ \left(\frac{\mu_1{}^{i}}{\lambda_d{}^{i} + \alpha^{i}} + \mu_3{}^{i} \right) \left(1 + \frac{\lambda_d{}^{i}}{e^{i}} \right) + \frac{\mu_2{}^{i}}{e^{i}} \right\}} \tag{35}$$

The initial conditions to be used in solving the equation set (32) are

$$P_k[0] = C_k{}^{9} (1 - P_u{}^{G}[\infty] \, P_u{}^{H}[\infty])^{k} \, (P_u{}^{G}[\infty] \, P_u{}^{H}[\infty])^{9-k} \tag{36}$$

5.5.2 Subsystem Models

5.5.2.1 Re-entry Vehicle (Subsystem A)

The Weapon System Summary (2.14) indicates that the re-entry vehicle is maintained independently of the other subsystems on a strictly calendar basis of remove and replace. Accordingly, the availability of this sub- [illegible] at a random point in time is given by;

$$\text{[illegible]}[\infty] = \frac{\mu_1{}^{A} \left\{ 1 - e^{-\lambda^{A}(T - T_r{}^{A})} \right\}}{T \lambda^{A}} \tag{37}$$

T = Time between recycles = one year

T_r^A = Replacement time (constant) = one day

μ_1^A = Probability that re-entry vehicle is nonfailed at time of installation

λ^A = Failure rate of re-entry vehicle

5.5.2.2 Guidance (Subsystem B)

The guidance subsystem is maintained independently of the other subsystems on a slipped schedule basis. Only ten minutes of the checkout time is system down time. The expression for availability is given by:

$$A_B[\infty] = \frac{\overline{P_B[G;t_{s_k}]}\left\{(1-P_{d_s}^B P_{u_s}^B)/(\lambda_{d_s}^B+\lambda_{u_s}^B) + P_{u_s}^B P_{d_s}^B\left[\frac{1-P_{u_c}^B P_{d_c}^B}{\lambda_{d_c}+\lambda_{u_c}} - (T_c^B)'\right]\right\}}{T_s^B + T_c^B + \overline{P_B[F]}\,(T_r^B - T_{c_2}^B)} \tag{38}$$

$$P_{d_s}^B = e^{-\lambda_{d_s}^B T_s^B} \tag{39}$$

$$P_{u_s}^B = e^{-\lambda_{u_s}^B T_s^B} \tag{40}$$

$$\overline{P_B[G;t_{s_k}]} = \frac{\mu_1^B(1-\beta^B)\left[1-(1-\alpha^B)P_{d_s}^B P_{d_c}^B\right]}{1 - P_{d_s}^B P_{d_c}^B P_{u_s}^B P_{u_c}^B (1-\alpha^B)\left[1 + (1-\alpha^B-\beta^B)(1-\mu_2^B) - (1-\alpha^B)P_{d_{c_2}}^B\right] P_{d_s}^B P_{d_{c_1}}^B} \tag{41}$$

$$\overline{P_B[F]} = \frac{(1-\beta^B)\left\{1-(1-\alpha^B)P_{d_s}^B P_{d_c}^B\right\}}{1 + \left\{(1-\alpha^B-\beta^B)(1-\mu_2^B) - (1-\alpha^B)P_{d_{c_2}}^B\right\} P_{d_s}^B P_{d_{c_1}}^B} \tag{42}$$

5.5.2.3 Autopilot (Subsystem C)

5.5.2.4 Propulsion (Subsystem D)

5.5.2.5 Structure (Subsystem E)

5.5.2.6 Overhead Door (Subsystem F)

These subsystems are treated as a group for periodic checkout. However, each one could be treated seperately. It is in this sense that we may consider the availability of each subsystem. Utilizing the proper superscript we have for each;

$$A_i[\infty] = \frac{\overline{P_i[G;t_{s_k}]}\,(1 - P_{d_s}{}^{i} P_{u_s}{}^{i})}{(\lambda_{d_s}{}^{i} + \lambda_{u_s}{}^{i})\, T_s{}^{i} + T_c{}^{i} + \overline{P_i[F]}\,(T_r{}^{i} - T_{c_2}{}^{i})} \tag{43}$$

$$P_{d_s}{}^{i} = e^{-\lambda_{d_s}{}^{i} T_s{}^{i}} \tag{44}$$

$$P_{u_s}{}^{i} = e^{-\lambda_{u_s}{}^{i} T_s{}^{i}} \tag{45}$$

$$\overline{P_i[G;t_{s_k}]} = \frac{\mu_1{}^{i}(1 - \beta^{i}) \left\{1 - (1 - \alpha^{i})\, P_{d_s}{}^{i} P_{d_c}{}^{i}\right\}}{\left\{1 - P_{d_s}{}^{i} P_{u_s}{}^{i} P_{d_c}{}^{i} P_{u_c}{}^{i} (1 - \alpha^{i})\right\} \left\{1 + [(1 - \alpha^{i} - \beta^{i})(1 - \mu_2{}^{i}) - (1 - \alpha^{i})\, P_{d_{c_2}}{}^{i}]\, P_{d_s}{}^{i} P_{d_{c_1}}{}^{i}\right\}} \tag{46}$$

$$\overline{P_i[F]} = \frac{(1-\beta^{i}) \left\{1 - (1-\alpha^{i})\, P_{d_s}{}^{i}\, P_{d_c}{}^{i}\right\}}{1 + \left\{(1-\alpha^{i}-\beta^{i})(1-\mu_2{}^{i}) - (1-\alpha^{i})\, P_{d_{c_2}}{}^{i}\right\} P_{d_s}{}^{i}\, P_{d_{c_1}}{}^{i}} \tag{47}$$

5.5.2.7 <u>Air Conditioning (Subsystem G)</u>

5.5.2.8 <u>Power Generation and Distribution (Subsystem H)</u>

These two subsystems are maintained independently of each other and the other subsystems. They are continuously monitored, hence;

$$A_i[\infty] = \frac{\mu_1^{i}\, e^{i}(\lambda_d^{i} + \alpha^{i})}{(\alpha^{i}+\lambda_d^{i}+\lambda_u^{i})\left\{e^{i}(\alpha^{i}+\lambda_d^{i}+\mu_1^{i}+\mu_3^{i}) + \alpha^{i}\mu_2^{i}+\lambda_d^{i}(\mu_1^{i}+\mu_2^{i}+\mu_3^{i})\right\}} \qquad (48)$$

where i is "G" or "H".

This may also be expressed as:

$$A_i[\infty] = \frac{\bar{t}_u^{\,i}\; P_{g/u}^{\,i}}{\bar{t}_u + \bar{t}_d} \qquad (49)$$

where

$$\bar{t}_d = \frac{1}{\mu_1 + \mu_2 + \mu_3} \qquad (50)$$

$$\bar{t}_u = \frac{1}{\mu_1 + \mu_2 + \mu_3}\left\{\left(\frac{\mu_1}{\lambda_d + \alpha} + \mu_3\right)\left(1 + \frac{\lambda_d}{e}\right) + \frac{\mu_2}{e}\right\} \qquad (51)$$

$$P_{g/u} = \frac{\mu_1}{(\lambda_d + \lambda_u + \alpha)\left\{\left(\frac{\mu_1}{\lambda_d + \alpha} + \mu_3\right)\left(1 + \frac{\lambda_d}{e}\right) + \frac{\mu_2}{e}\right\}} \qquad (52)$$

with appropriate superscripts on the parameters.

5.5.3 Apparent Availability

The expressions developed to this point yield true availability. To the casual observer, however, the apparent availability is given by,

$$A_u[\infty] = \frac{\bar{t}_u}{\bar{t}_u + \bar{t}_d} \tag{53}$$

$A_u[\infty] \triangleq$ apparent availability

$\bar{t}_u$ = mean time assigned to alert

$\bar{t}_d$ = mean time down in checkout and/or repair

Referring to the various subsystems;

$$A_u^A[\infty] = \frac{T^A - T_c^A}{T^A} \tag{54}$$

$$A_u^B[\infty] = \frac{T_s^B + T_c^B - (T_c^B)'}{T_s^B + \bar{t}_d^B} \tag{55}$$

$$A_u^{CDEF}[\infty] = \frac{T_s^C}{T_s^C + \bar{t}_d^C} \tag{56}$$

$$A_u^G[\infty] = \frac{\bar{t}_u^G}{\bar{t}_u^G + \bar{t}_d^G} \tag{57}$$

$$A_u^H[\infty] = \frac{\bar{t}_u^H}{\bar{t}_u^H + \bar{t}_d^H} \tag{58}$$

where $\bar{t}_u^{G,H}$ and $\bar{t}_d^{G,H}$ are defined by Equations (50) and (51).

5.6 Dependability

5.6.1 System Models

5.6.1.1 The System Dependability Matrix

The system dependability matrix accounts for that portion of the mission following receipt of the execution directive. In the case of an ICBM, the matrix must account for the following factors,

- Reliability aspects of communication and verification of the launch directive (P_C)
- Countdown (launch) reliability (P_L)
  - Repair potential on aborted launch attempt
- Flight reliability (P_f)

It is assumed that each of these factors is independent of the others, hence we write,

$$R = P_C P_L P_f \tag{59}$$

Then the elements d_{ij} of the dependability matrix $[D]$ become;

$$\begin{aligned} d_{ij} &= C^{10-i}_{10-j} \, R^{10-j} \, (1 - R)^{j-i} \quad i = 1, 2, \ldots 10; \quad j \geq i \leq 10 \\ d_{ij} &= 0, \; j < i \end{aligned} \tag{60}$$

These elements are the probabilities that exactly 10-j missiles of the squadron will survive countdown and flight; given that exactly 10-i are available.

5.6.1.2 Communication Reliability

This factor is unity by assumption.

5.6.1.3 Countdown Reliability

The probability of successfully completing countdown is assumed to be expressible in the form,

$$R_{CD}[t] = P_{CD}[\infty] \; P_{CD}[t/CD] \tag{61}$$

$P_{CD}[\infty]$ = The probability of successfully completing countdown without specific regard for the duration of countdown.

$P_{CD}[t/CD]$ = The probability of completing a countdown in time t or less; given that the countdown is successfully completed.

The probability of aborting a countdown is assumed to be expressible in the form,

$$1 - R_{CD}[t] = (1 - P_{CD}[\infty]) \; P_{CD}[t/\overline{CD}] \tag{62}$$

$P_{CD}[t/\overline{CD}]$ = The probability of completing a countdown in time t or less; given that the countdown is aborted

Provided that the launch site is in an apparently ready state (no known failures), it may enter countdown on demand. If countdown is successful, the missile will be launched in a time τ_c or less after the initiation of countdown. If the first countdown is aborted, it will enter repair at a time τ_c or less after the initiation of countdown. The repair will be effected at a mean rate μ_c and a second countdown will then be attempted. Failure to successfully complete the second countdown terminates the attempt sequence under the given assumptions. The possible state transitions are indicated in Figure B-1 of Appendix II.

From Appendix II,

$$
\begin{aligned}
P_L[\tau_c] = \Bigg\{ & A[\infty]\, P_{CD}[\infty]\, \left(1 - e^{-(\lambda_L+L)(\tau_c-\theta)}\right) \\
& + (1 - P_u[\infty])\, P_{CD}[\infty]\, \Bigg(1 - \frac{\mu_c}{(\mu_c-L-\lambda_L)}\, e^{-(\lambda_L+L)(\tau_c-\theta)} \\
& + \frac{L+\lambda_L}{(\mu_c-L-\lambda_L)}\, e^{-\mu_c(\tau_c-\theta)}\Bigg)\Bigg\}\, U[\tau_c-\theta] \\
& + A[\infty]\, P_{CD}[\infty] \Bigg\{ 1 - \frac{L(\lambda_L+L)}{(\mu_c-L)(\mu_c-L-\lambda_L)}\, e^{-\mu_c(\tau_c-2\theta)} \\
& + \frac{\mu_c L}{\lambda_L(\mu_c-L-\lambda_L)}\, e^{-(L+\lambda_L)(\tau_L-2\theta)} - \frac{\mu_c(L+\lambda_L)}{\lambda_L(\mu_c-L)}\, e^{-L(\tau_c-2\theta)}\Bigg\}\, U[\tau_c-2\theta] \\
& - A[\infty]\, P_{CD}^2[\infty] \Bigg\{ 1 - \frac{(L+\lambda_L)^2}{(\mu_c-L-\lambda_L)^2}\, e^{-\mu_c(\tau_c-2\theta)} \\
& - \frac{(\tau_c-2\theta)\,\mu_c(L+\lambda_L)}{(\mu_c - L - \lambda_L)}\, e^{-(L+\lambda_L)(\tau_c-2\theta)} \\
& - \frac{\mu_c\,(\mu_c-2L-2\lambda_L)}{(\mu_c-L-\lambda_L)^2}\, e^{-(L+\lambda_L)(\tau_c-2\theta)}\Bigg\}\, U[\tau_c-2\theta] \\
& + (P_u[\infty] - A[\infty])\, P_{CD}[\infty] \Bigg\{ 1 + \frac{(L+\lambda_L)}{(\mu_c-L-\lambda_L)}\, e^{-\mu_c(\tau_c-2\theta)} \\
& - \frac{\mu_c}{(\mu_c-L-\lambda_L)}\, e^{-(L+\lambda_L)(\tau_c-2\theta)}\Bigg\}\, U[\tau_c-2\theta]
\end{aligned}
\qquad (63)
$$

Assuming subsystem independence,

$$P_{CD}[\infty] = \prod_{i=A}^{H} R_{CD}^{\;i} \tag{64}$$

where

$R_{CD}^{\;i}$ = Reliability of the ith subsystem for the mean length of countdown.

The factor $P_{CD}[t/CD]$ is usually empirically determined from demonstration launch attempts.

5.6.1.4 Flight Reliability

The probability of successfully completing a flight is assumed to be expressible in the form,

$$P_f[t] = \prod_{i=A}^{H} R_f^{\,i}[t] \tag{65}$$

$R_f^{\,i}[t]$ = The reliability of the ith subsystem for a flight duration of the length t.

The $R_f^{\,i}[t]$ are given by the subsystem models.

5.6.2 Subsystem Reliability Models

5.6.2.1 Countdown Models

We shall illustrate the principle of subsystem modeling for only one typical subsystem, namely, the re-entry vehicle reliability in countdown.

The reliability functional block diagram for this subsystem is illustrated in Figure 6. The time line analysis of a standard countdown is shown in Figure 3. The appropriate failure rates are listed in Figure 5.

- Reliability Model
  - The reliability of the re-entry vehicle during countdown is given by the product of the reliabilities of the subsystem functions.
  - The reliability of a subsystem function is determined from the physical organization of its reliability functional blocks. By inspection of Figure 6,
  - $$R^{(A)} = R_{A.1} \cdot R_{A.2} \cdot R_{A.3} \cdot R_{A.4} \cdot R_{A.5} \cdot R_{A.7} \cdot R_{A.8} \cdot R_{A.9} \cdot [1 - (1 - R_{A.10.1})(1 - R_{A.10.2})] \cdot R_{A.12} \tag{66}$$
- Failure Distribution
  - The exponential function best describes the failure pattern of the reliability functional blocks of Figure 6.

$$\text{Typically:} \quad R_{A.3} = \epsilon^{-\lambda_{A.3} t_{A.3}} \tag{67}$$

$$\lambda_{A.3} = 0.8 \times 10^{-6} \quad \text{(from Figure 5)} \tag{68}$$

$$t_{A.3} = 5.5 \times 60 \quad \text{(from Figure 3)} \tag{69}$$

These results hold only for a standard countdown of fixed duration. When the countdown duration is variable a modified procedure must be used. Let $p_c[t_c \leq t]$ be the density distribution of the durations t_c of countdown. Let

$R_{A.i}[t_c - \gamma_i]$ = Reliability of ith RFB in a countdown of duration t_c.

γ_i = Non-operating time in countdown.

Let $R^A[t_c, \gamma] = f[R_{A.i}]$ be the total subsystem reliability function. Then,

$$R_{CD}{}^A[\infty] = \int_0^\infty p_c[t_c]\, R^A[t_c, \gamma]\; dt_c \tag{70}$$

and

$$R_{CD}{}^A[t/CD] = \frac{1}{R^A_{CD}[\infty]} \int_0^t p_c[t_c]\, R^A[t_c, \gamma]\, dt_c \tag{71}$$

5.6.2.2 Flight Models

These are handled in a manner completely analogous to the countdown. (See Section 5.6.2).

5.7 Design Capability

5.7.1 System Models

5.7.1.1 Capability Vector

Although a system may be available and function as designed during the mission, the system may still fail to accomplish its intent due to a variety of factors. In the case of an ICBM such factors may include,

- Communication interferences (noise, blanking, etc.)
- Ground vulnerability.
- Penetration probability.
- Propellant depletion probability.
- Guidance dispersion .
- Warhead yield (overpressure versus target hardness, area, etc.)

It is convenient to treat these factors from the standpoint of a design capability vector $\overline{C}$ in assessing system effectiveness. In the present example we shall restrict ourselves to a treatment of guidance dispersion and the target damage function, i.e., the probability of target damage expressed as a function of war head yield, miss distance and target hardness. This will illustrate the nature of $\overline{C}$, but it should be carefully noted that the situation depicted is a considerably oversimplified one.

We shall define a design capability vector $\overline{C}$ as follows,

$$\overline{C} = \begin{bmatrix} C_9 \\ C_8 \\ \cdot \\ \cdot \\ \cdot \\ C_1 \\ C_0 \end{bmatrix} \tag{72}$$

where the C_i are the expected number of sites destroyed; given that i missiles of the squadron are delivered to the target areas.

In the example chosen for illustration here it was assumed that the nine sites are targeted against three objectives, three missiles to a target. One successful war head detonation within a lethal radius R_L will destroy a target.

5.7.1.2 Per Unit Kill Probability

We define the per unit probability of target destruction as follows. Let the ensemble average of the probability of target destruction when one missile is targeted per objective be P_k,

$$P_k = \int_0^\infty p_D [R_0] \; f_g [R_0] \; dR_0 \tag{73}$$

where

$p_D [R_0]$ is a target damage density function.

$f_g [R_0]$ is the probability that the warhead is delivered within a distance R_0 of the target with successful warhead detonation and planned yield.

$$f_g [R_0] = P_{NPD}[r_0] \, P_P \, P_g [R_0] \, P_{WH} \tag{74}$$

P_{NPD} = Probability of no propellant depletion (a function of target range, launch error, propellant reserve, etc.)

P_p = Penetration probability (function of decoys and effectiveness of counter measures).

P_g = Guidance accuracy dispersion.

P_{WH} = Re-entry vehicle/war head yield dispersion.

TABLE I. EXPECTED KILL AS A FUNCTION OF TARGETING

No. of Detonated Missiles	No. of Missiles/Target			Expected Kill
9	111	111	111	$3[1-(1-P_k(x))^3]$
8	111	111	11	$2[1-(1-P_k(x))^3] + [1-(1-P_k(x))^2]$
7	111	111	1	$1.25[1-(1-P_k(x))^3] + 1.5[1-(1-P_k(x))^2]$
	111	11	11	$+ .25P_k(x)$
6	111	111	0	$\frac{5}{7}[1-(1-P_k(x))^3] + \frac{45}{28}[1-(1-P_k(x))^2]$
	111	11	1	$+ \frac{2}{14} P_k(x)$
	11	11	11	
5	111	11	0	$\frac{5}{14}[1-(1-P_k(x))^3] + \frac{10}{7}[1-(1-P_k(x))^2]$
	111	1	1	$+ \frac{15}{14} P_k(x)$
	11	11	1	
4	111	1	0	$\frac{1}{7}[1-(1-P_k(x))^3] + \frac{15}{14}[1-(1-P_k(x))^2]$
	11	11	0	$+ \frac{10}{7} P_k(x)$
	11	1	1	
3	111	0	0	$\frac{1}{28}[1-(1-P_k(x))^3] + \frac{9}{14}[1-(1-P_k(x))^2]$
	11	1	0	$+ \frac{45}{28} P_k(x)$
	1	1	1	
2	11	0	0	$.25[1-(1-P_k(x))^2] + 1.5P_k(x)$
	1	1	0	
1	1	0	0	$P_k(x)$
0	0	0	0	0

For example when [12/]

$$p_D[R_0] = \delta[R_0 - R_L] \tag{75}$$

$$R_L = \text{Lethal radius}$$

$$f_g[R_0] = 1 - e^{-R_0^2/2\sigma^2} \tag{76}$$

Then

$$\begin{aligned} P_k &= \int_0^\infty (1 - e^{-R_0^2/2\sigma^2})\, \delta[R_0 - R_L]\, dR_0 \\ &= 1 - e^{-R_L^2/2\sigma^2} \end{aligned} \tag{77}$$

For the assumed targeting plan described earlier, the C_i may be defined in terms of P_k as follows; [13/]

$$\begin{aligned}
C_9 &= 3[1 - (1-P_k)^3] \\
C_8 &= 2[1 - (1-P_k)^3] + 1 - (1-P_k)^2 \\
C_7 &= 1.25[1 - (1-P_k)^3] + 1.5[1 - (1-P_k)^2] + .25\, P_k \\
C_6 &= \tfrac{5}{7}[1 - (1-P_k)^3] + \tfrac{45}{28}[1 - (1-P_k)^2] + \tfrac{9}{14} P_k \\
C_5 &= \tfrac{5}{14}[1 - (1-P_k)^3] + \tfrac{10}{7}[1 - (1-P_k)^2] + \tfrac{15}{14} P_k \\
C_4 &= \tfrac{1}{7}[1 - (1-P_k)^3] + \tfrac{15}{14}[1 - (1-P_k)^2] + \tfrac{10}{7} P_k \\
C_3 &= \tfrac{1}{28}[1 - (1-P_k)^3] + \tfrac{9}{14}[1 - (1-P_k)^2] + \tfrac{45}{28} P_k \\
C_2 &= .25[1 - (1-P_k)^2] + 1.5\, P_k \\
C_1 &= P_k \\
C_0 &= 0
\end{aligned} \tag{78}$$

12/ See 5.7.2.1 and 5.7.2.2 for the development of these functions.

13/ See Table I.

5.7.2 Subsystem Models

We shall illustrate the development of the per unit kill probability by considering only two of the factors of P_k, namely, guidance dispersion, and war head effects. We shall assume that,

$$P_{NPD} = P_P = P_{WH} = 1 \tag{79}$$

5.7.2.1 Guidance Dispersion P_g

Consider the coordinate system of Figure 14. The variables y and $\bar{y}$ are the down range miss distance and bias error respectively. They are measured in the "plane of fire" along a line tangent to the earth at the planned impact point. The plane of fire is that plane which is defined by the three points; the earth's geometric center, the launch site, and the planned impact point.

The variables x and $\bar{x}$ are the cross range miss distance and cross range bias error, respectively. They are measured along a line orthogonal to the plane of fire and passing through the planned impact point.

It is usually assumed (or demonstrated) that $x - \bar{x}$ and $y - \bar{y}$ are independent, gaussian variables of zero mean. If σ_x and σ_y are the respective standard deviations of these variables, then the miss distance defined as,

$$R \triangleq \sqrt{x^2 + y^2} \tag{80}$$

is distributed as follows,

$$P[R \leq R_0] = \frac{1}{2\pi\sigma_x\sigma_y} \int_{-R_0}^{R_0} \int_{-\sqrt{R_0^2-y^2}}^{\sqrt{R_0^2-y^2}} e^{-\frac{1}{2}\left[\left(\frac{x-\bar{x}}{\sigma_x}\right)^2 + \left(\frac{y-\bar{y}}{\sigma_y}\right)^2\right]} dxdy \tag{81}$$

This function cannot be expressed in closed form for the general case although it is widely tabulated for specific choices of the variables.

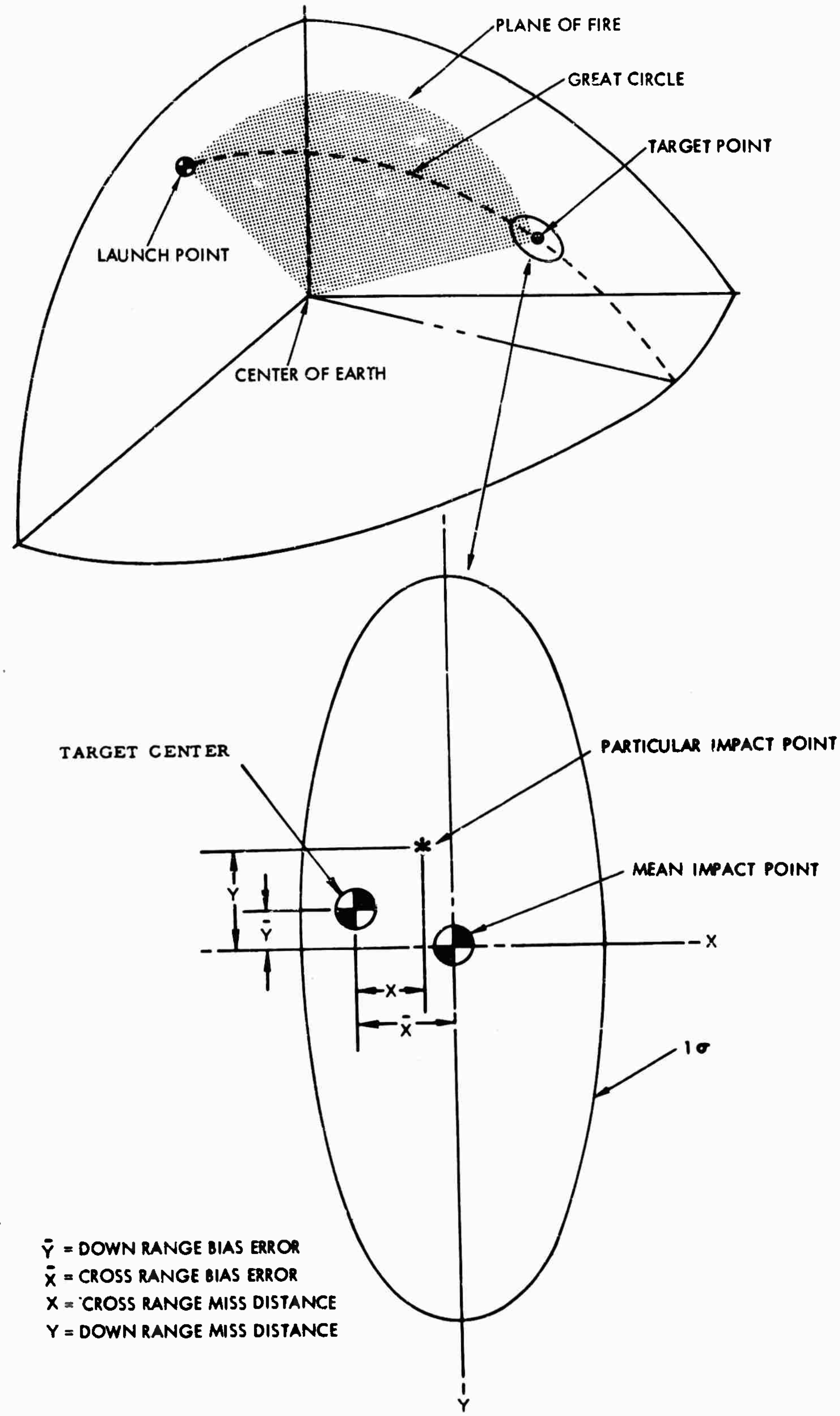


FIGURE 14. COORDINATE SYSTEM OF MISSILE IMPACT DISPERSION

For the present illustration, therefore, we shall set,

$$\sigma_x = \sigma_y \qquad \bar{x} = \bar{y} = 0 \tag{82}$$

Then using

$$dxdy = rdrd\theta \qquad x^2 + y^2 = r^2 \tag{83}$$

we arrive at the circular error function,

$$P[R \leq R_0] = 1 - e^{-R_0^2/2\sigma^2} \tag{84}$$

5.7.2.2 <u>Point Target Blast Damage Function</u>

We shall assume that the fleet is targeted upon point targets; that is, targets whose area is small compared to the total area of weapon effect. We shall also assume that of the three possible weapon effects,

- Heat
- Radiation
- Overpressure (blast damage)

only the latter has appreciable affect on the target.

Under these assumptions, the target damage function may be expressed as a function of three parameters

- Overpressure
- Target hardness
- Miss distance

In order to simplify the example, we shall assume the unity damage function; that is, the probability of target destruction P_D is given by

$$P_D[R_0 \leq R] = \begin{cases} 1 \; ; & 0 \leq R_0 \leq R_L \\ 0 \; ; & R_0 > R_L \end{cases} \tag{85}$$

where R_L is the so called "lethal radius." Note that the damage density function is,

$$p_D[R_O] = \delta[R_O - R_L] \tag{86}$$

6.0 DATA ACQUISITION

6.1 Specification of Data Elements

The basic information from the field which is required to estimate the parameters associated with availability and countdown reliability[14/] is a chronological listing of the time (number of maintenance cycles from repair to repair) by site and by subsystems; that is, the time (number of maintenance cycles) between a repair and the next no-go checkout. This data may be called apparent failure data.

In addition to this apparent failure data, it is necessary to record the total down time resulting from each apparent failure. Total down time per failure is defined as starting from the instant that the system is declared to be failed and continuing until reassignment of the system (subsystem) to alert.

Also it is necessary to record

- alert duration
- the duration of an all-go checkout
- the duration of a no-go checkout
- the duration of time from the start of a test to the point in checkout at which test decision is made.

The evaluation of test coverage requires a detailed failure analysis of a sample of rejected equipment. Such an analysis must be conducted against the Technical Order or test equipment which led to the rejection in order to ascertain if any of the failures which are noted during the failure analysis could have been missed and were, in fact, not responsible for the rejection.

14/ Certain portions of the launch sequence are not estimable from field data. Flight reliability is not estimable until a correlation has been made between ground environmental stresses and flight environmental stresses.

In brief, the following information is required in order to evaluate and improve the readiness of systems and subsystems:

- location (by site number and base).
- name (description) of checkout.
- name (description) of subsystem or items or components etc.
- time and data of assignment to EWO status.
- time and date of entry into checkout (failure).
- time and date of each problem encountered in checkout.
- description of each problem encountered in checkout.
- date of bench test of rejected parts.
- results of bench test.
- date of tear-down failure analysis of rejected parts.
- results of failure analysis.

6.2 Specification of Test Methodology

Because the ICBM fleets are operational, no discussion will be given of test methodology in the conceptual, definition, and acquisition phases. The current document will restrict itself to a discussion of a suitable test methodology for the operational phase of system life.

In principle, it is possible to obtain all the information required to implement the effectiveness model developed herein. All that is required is a complete system exercise. That is a practical impossibility.

The second best approach involves a combination of

- field testing
  - normal maintenance actions
  - impromptu survey inspections
  - special field exercises
- depot analysis
  - bench test results
  - tear down failure analysis
- special non-field tests

- recycle to depot
- VAFB launches
- lot acceptance tests

The point of view adopted with respect to field tests is one of practical necessity. The majority of the data is obtained in the course of normal maintenance actions and the impromptu inspections which are part of the current field practices.

The validity of this data as a true measure of the actual state of the fielded system is not an assumption which can be tolerated. Therefore, the parameter estimation methods of section 7.0 make no such assumptions.

However, those methods are workable only when they are supported by a limited number of "special field exercises" and the results of depot analyses and special tests not conducted in the field.

The basic test methodology to be employed in the field is as follows:

- the time between successive "periodic" inspections on each subsystem must be variable, involving at least three different standby periods.
- a limited number of checkouts must be repeated twice (three in a row) in a back to back to back fashion without regard for the intermediate tests results, i.e., repair is not initiated between tests. This is done, of course, only when safety permits.

6.3 Specification of Data Reporting System

Data on the ICBM status, maintenance actions, and countdown results are currently reported in the U-82, AFM 66-1, and U-86 data reporting systems respectively. A realistic approach to data collection requires a consideration of these systems.

The data elements specified in Section 6.1 above are currently obtained haphazardly by means of these reporting systems in accordance with fluctuating schedules, indicated failures, and impromptu inspections. This erraticness is frequently an asset to the calculation of parameters. However, a methodical variation in scheduled inspections is highly desirable from the standpoint of accuracy of parameter estimation. In any event, whether they are scheduled, or unscheduled, equipment inspections provide information. The limitations and uses of this information in estimating parameters are given in Table II.

TABLE II. DATA AVAILABLE FROM CURRENT AF DATA REPORTING SYSTEMS

Items of Information	U-82[1,6]	U-86[3,7]	AFM 66-1[8]
Location (by site number and base)	yes	yes	yes
Name of checkout	no[2]	yes	no[9]
Name of subsystem	yes[4]	yes	yes[4,10]
Time and date of assignment to EWO	yes	no	no
Time and date of entry to checkout	yes	yes	date of completion only
Time and date of each problem encountered in checkout	yes	yes	problem only[11]
Description of each problem encountered in checkout	yes[13]	yes	yes[11]
Date of bench test of rejected parts	no	no	date only[5]
Results of bench test	no	no	yes[5]
Date of tear-down failure[12] analysis of rejected parts	no	no	no
Results of failure analysis[12]	no	no	no

1 "by exception" reporting, i.e., only when condition takes site off alert.

2 was removed from data system recently (November 1963). Is scheduled for return to data system when checkout S.G.C. are detailed in -06 code books.

3 reports countdown only.

4. through Work Unit Code correlation only.

5 available for recoverable items only.

6 key punched for machine processing.

7 not keypunched.

8 partially keypunched.

9 requires support general code of -06 code and changes to T.O.-0020E-1.

10 no Work Unit Code when checkout only.

11 cannot correlate checkout AFTO forms and resulting maintenance problem.

12 can be directed by responsible AMA as a special task for problem areas.

13 problem -- frequently cannot be correlated to checkout data.

The following are currently known deficiencies of these data systems:

- Alert Status (up-time) is not reported. Instead, the site is assumed on alert unless specifically reported off alert (SAC Regulation 66-7, paragraph 4a). This leads to erroneous "up" time data; e.g., Walker I was assumed "on alert" for some time because no one reported it off alert. The data files were eventually corrected in this case. In October 1963, the F Series data for June and July had to be corrected because two weeks data from one squadron was received three months late (it had been assumed that the facility was on alert since it hadn't been reported otherwise). Solution: Go closed loop by having the site report the "Total Clock Hours on Alert" in columns 28 - 31 of SAC Form 127, each week. This will make it mandatory for each site to report the on-off status each week, and eliminate the guess work.

- There is no clearly defined relationship between the "status" categories and the portion of the system being tested. Also the same "status" category appears to be used for both a partial test and a complete test.

- Inconsistencies have been frequently noted between data reported on the SAC Form 127 (U-82) and the U-86. In one instance the U-86 reported 24 countdowns while the U-82 reported 41 countdowns for the same time period. Only ten of these countdowns were common to the two data systems.

- SAC Form 127 reports have had frequent occurrances of the following types of errors.

  - "Total clock hours off alert" for the system does not equal the sum of the individual alert degradation times (must be equal by definition of alert degradation time).

  - Time gaps exist between end of one category and the beginning of the next category.

- System down time periods overlap each other.
- System is returned to alert without accounting for all of the alert time.
- Cards submitted with incomplete information.

It is our understanding that steps are currently being taken by SAC to minimize these errors by having audits made at different reporting levels and in greater depth. The results of the SAC auditing procedures on the quality of the data has not yet been determined. (May 1964)

7.0 SPECIFICATION OF PARAMETER ESTIMATION METHODS

7.1 Point of View

There are two fundamental difficulties associated with field data with which all practical methods of parameter estimation must successfully cope; (1) the data usually arises from fortuitous system exercise as opposed to the careful planning associated with controlled experiments, and (2) the judgments made regarding the true state of the equipment are not necessarily complete, accurate, or timely.

The first difficulty is frequently a virtue in disguise; the irregularity of the schedule of system exercises may be utilized to obtain a separation of the time dependent system parameters from those which are time invariant. Indeed, a variation in the time between exercises is absolutely essential, although a planned variation would yield better parameter estimates with less data.

The second difficulty is surmountable only in a limited sense. Lack of test coverage may be compensated for by means of a tear down failure analysis program.

Mistaken judgments may be explicitly accounted for and estimated in a manner to be illustrated later. Lack of system exercise may be circumvented by recycle of field equipment to a base depot for special test on a scheduled basis.

On the other hand, failure to report, erroneous entries, inconsistent interpretations of events, and misinterpretation of codes, procedures, et al, require direct action.

In the light of the above comments, the basis of the parameter estimation methods to be developed here may be summarized as follows.

- a checkout passes or rejects an item of equipment; a rejection is not necessarily a failure, nor does a pass necessarily guarantee a non-failed system.
- test coverage is incomplete.
- failures are not necessarily observable at the instant of occurrance.

7.2 Techniques of Parameter Estimation from Field Data on Periodically Checked Systems

7.2.1 Introduction

Equipment operation for "periodically" checked systems is characterized by time variable modes of operation defined by the system maintenance policy. In general, equipment stress levels and failure distributions tend to differ between modes of operation.

Checkout survival probabilities are ordinarily best considered without specific regard for checkout duration since equipment tests undoubtedly imply a mixture of binomial (event) probabilities and time delayed exponentials. On the other hand, it is necessary to assume (or determine) a specific distribution of failures for the standby mode of operation in order to be able to compute the expected time to failure in that mode of operation. There are substantial grounds for selecting the exponential failure distribution when the population is a heterogeneous mixture, as is likely the case in complex equipment.

The specific techniques of parameter estimation developed here hinge on the variability of system maintenance policies/procedures. Time variable system parameters may be separated from time invariant system parameters by utilizing the results of checkouts performed subsequent to standby durations of irregular length. The results of a sequence of back to back checkouts performed without regard for the results of the earlier checkouts yields information of use in separating certain time invariant system parameters. In the following, the basic technique is illustrated.

7.2.2 Time Line Sequence of the Basic Periodic Maintenance Policy

Figure 8 illustrates a typical sequence of information as described by field data. The equipment is assigned to standby for a time duration T_s (which may vary widely between successive assignments), at the end of which it enters checkout. During the checkout the equipment is "tested" and passed or rejected. The set up, warmup, and test performance take a time T_{c_1} if the system is "go" and a time T_{c_1}' if the system is "no-go". These times are not necessarily equal nor constant from test to test. Demating, cleanup, etc., take a time T_{c_2} for a "go" test. This time may also be variable from test to test. A "no-go" leads to a repair time T_r which is defined as commencing at the first "no-go" indication and continuing until all necessary repairs and rechecks are completed.

7.2.3 The Concept of a Test

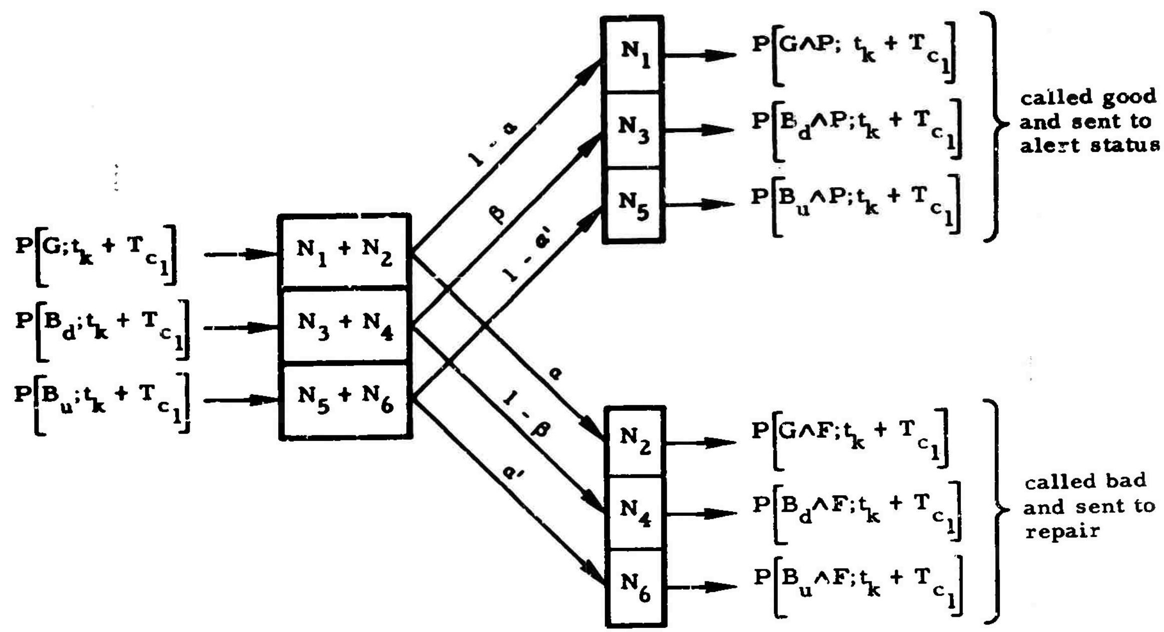
The point of view adopted here requires that the nature of a test be carefully delineated. Specifically, the test will have four basic properties. First, it will "pass" or "reject" an equipment at a specific point in time. That is, it is assumed that the test decision occurs at a well defined point in time. Second, it will on occasion "false alarm" a nonfailed characteristic of the equipment; i.e., it will call a good system bad. Third, the test will sometimes pass a failed characteristic. That is, it will not always reject a failure which it presumably is designed to detect. A test which does this too frequently is one of poor "quality". The quality of a test we shall define as the probability of detecting a failed system, given that the system is failed on or before the time that the point of test decision is reached. The fourth and last property of a test is "coverage". By coverage we shall mean that not all the possible equipment functional characteristics are examined by the test. It is assumed that the failure of such a characteristic cannot cause the test to reject the equipment, since its effect on the equipment is indeterminate from the test. However, we shall further assume that all equipments which have failed in this manner will be eventually rejected by either a false alarm or by the detection of a failure of an observed characteristic of the equipment.

These concepts may be readily formalized as follows. Assume that at the point of test decision the test acts instantaneously to partition failed equipments from nonfailed equipments as indicated by the partition of Figure 15. Consider the action of the test at this point in time on a total of $N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6$ equipments all of one kind. (Alternatively, one equipment may be tested N times in succession and may be either good or bad at each test.) Let $N_1 + N_2$ be the true number of nonfailed equipments. Let $N_3 + N_4$ equipments have one or more failures which are inherently detectable; that is, they contain failures among the essential characteristics which are examined by the test, but they will not necessarily all be detected because of "noise" or because some of the failures may be marginal. In addition let there be $N_5 + N_6$ equipments which contain no failures among the characteristics which the test examines, but which do contain one or more failures among the characteristics which are not examined. We may now make the following definitions if the total number of equipments being examined is very large

$$P[G;t_k + T_{c_1}] \doteq \frac{N_1 + N_2}{\sum_{i=1}^{6} N_i} = \text{probability of no failures of any kind at } t = t_k + T_{c_1}.$$

$$P[B_d;t_k + T_{c_1}] \doteq \frac{N_3 + N_4}{\sum_{i=1}^{6} N_i} = \text{probability of one or more failures of the "detectable in principle" type at } t = t_k + T_{c_1}.$$

$$P[B_u;t_k + T_{c_1}] \doteq \frac{N_5 + N_6}{\sum_{i=1}^{6} N_i} = \text{probability of one or more failures that are "inherently undetectable" at } t = t_k + T_{c_1}.$$



$$\alpha = \frac{N_2}{N_1 + N_2}$$

$$\alpha' = \frac{N_6}{N_5 + N_6}$$

$$1 - \beta = \frac{N_4}{N_3 + N_4}$$

FIGURE 15. ACTION OF A CHECKOUT (TEST) AT THE "POINT OF TEST DECISION," $t = t_k + T_{c_1}$ ON A SYSTEM THAT IS DISCRETELY MONITORED

These partitions of the true facts concerning the states of the equipments at the instant of test decision are shown in the left hand boxes of Figure 15. The right hand boxes of Figure 15 show the partition of the equipments which result as a consequence of test decisions. If α is the probability of calling any nonfailed characteristic bad, given that it is examined by the test, then we may write

$$\alpha \doteq \frac{N_2}{N_1 + N_2} = \text{false alarm probability}$$

and

$$\alpha' \doteq \frac{N_6}{N_5 + N_6} = \text{false alarm probability}$$

There is no physical reason to presume that $\alpha \neq \alpha'$, hence, we shall write without further justification

$$\alpha = \alpha'.$$

We further define a parameter β

$$1 - \beta \doteq \frac{N_4}{N_3 + N_4} = \text{probability of catching an "inherently detectable" failure.}$$

Notice that the question of test coverage has been accounted for in the above by introducing the notion of an "inherently undetectable failure."

7.2.4 The Probability of Passing a Test

We are now in a position to rigorously define the probability of passing a test. Let,

P_{d_g} = The probability that the inherently detectable equipment characteristics are good at the beginning of the standby period.

P_{d_s} = The probability that the inherently detectable equipment characteristics survive the standby period; given that they were unfailed at the time of assignment to standby.

$P_{d_{c_1}}$ = The probability that the inherently detectable equipment characteristics survive checkout up to the point of test decision; given that they are nonfailed at entrance to checkout.

P_{u_g} = The probability that the inherently undetectable equipment characteristics are good at the beginning of the standby period.

P_{u_s} = The probability that the inherently undetectable equipment characteristics survive the standby period; given that they were nonfailed at the time of assignment to standby.

$P_{u_{c_1}}$ = The probability that the inherently undetectable equipment characteristics survive checkout up to the point of test decision; given that they were nonfailed at entrance to checkout.

The probability that the equipment will be nonfailed at the point of test decision is,

$$P_{d_g} \; P_{d_s} \; P_{d_{c_1}} \cdot P_{u_g} \; P_{u_s} \; P_{u_{c_1}} \tag{87}$$

The probability that the equipment will be failed undetectably at the point of test decision is,

$$P_{d_g} \; P_{d_s} \; P_{d_{c_1}} \left(1 - P_{u_g} \; P_{u_s} \; P_{u_{c_1}}\right) \tag{88}$$

The proability that the equipment will be failed detectably at the point of test decision is,

$$1 - P_{d_g} \; P_{d_s} \; P_{d_{c_1}} \tag{89}$$

The probability of passing the test P[P] is, in general,

P[P] = (probability of being good at point of test decision) x (probability of no false alarm) + (probability of being failed undetectably at the point of test decision) x (probability of no false alarm) + (probability of being failed detectably at the point of test decision) x (probability of not catching the inherently detectable failure)

$$P[P] = P_{d_g} P_{d_s} P_{d_{c_1}} P_{u_g} P_{u_s} P_{u_{c_1}} (1 - \alpha)$$

$$+ P_{d_g} P_{d_s} P_{d_{c_1}} (1 - P_{u_g} P_{u_s} P_{u_{c_1}}) (1 - \alpha)$$

$$+ (1 - P_{d_g} P_{d_s} P_{d_{c_1}}) \beta$$

$$P[P] = \beta + P_{d_g} P_{d_s} P_{d_{c_1}} (1 - \alpha - \beta) \tag{90}$$

7.2.5 Maximum Likelihood Estimate of the Probability of Passing a Test

Let it be supposed that we have N field reports listing

- the duration of standby
- the results of the subsequent checkout

The expected number $(\bar{s})$ of these reports that indicate a test success is given by

$$\bar{s} = N\, P[P]$$

$$= N \left\{ \beta + P_{d_g} P_{d_s} P_{d_{c_1}} (1 - \alpha - \beta) \right\} \tag{91}$$

If s is the actual number of test successes in the N attempts, then the maximum likelihood estimate $\hat{P}[P]$ of $P[P]$ is,

$$s = N\, \hat{P}[P]$$

$$\hat{P}[P] = \frac{s}{N} \tag{92}$$

Unfortunately this estimate does not separate the variables.

7.2.6 Use of Variable Standby Duration as a Means of Variable Separation

The probabilities of Equation (90) may be separated if control can be exerted over some factor upon which the probabilities are dependent. A likely candidate is the duration of standby. If the failure distribution in standby is exponential, then

$$P_{d_s} = e^{-\lambda_{d_s} T_s} \tag{93}$$

and if T_s is variable, then λ_{d_s} may be estimated. For example, suppose that the field data can be subdivided into three groups for each of which T_s is constant but unequal between groups. Specifically, if the various values of T_s are T, $2T$, and $3T$, and if there are M_1, M_2, and M_3 reports in each group, then;

$$\frac{s_1}{M_1} = \beta + e^{-\lambda_{d_s} T} P_{d_{g_1}} P_{d_{c_1}} (1 - \alpha - \beta) \tag{94}$$

$$\frac{s_2}{M_2} = \beta + e^{-\lambda_{d_s} 2T} P_{d_{g_2}} P_{d_{c_1}} (1 - \alpha - \beta) \tag{95}$$

$$\frac{s_3}{M_3} = \beta + e^{-\lambda_{d_s} 3T} P_{d_{g_3}} P_{d_{c_1}} (1 - \alpha - \beta) \tag{96}$$

where s_1, s_2, and s_3 are the number of successful tests in each group and where $P_{d_{g_1}} = P_{d_{g_2}} = P_{d_{g_3}}$ if there is no regularity in the succession of T, $2T$, and $3T$ on any given equipment. Equations (94), (95), and (96) may be readily combined to give;

$$\frac{\frac{s_1}{M_1} - \frac{s_2}{M_2}}{\frac{s_2}{M_2} - \frac{s_3}{M_3}} = e^{+\hat{\lambda}_{d_s} T} = \frac{1}{P_{d_s}[T]} \tag{97}$$

provided that λ_{d_s}, s_1, s_2, and s_3 are not equal to zero.

So that, Step 1

$$\hat{\lambda}_{d_s} = \frac{1}{T} \ln \frac{\frac{s_1}{M_1} - \frac{s_2}{M_2}}{\frac{s_2}{M_2} - \frac{s_3}{M_3}} \tag{98}$$

and Step 2

$$\hat{\beta} = \frac{\frac{s_2}{M_2} - \frac{s_1}{M_1} \hat{P}_{d_s}}{1 - \hat{P}_{d_s}} \tag{99}$$

and Step 3

$$\hat{C} \triangleq P_{d_g} P_{d_{c_1}} (1 - \alpha - \beta) = \left(\frac{s_1}{M_1} - \hat{\beta} \right) \frac{1}{\hat{P}_{d_s}} \tag{100}$$

Thus, three different values of T_s in the ratios 1, 2, 3 allows a unique separation of the standby failure rate and the Type II statistical error of the test and a certain composite parameter. No further separation can be achieved by variation of the standby duration. The method just considered is, of course, hopelessly optimistic. It is too much to expect the nice neat ratios T, 2T, and 3T for T_s. The method must be generalized. This can be done. Let N_i be the number of test failures out of M_i attempts where $T_s = T_i$. Then it is reasonable to search for a value of λ_{d_s} which will minimize

$$G = \sum_{i=1}^{n} [F_i - N_i \{1 - \beta - P_{d_s}[T_i] P_{d_g} P_{d_{c_1}} (1 - \alpha - \beta)\}]^2 \quad (101)$$

That is, we seek that value of λ_{d_s} which minimizes the sum of the squares of the differences between the observed number of test failures and the predicted number of test failures.

The mathematical problem is to minimize the function G, regarded as a function of β, λ_{d_s}, and the composite parameter $P_{d_{c_1}} P_{d_g} (1 - \alpha - \beta)$ subject to the constraints

$$0 \leq \beta \leq 1$$

$$0 \leq P_{d_{c_1}} P_{d_g} (1 - \alpha - \beta) \leq 1 \quad (102)$$

$$\lambda_{d_s} \geq 0$$

A specific satisfactory approach to the solution of this problem is to keep the partial derivatives of G equal to zero and test the function values of G for a minimum while increasing λ_{d_s} from zero. It should be noted however, that at least three different values of the T_i are required, but they may be in any ratio one to another.

7.2.7 Use of Back to Back Checkouts in Effecting Separation of the Variables

Additional separation of the system parameters may be accomplished if a sequence of back to back checkouts is performed without regard for the intermediate test results. Consider, for example, the situation depicted in Figure 16.

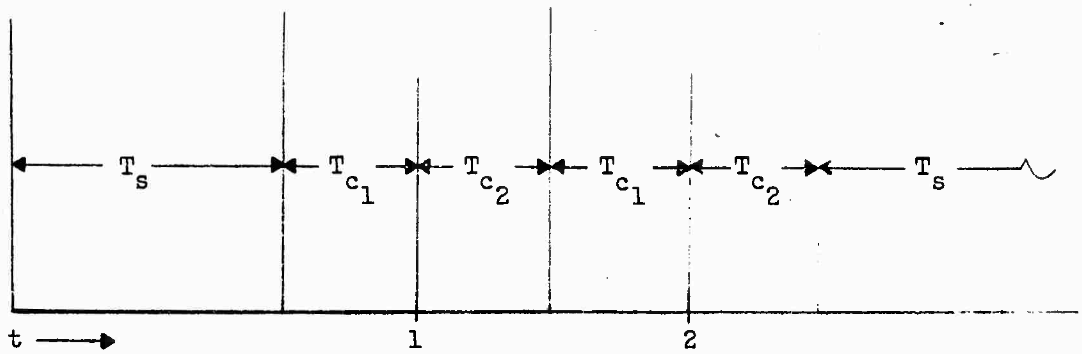


FIGURE 16. A SEQUENCE OF TWO-BACK-TO-BACK CHECKOUTS OUT OF STANDBY WITHOUT REGARD FOR THE FIRST TEST RESULTS

The probability of passing the first checkout is given by;

$$\frac{s_1}{M} = \beta + P_{d_g} P_{d_s} P_{d_{c_1}} (1 - \alpha - \beta) \tag{103}$$

If the test results of this checkout are ignored (but noted) and the equipment is immediately retested, the probability of passing the second checkout is given by;

$$\frac{s_2}{M} = \beta + P_{d_g} P_{d_s} P_{d_{c_1}}^2 P_{d_{c_2}} (1 - \alpha - \beta) \tag{104}$$

If we now utilize the estimate of β obtained from Equation (99), then we have Step 4.

$$\hat{x} = x \triangleq P_{d_{c_1}} P_{d_{c_2}} = P_{d_c} = \frac{\frac{s_2}{M} - \hat{\beta}}{\frac{s_1}{M} - \hat{\beta}} \tag{105}$$

It should be carefully noted that T_{c_2} and $P_{d_{c_2}}$ are assumed to be a necessary preliminary physical fact of retest.

Now consider Figure 17 which illustrates the situation of three checkouts back to back without regard for the intermediate test results.

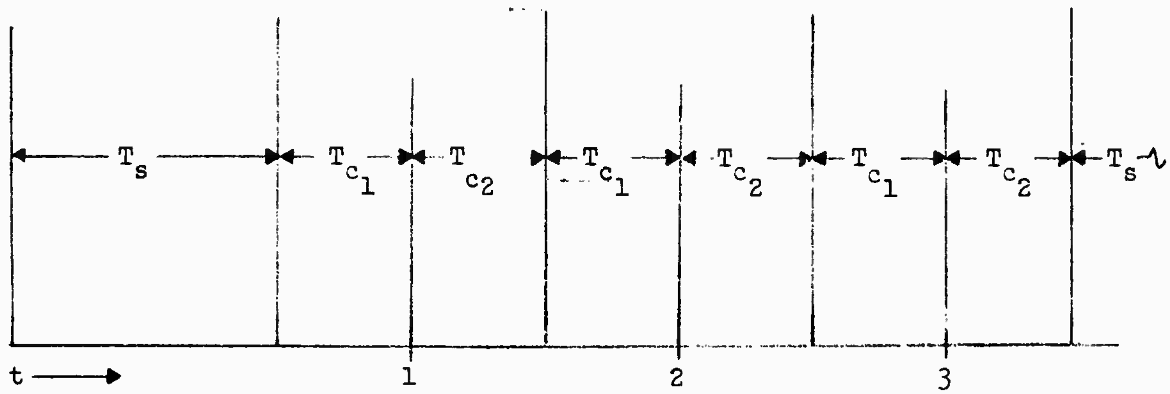


FIGURE 17. A SEQUENCE OF THREE CHECKOUTS BACK-TO-BACK WITHOUT REGARD FOR THE INTERMEDIATE TEST RESULTS

The probability of passing the first test is given by

$$\frac{S_1}{N} = \beta + P_{d_g} P_{d_s} P_{d_{c_1}} (1 - \alpha - \beta) \qquad (106)$$

The probability of passing the second test is given by,

$$\frac{S_2}{N} = \beta + P_{d_g} P_{d_s} P_{d_{c_1}}^{2} P_{d_{c_2}} (1 - \alpha - \beta) \qquad (107)$$

The probability of passing the third test is given by;

$$\frac{s_3}{N} = \beta + P_{d_g} P_{d_s} P_{d_{c_1}}^{\;3} P_{d_{c_2}}^{\;2} (1 - \alpha - \beta) \tag{108}$$

The probability of passing the first test and failing the second test is given by;

$$\frac{K}{N} = (1 - \beta) \left\{ \beta + P_{d_g} P_{d_s} P_{d_{c_1}} (1 - \alpha - \beta) \right\}$$

$$- P_{d_g} P_{d_s} P_{d_{c_1}}^{\;2} P_{d_{c_2}} (1 - \alpha)(1 - \alpha - \beta) \tag{109}$$

where N is the number of such successive attempts and K is the number which pass the first and fail the second attempt. Combining Equations (106), (107), and (108) we have <u>Step 5.</u>

$$\hat{y} = \hat{P_{d_{c_1}} P_{d_{c_2}}} = \hat{P_{d_c}} = \frac{\frac{s_2}{N} - \frac{s_3}{N}}{\frac{s_1}{N} - \frac{s_2}{N}} \tag{110}$$

and an alternative to <u>Step 2.</u>

$$\hat{\beta} = \frac{\frac{s_1}{N}\frac{s_3}{N} - \left(\frac{s_2}{N}\right)^2}{\frac{s_1}{N} - 2\frac{s_2}{N} + \frac{s_3}{N}} \tag{111}$$

and an alternative to Step 3.

$$\hat{x} = [\widehat{P_{d_g} P_{d_s} P_{d_{c_1}} (1 - \alpha - \beta)}] = \frac{(\frac{s_1}{M} - \frac{s_2}{M})^2}{\frac{s_1}{M} - 2\frac{s_2}{M} + \frac{s_3}{M}} \tag{112}$$

Combining these estimates with Equation (109) yields an estimate of α, Step 6.

$$\hat{\alpha} = 1 + \frac{\frac{K}{M} - (1 - \hat{\beta})(\hat{\beta} + \hat{x})}{\hat{x}\,\hat{P}_{d_c}} \tag{113}$$

Also note that,

$$[\widehat{P_{d_g} P_{d_s} P_{d_{c_1}}}] = \frac{\hat{x}}{1 - \hat{\alpha} - \hat{\beta}} \tag{114}$$

If the three checkouts are conducted immediately following repair then $P_{d_g} P_{d_s}$ is replaced with $(\mu_1 + \mu_3)$ so that

$$\hat{x} \longrightarrow [\widehat{(\mu_1 + \mu_3) P_{d_{c_1}} (1 - \alpha - \beta)}] \tag{115}$$

and

$$\left[\widehat{P_{d_g} P_{d_s} P_{d_{c_1}}}\right] \longrightarrow \left[\widehat{(\mu_1 + \mu_3) P_{d_s} P_{d_{c_1}}}\right] \tag{116}$$

If we make the assumption that

$$P_{d_{c_1}} = r P_{d_c} \tag{117}$$

Where r has a known or assumed value, then Step 7.

$$\widehat{\mu_1 + \mu_3} = \frac{[(\mu_1 + \mu_3)\hat{P}_{d_s} P_{d_{c_1}}]}{r\,\hat{P}_{d_c}\,\hat{P}_{d_s}} \tag{118}$$

or

$$\hat{P}_{c_g} = \frac{[P_{d_g}\,\hat{P}_{d_s}\,P_{d_{c_1}}]}{r\,\hat{P}_{d_c}\,\hat{P}_{d_s}} \tag{119}$$

7.2.8 The Role of Failure Analysis

Evaluation of μ_3 P_{u_s} P_{u_c} and λ_u requires a limited amount of "complete failure analysis" of field rejected items. This must be accomplished in conjunction with examination of the procedures used to accomplish field testing. By "complete failure analysis" it is meant that field rejected items are to be examined for every functional characteristic that is considered to be essential for proper operation of the equipment. In general, of course, one or more of the examined characteristics will indeed be failed. To determine whether any given failed characteristic falls in the "inherently undetectable" category it is necessary to answer a specific question: If this characteristic failed in the field, would field testing discover the failure? A negative answer implies that the failure is inherently undetectable in the field.

To quantify test coverage, which is a function of μ_3, λ_u, and P_{u_s} P_{u_c} two items of information are required:

- The number of maintenance cycles from previous replacement/repair to the current rejection.
- Failure analysis of the rejected item to determine the existence or non-existence of inherently undetectable failures. (Multiple failures of this type in a rejected item are counted as a single failure).

The specific data to be collected is as follows:

- The number (M) of items rejected from the field on the kth checkout subsequent to repair.
- The number (r) of the M rejected items which have an inherently undetectable failure.

In general, if the counts for the kth checkout are denoted by M [k] and r [k] and the counts for the jth checkout are denoted by M[j] and r [j], then;

$$P_{u_s} P_{u_c} = \left\{ \frac{M[k] - r[k]}{M[k]} \quad \frac{M[j]}{M[j] - r[j]} \right\}^{\frac{1}{k-j}} \tag{120}$$

For example, if we have information on items rejected after one (j=1) and two (k=2) maintenance cycles, then we have Step 8.

$$[P_{u_s} \hat{P}_{u_c}] = \frac{M[2] - r[2]}{M[2]} \quad \frac{M[1]}{M[1] - r[1]} \tag{121}$$

$$\hat{\lambda}_u \leq \frac{1}{T_s + T_c} \times \ln \frac{1}{[P_{u_s} \hat{P}_{u_c}]} \tag{122}$$

if information is available for several k and j, an average may be taken of the $P_{u_s} + P_{u_c}$ or the maximum likelihood value for the entire set of k may be used.

A conservative (pessimistic) estimate of the probability of leaving repair with an inherently undetectable failure (μ_3) is also readily found. We have, Step 9.

$$\hat{\mu}_3 \cong \frac{M[k] - r[k]}{M[k]} \left\{ [P_{u_s} \hat{P}_{u_c}] \right\}^{-k} \qquad k = 1, 2, \text{etc.} \tag{123}$$

The estimate of the probability leaving repair is now obtained from Step 10.

$$\hat{\mu}_1 = [(\mu_1 \hat{+} \mu_3] - \hat{\mu}_3 \tag{124}$$

7.2.9 Estimation of $P_{CD}[\infty]$

Actual launch attempts are not likely to be initiated from operational sites for a variety of reasons. Therefore, a secondary source of data must be used. Simulated countdowns in the field may be used as this source, the data being processed in accordance with Step 5 of the previous section.

However, it must be recognized that the estimate obtained in this way must be corrected by data on pyrotechnics and the engines.

7.2.10 Estimation of $P_L[t/L]$

The probability of completing a launch in time t or less; given that it is completed successfully may be determined directly from field data.

Order the M observed durations t_L of successful attempts in increasing order,

$$t_{L_1} < t_{L_2} < t_{L_3} < \ldots < t_{L_M}$$

Associate with each t_{L_i} an empirical probability number $\left\{\frac{M+1-i}{M+1}\right\}$. Plot the number pairs

$$\left\{ t_{L_i} ; \frac{M+1-i}{M+1} \right\}$$

on any convenient type of graph paper. This plot is the empirical probability that a successful launch attempt will take a time t or greater to complete. One minus the plot is the empirical probability that it will take a time t or less to complete a launch; given that it is completed successfully.

7.3 Summary of Estimates in Terms of Test Methodology

Table III summarizes the conditions of test and the parameters which may be estimated from the tests. It should be noted that there is a considerable degree of useful overlap between the various tests. It should also be noted that $P_{d_{c_1}}$ and $P_{d_{c_2}}$ are not separable by any of the test methodologies considered.

TABLE III. PARAMETERS THAT MAY BE ESTIMATED

Test Methodology	α	β	P_{d_s}	P_{d_c}	$\mu_1 P_{d_{c_1}}(1-\alpha-\beta)$	$\mu_1 P_{d_s} P_{d_{c_1}}(1-\alpha-\beta)$	$\mu P_{d_{c_1}}$	$P_{d_{c_1}}$	$\mu_1+\mu_3$	$P_u P_{d_c}$	μ_1	μ_2	μ_3
. 3 different T_s test results on first c/o after repair		x	x		x								
. 1 value for T_s test on first c/o after repair . 3 back to back c/o immediately after repair	x	x		x		x							
. 3 back to back c/o immediately after repair	x	x		x			x						
. 2 different T_s number of cycles to re-entry to repair . 2 back to back c/o immediately after repair	x	x	x	x					x				x
. failure analysis										x			
. failure analysis . 2 T_s . 2 bb checkout . assume $\frac{P_{d_{c_1}}}{P_{d_c}} = r$	x	x	x	x				x	x	x	x	x	x

8.0 MODEL EXERCISE

8.1 Numerical Evaluation

It is assumed that those steps upon which we have touched only briefly (1.0 through 7.0) have been successfully completed. We are at this point in possession of estimates of all system parameters and are ready to exercise the model.

8.1.1 List of Parameter Values

It is assumed that tasks 1.0 through 7.0 have been accomplished with the results listed in Table IV. These are initial best estimates for the system.

8.1.2 Availability

8.1.2.1 Steady State Values

Table V lists the results of the availability model exercise by subsystem (and by system) for the initial best estimates tabulated in Table IV. It should be noted that the true availability is considerably less than the apparent availability. The availability vector, based on the true availability is given by;

$$\overline{A_s[\infty]} = \begin{bmatrix} 5 \times 10^{-6} \\ 1.39 \times 10^{-4} \\ 1.59 \times 10^{-3} \\ 1.052 \times 10^{-2} \\ 4.49 \times 10^{-2} \\ .1278 \\ .242 \\ .296 \\ .210 \\ .067 \end{bmatrix} \qquad (125)$$

TABLE IV. PA

SUBSYSTEM	AVAILABI								
	α	β	μ_1	μ_2	μ_3	τ_r	τ_{c_1}	τ_c	
A	—	—	0.99	—	—	—	—	1	
B	0.1	0.01	0.9	0.1	0	0.333	0.04	0.04	6
C	0.05	0.01	0.99	0.005	0.005	0.1	0.05	0.1	
D	0.01	0.01	0.985	0.01	0.005	3	0.05	0.1	
E	0	0	1.0	0	0	10	0.1	0.2	
F	0.01	0	1.0	0	0	1	0.5	0.05	
G	0.002	—	1*	0	0	—	—	—	
H	0.1	—	1*	0	0	—	—	—	
(CDEF)	← SAME AS INDIVIDUAL SUBSYSTEMS →						C = 0.1 E=0.3 D = 0.1 F = 0.6	0.6	

MISCELLANEOUS SYSTEM PARAMETERS

- TCTO

  $\frac{1}{\lambda_o} = 60$

  $\frac{1}{\mu_o} = 1$

- WARHEAD YIELD

  $R_{L/\sigma} = 1$

*UNITS ARE DAYS, PER DAY, AND PROBABILITY

TABLE IV. PARAMETER VALUES*

	AVAILABILITY										COUNTDOWN				FLIGHT	
	r_{c_1}	T_c	$(T_c)'$	$P_{D_{c_2}}$	λ_{d_s}	λ_c	$\lambda_{u_s}, \lambda_{u_i}$	e	λ_d	T_s	λ_{CD}	L	μ_C	θ	T_f	λ_f
	—	1	—	—	9.13×10^{-4}	—	—	—	—	364		↑	↑	↑	0.125	↑
3	0.04	0.04	6.94×10^{-3}	1	0.02	0.2	0	—	—	10					0.125	
	0.05	0.1	—	0.9	0.02	0.2	0.002	—	—	—					0.125	
	0.05	0.1	—	1.0	0.005	0.05	0.0005	—	—	—		0.125	3	0.0625	0.125	
	0.1	0.2	—	1.0	0.0005	0.0005	0	—	—	—	SEE λ_c				0.125	
	0.5	0.05	—	1.0	0.002	0.02	0	—	—	—					—	$3\lambda_c$
	—	—	—	—	—	—	0	24	0.02	—					—	
	—	—	—	—	—	—	0.01	24	0.2	—		↓	↓	↓	—	↓
→	C = 0.1 E=0.3 D = 0.1 F = 0.6	0.6	—	← SAME AS INDIVIDUAL SUBSYSTEMS →				—	—	30					—	

2

TABLE V. AVAILABILITY OF THE SYSTEM BY SUBSYSTEM, et al.

Subsystem	$A^i[\infty]$	$A_u[\infty]$	T_s (days)
TCTO	0.9 (84)	0.984	-
G	0.9 (78)	0.978	-
B	0.8 (50)	0.991	10
A	0.8 (41)	0.997	364
H	0.7 (40)	0.771	-
<CDEF>	0.5 (11)	0.962	30

System true availability $A_s[\infty]$ = 0.2 (60)

System apparent availability $A_{u_s}[\infty]$ = 0.7 (05)

8.1.2.2 Augmented Availability

Numerical integration of Equation (32) provides a curve of augmented apparent availability $P_u[\tau]$.

When this curve is multiplied by the factors indicated in Equation (34), the true augmented availability is obtained. These two curves are shown in Figure 18. It will be noted that the alert status is not markedly improved by the change in policy ($A_s[\infty]$ = .260 as opposed to $A_s[\tau]\Big|_{\substack{max \\ (\tau \cong 1 \text{ day})}} \cong .31$).

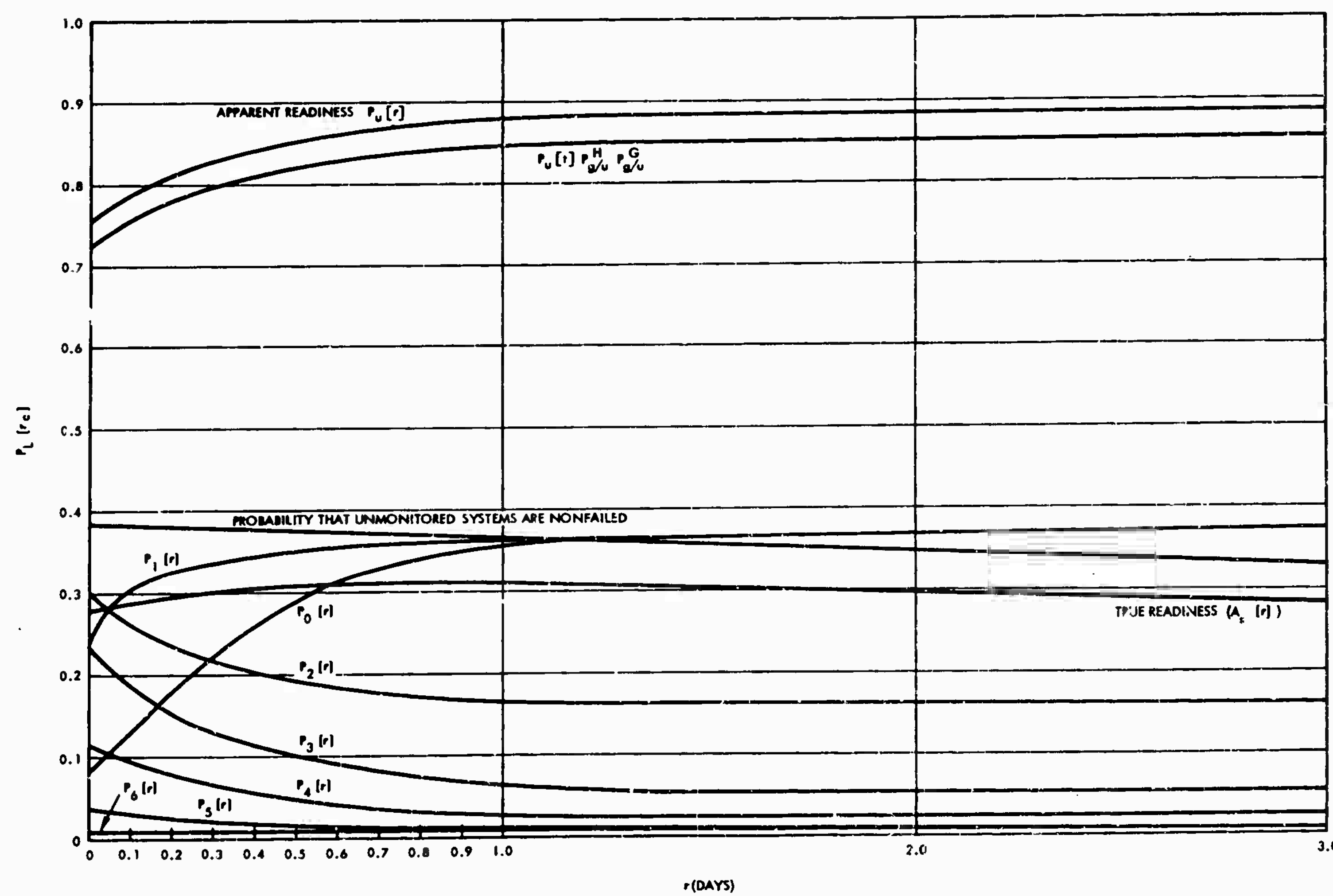


FIGURE 18. AUGMENTED AVAILABILITY

8.1.3 Countdown Reliability

We shall demonstrate the technique of countdown reliability prediction for the re-entry vehicle only. All other subsystems are handled in an analogous manner. Referring to Figure 6 we have;

$$\begin{aligned} R(t) = e^{-3.8 \times 10^{-6} t} & \left\{ U(t) - U(t-4)\left(1 - e^{-1.6 \times 10^{-6}(t-4)}\right) \right\} \\ & \times \left\{ U(t) - U(t-6)\left(1 - e^{-15 \times 10^{-6}(t-6)}\right) \right\} \\ & \times \left\{ U(t) - U(t-6.5)\left(1 - e^{-10^{-6}(t-6.5)}\right) \right\} \\ & \times \left\{ U(t) - U(t-7.5)\left(1 - e^{-20 \times 10^{-6}(t-7.5)}\right)^2 \right\} \end{aligned} \tag{126}$$

Where t is in seconds.

From the weapon system summary, the density distribution of the duration of countdown is given by,

$$p(t_{CD}) = L\, e^{-L(t-9)}\, U(t-9) \tag{127}$$

$$\frac{1}{L} = \text{twenty minutes}$$

Then,

$$R(t_{CD}) = \int_0^{\infty} p(t_{CD}) \; R(t_{CD}) \, d\, t_{CD} = 0.9(53) \qquad (128)$$

Similarly, under the assumption that checkout failure rates hold for countdown, we have the results shown in Table VI. The figures shown are best estimates. The last two digits are not considered to be significant but are retained to reduce round off errors in calculating the net countdown reliability. Assuming subsystem independence;

$$P_{CD}[\infty] = \prod_{i=A}^{H} R_{CD}^{\;i} = 0.9(44) \qquad (129)$$

TABLE VI. SUMMARY OF COUNTDOWN RELIABILITY PREDICTION BY SUBSYSTEM

Subsystem Designator	Subsystem Countdown Reliability ($R_{CD}^{\;i}$)
A	0.95(300)
B	0.99(720)
C	0.99(720)
D	0.99(935)
E	1.00(0000)$^{-}$
F	0.99(975)
G	0.99(975)
H	0.99(720)

8.1.3 Flight Reliability

As a result of ground tests conducted under flight similar conditions of vibration and temperature, it is estimated that the flight stresses exceed normal checkout stresses by a factor of three, except for structure and propulsion which are markedly greater. Performing calculations similar to those illustrated for the re-entry vehicle in countdown we have the results tabulated in Table VII. Assuming subsystem independence,

$$P_f = \prod_{i=a}^{E} R_f^i = 0.7\ (84) \tag{130}$$

TABLE VII. SUMMARY OF FLIGHT RELIABILITY PREDICTION BY SUBSYSTEM

Subsystem Designator	Subsystem Flight Reliability (P_f^i)
A	0.9 (20)
B	0.9 (52)
C	0.9 (71)
D	0.9 (89)
E	0.9 (32)

8.1.5 Dependability Matrix

The components of the dependability matrix are given by,

$$d_{ij} = C_{10-j}^{10-i}\ R^{10-j}\ (1-R)^{j-i};\quad i = 1, 2, \ldots, 10\ \ j \geq i \leq 10$$
$$d_{ij} = 0\ ;\ j < i \tag{131}$$

where for the specified reaction time of two and one half hours we have;

$$R \cong P_{CD}[\infty]\ P_f = 0.9\ (44) \times 0.7\ (84)$$
$$= 0.7\ (40) \tag{132}$$

For shorter time periods and permitting only one countdown attempt,

$$R(t) = 0.740\left(1 - e^{-\frac{(t-9)}{30}}\right) u[t-9] \tag{133}$$

This function is plotted in Figure 19a.

If two countdown attempts are permitted, and if the site survives enemy counter measures, then the probability of launch is given by,

$$\begin{aligned} P_L[\tau_c] = &\left\{0.524 - .234\, e^{-3.18(\tau_c-.15)} - .290\, e^{-.125(\tau_c-.15)}\right\} \\ &\times U[\tau_c-.15] + \left\{0.434 - .453\, e^{-.125(\tau_c-.3)} - .131\, e^{-3.18(\tau_c-.3)}\right. \\ &\left. + .189\, e^{-3(\tau_c-.3)} - .030(\tau_c-.3)\, e^{-3.18(\tau_c-.3)}\right\} U[\tau_c-.3] \end{aligned} \tag{134}$$

This function is plotted in Figure 19b.

It should be carefully noted that it has been necessary to combine countdown with the readiness vector in order to perform this computation. Since this violates the original intent of this memorandum, Equation (134) will not be used.

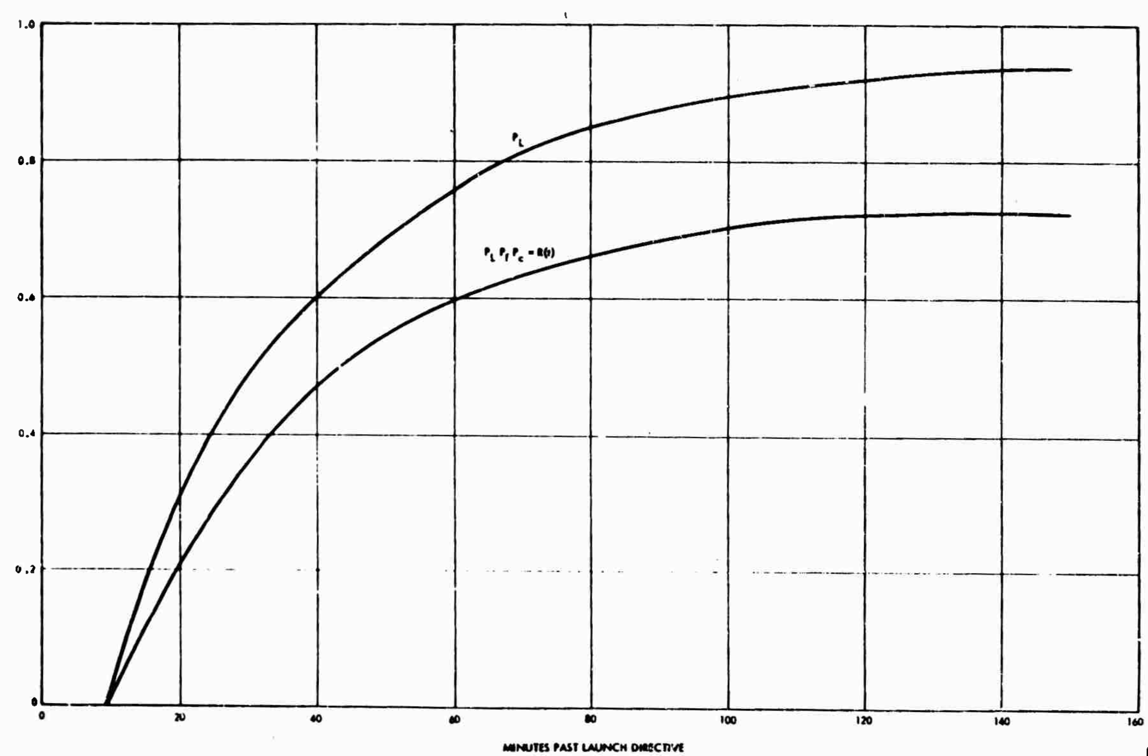


FIGURE 19a. LAUNCH RELIABILITY WHEN ONLY ONE ATTEMPT IS PERMITTED

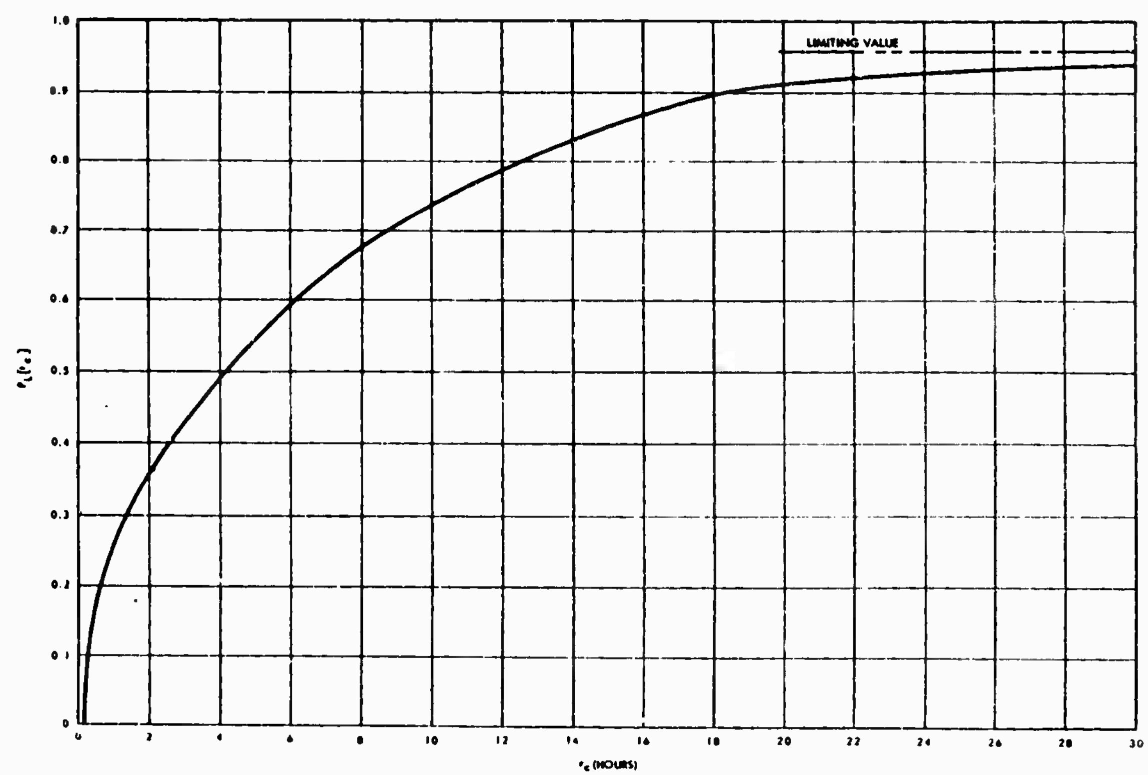


FIGURE 19b. LAUNCH PROBABILITY WHEN TWO ATTEMPTS (WITH REPAIR OF ABORTS) IS PERMITTED

The reliability matrix based on Equation (134) becomes;

$$[D] = \begin{bmatrix} .0666 & .2106 & .2958 & .2424 & .1277 & .0448 & .0105 & .0016 & .0001 & .0000 \\ 0 & .0900 & .2529 & .3108 & .2183 & .0958 & .0269 & .0047 & .0005 & .0000 \\ 0 & 0 & .1216 & .2990 & .3150 & .1844 & .0647 & .0136 & .0016 & .0001 \\ 0 & 0 & 0 & .1643 & .3463 & .3040 & .1423 & .0375 & .0053 & .0003 \\ 0 & 0 & 0 & 0 & .2221 & .3899 & .2738 & .0962 & .0169 & .0012 \\ 0 & 0 & 0 & 0 & 0 & .3000 & .4214 & .2220 & .0520 & .0046 \\ 0 & 0 & 0 & 0 & 0 & 0 & .4054 & .4270 & .1500 & .0176 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .5477 & .3847 & .0675 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .7401 & .2599 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{135}$$

8.1.6 Capability

The best estimate of the standard deviation of miss distance is one mile. The lethal radius for the targets under consideration is also one mile, and a unity damage function is considered to be a reasonable approximation to the weapon effects. Therefore, the per unit probability of kill is,

$$P_K = 1 - e^{-R_L^2/2\sigma^2} \tag{136}$$

$$= 1 - e^{-.5} = 0.394$$

Hence, the capability vector is from Equation (78)

$$\overline{C} = \begin{bmatrix} 2.3340 \\ 2.1890 \\ 2.0205 \\ 1.8263 \\ 1.6043 \\ 1.3522 \\ 1.0679 \\ 0.7492 \\ 0.3940 \\ 0.0000 \end{bmatrix} \tag{137}$$

8.1.7 Expected Kill (E)

The expected kill is given by,

$$E = \overline{A}' \; [D] \; \overline{C} \tag{138}$$

where $\overline{A}$ is defined by Equation (125), $[D]$ is defined by Equation (135) and $\overline{C}$ is defined by (137). Performing the indicated multiplication,

$$E = 0.332 \tag{139}$$

There is, therefore, less than a fifty-fifty chance of destroying one of the three targets on which the squadron of nine ICBM's is targeted.

III. APPLICATION OF MODEL RESULTS

1.0 COMPARATIVE SYSTEMS ANALYSIS

1.1 Comparison of Best Estimate with S.O.R.

Table VIII lists the system estimates obtained from the model. Table IX gives the apportionment of readiness and reliability from the S.O.R.

1.2 Flight Reliability Ranked by Subsystem

Table X lists the flight reliability of the subsystems in order of increasing reliability.

1.3 Countdown Reliability Ranked by Subsystem

Table XI lists the countdown reliability of the subsystems in order of increasing reliability.

1.4 Availability Ranked by Subsystem

Table V lists the availability of the subsystems in increasing order.

TABLE VIII. SOR REQUIREMENTS AND MODEL OUTPUTS

Parameter	SOR Requirements		Model Output
	Min. Accept.	Obj. Value	
A_s	0.5	0.9	0.260
P_{CD}	0.8	0.95	0.9 (44)
P_f	0.7	0.9	0.7 (84)
P_k	0.8	0.9	0.394

TABLE IX. SUBSYSTEM APPORTIONMENT AGAINST SOR

Parameter		SOR	Equal Partition
Availability	Min.	.5	0.9259
(9 subsystems)	Obj.	.9	0.9884
Countdown Reliability	Min.	.8	0.9657
(9 subsystems)	Obj.	.95	0.9943
Flight Reliability	Min.	.7	0.9312
(5 subsystems)	Obj.	.9	0.9791

TABLE X. FLIGHT RELIABILITY BY SUBSYSTEM

Subsystem	P_f^i
Re-entry Vehicle	0.9(20)
Structure	0.9(32)
Guidance	0.9(52)
Autopilot	0.9(71)
Propulsion	0.9(89)

TABLE XI. COUNTDOWN RELIABILITY BY SUBSYSTEM

Subsystem	P_{CD}^{i}
Re-entry Vehicle	0.95(300)
Guidance	0.99(720)
Autopilot	0.99(720)
Power generation and distribution	0.99(720)
Propulsion	0.99(935)
Air conditioning	0.99(975)
Overhead door	0.99(975)
Structure	1.0000⁻

2.0 Parameter Variation Study on Availability

Examination of the availability vector, Equation (125), the capability vector (137) and Tables VIII, IX, X, and XI leads to the conclusion that system availability and per unit probability of kill are weak as compared to system reliability. Accordingly, we institute a parameter variation analysis of these two factors in order to assess the potential for system improvement. We shall only perform a limited investigation here, stressing the importance of the proper checkout periodicity for availability and the effect of guidance accuracy on unit kill probability.

2.1 Subsystem Availability

2.1.1 Subsystem A

Figure 20 illustrates how the availability of the re-entry vehicle varies as a function of the replacement cycle length T^A. A substantial gain in

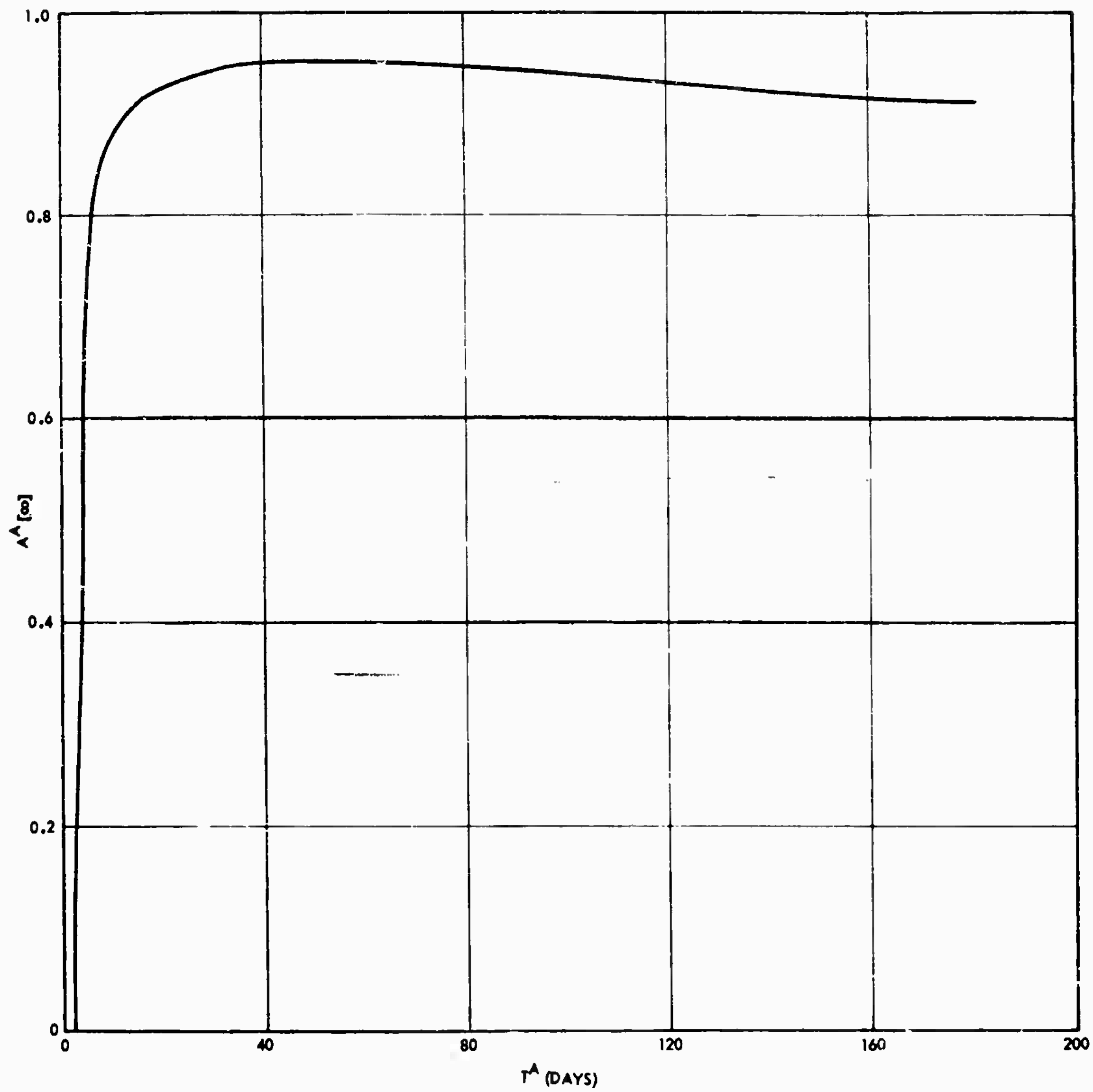


FIGURE 20. AVAILABILITY OF RE-ENTRY VEHICLE AS A FUNCTION OF THE REPLACEMENT INTERVAL

availability can be achieved by recycle of the re-entry vehicle every forty to fifty days as opposed to the planned recycle of one year.

2.1.2 Subsystem B

Figure 21 illustrates the variation of guidance availability as a function of standby status duration T_s^B. The optimum standby interval is of the order of 2.5 to 3.0 days as opposed to the planned duration of ten days.

2.1.3 Subsystems CDEF

Figures 22 through 26 illustrate the potential increase in availability of Subsystems C, D, E, and F. It is clear that Subsystem C and D should not be checked as infrequently as thirty days. Specifically, we have the following optimum standby periods for maximum availability.

TABLE XII. OPTIMUM STANDBY DURATION FOR SUBSYSTEMS C, D, E, F

Subsystem	(T_s) Alert Duration (in days)	A^1 [∞] Availability
C	3	.830
D	3-5	.882
E	30	.982
F	10	.982

2.1.4 Subsystem G

Subsystem G requires little or no improvement.

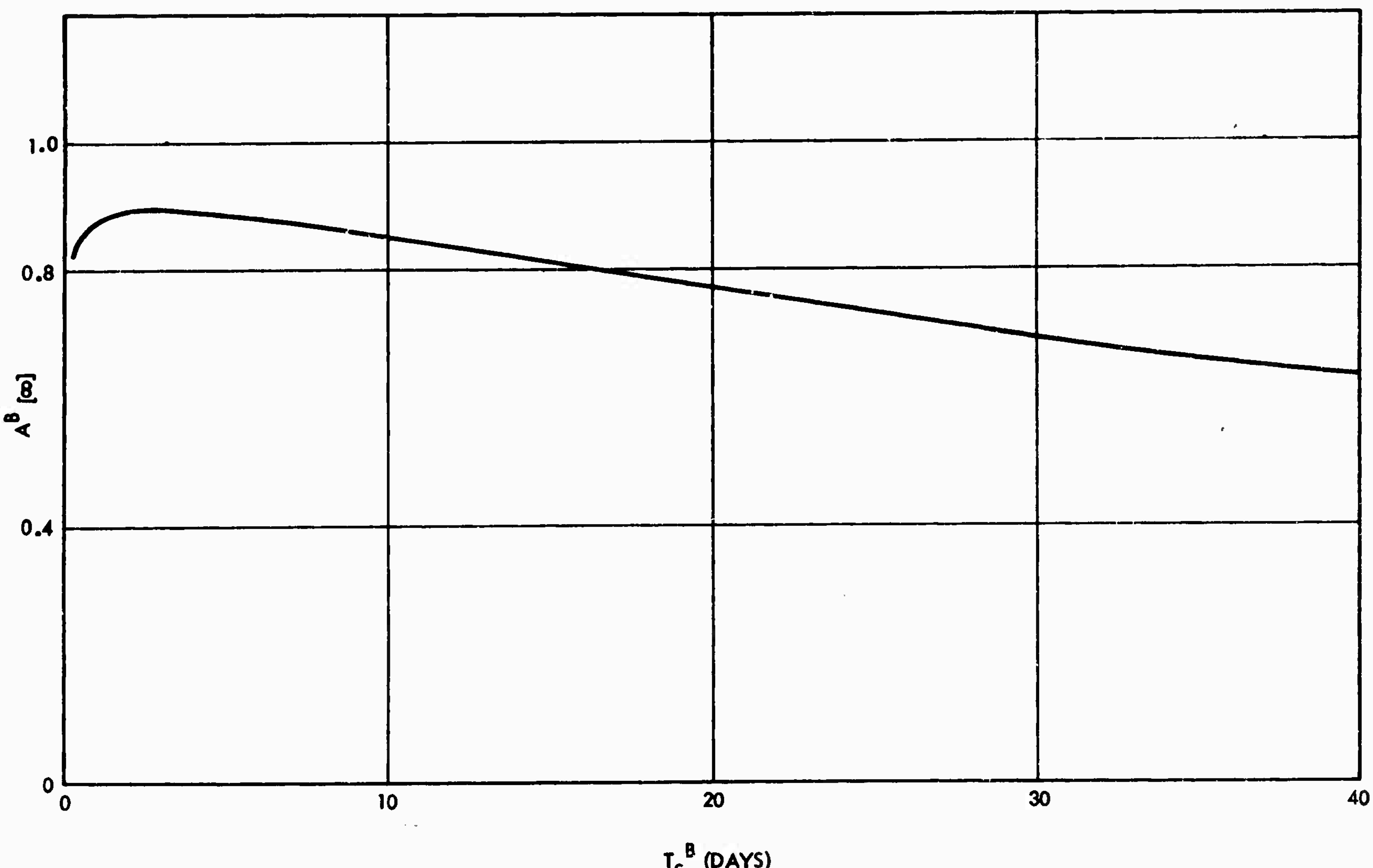


FIGURE 21. AVAILABILITY OF GUIDANCE SYSTEM AS A FUNCTION OF THE DURATION OF ALERT STATUS

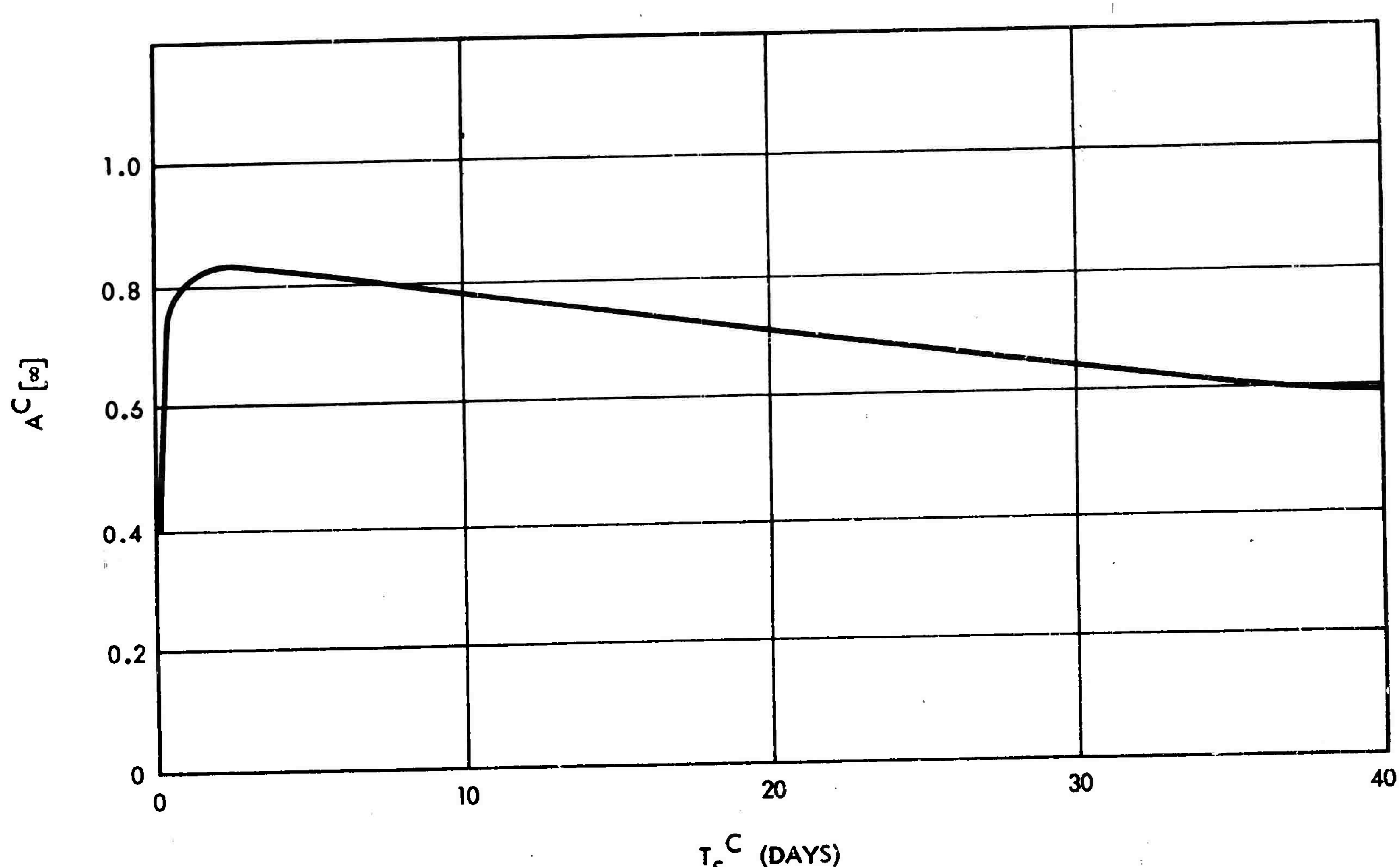


FIGURE 22. AVAILABILITY OF THE AUTOPILOT AS A FUNCTION OF THE DURATION OF ALERT STATUS

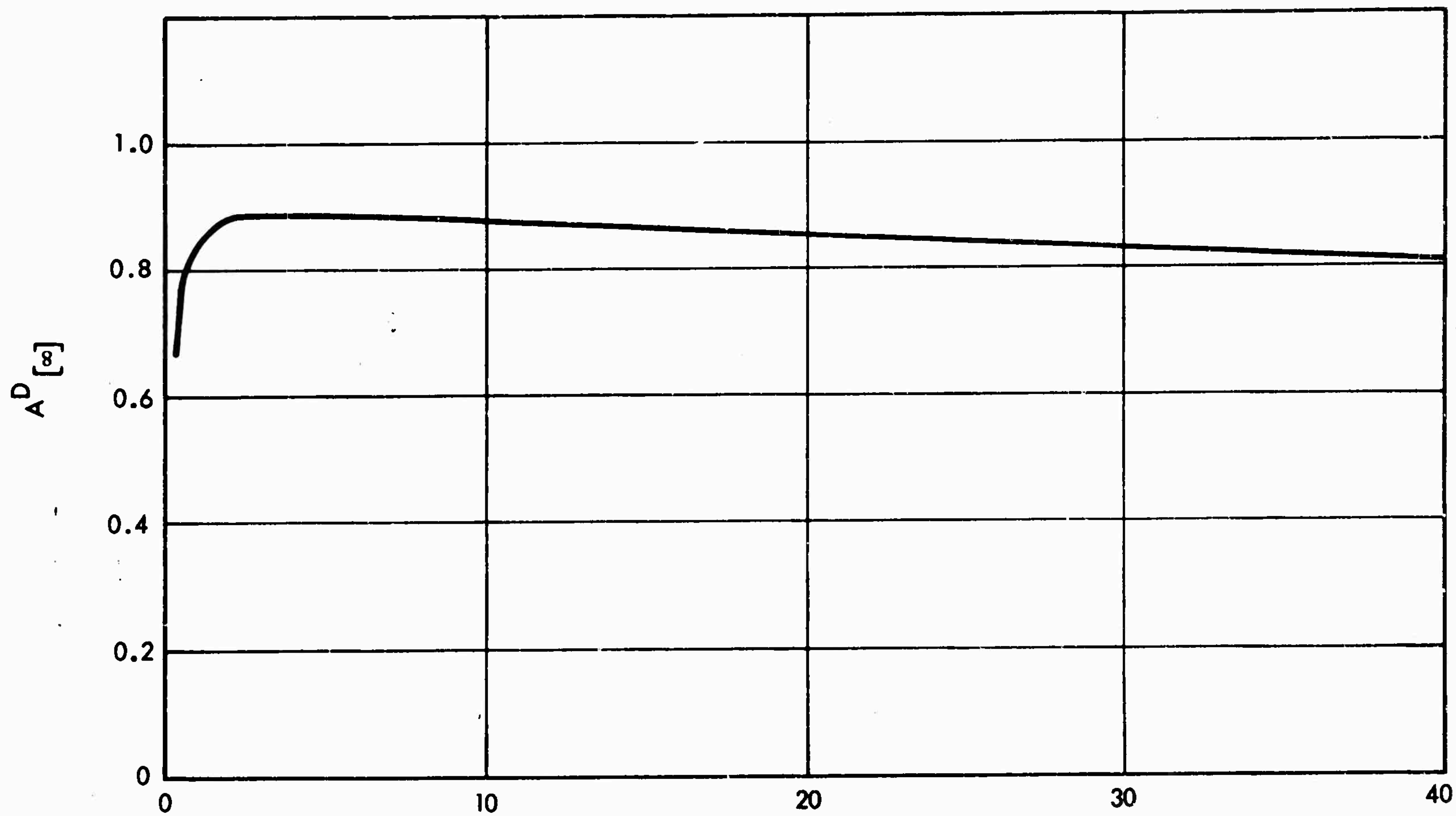


FIGURE 23. AVAILABILITY OF THE PROPULSION SUBSYSTEM AS A FUNCTION OF THE DURATION OF ALERT STATUS

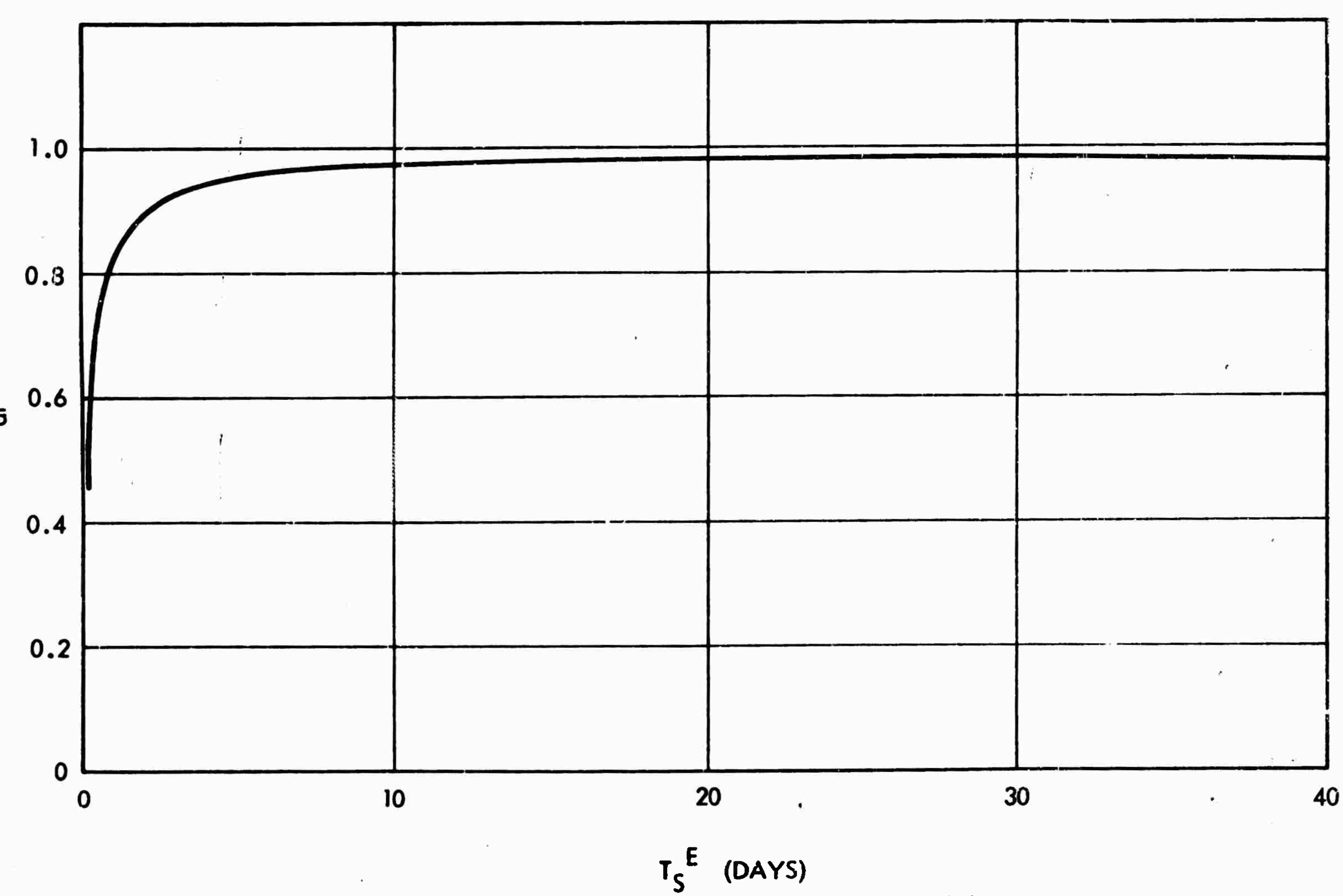


FIGURE 24. AVAILABILITY OF THE STRUCTURE AS A FUNCTION OF THE DURATION OF ALERT STATUS

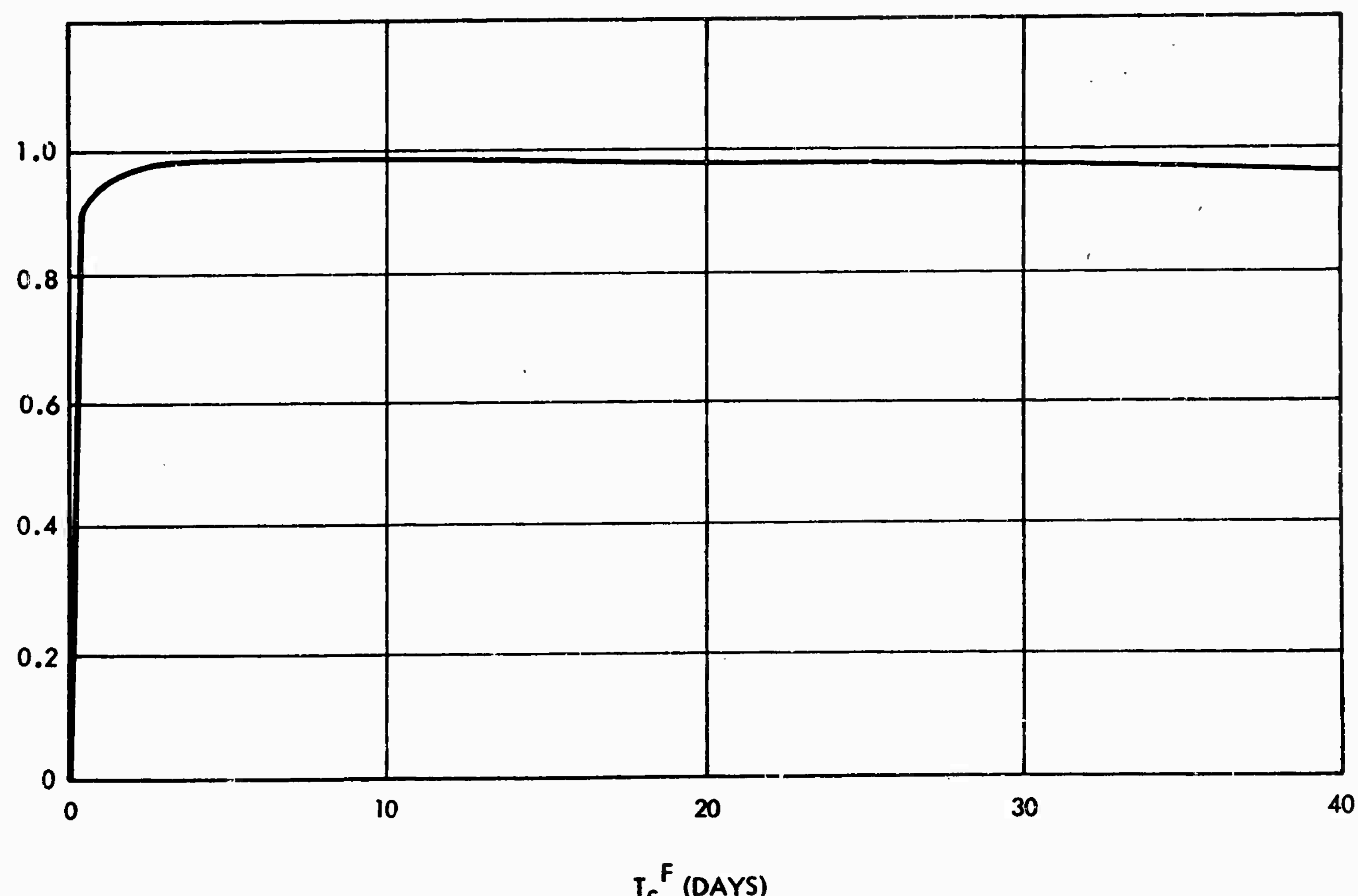


FIGURE 25. AVAILABILITY OF THE OVERHEAD DOOR AS A FUNCTION OF THE DURATION OF ALERT STATUS

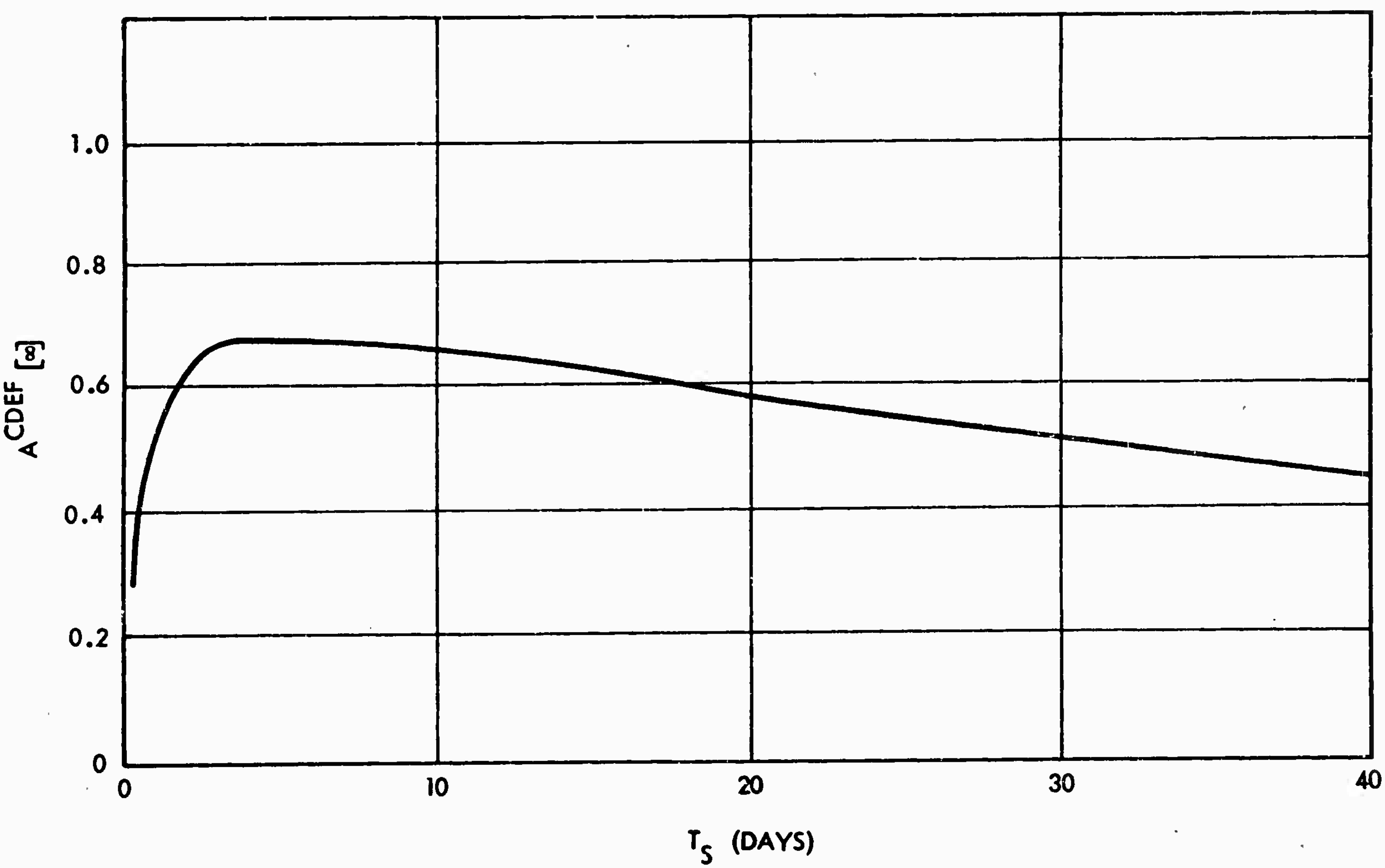


FIGURE 26. COMPOSITE AVAILABILITY OF SUBSYSTEMS C, D, E, F

2.1.5 Subsystem H

Table XIII illustrates the potential gains arising from various changes in the power generation and distribution. Substantial changes in false alarm rate (α), detectable failure rate (λ_d), and the undetectable failure rate (λ_u) will be required to achieve the apportionment (equal availability partition) cited in Table IX.

TABLE XIII. EFFECTS OF ALTERATIONS ON SUBSYSTEM H

Parameter*			$A^H[\infty]$	% Change
	Current estimate	Proposed change		
No change	-	-	0.740	-
$1/\lambda_u$	100	∞	0.765	2.03
$1/\alpha$	10	∞	0.788	6.49
$1/(\alpha+\lambda_u)$	9.09	∞	0.814	10.00
$1/\lambda_d$	5	50	0.825	11.50
$1/(\alpha+\lambda_d+\lambda_u)$	7.69	50	0.981	32.6

* Units are days or days between events.

2.2 Per Unit Kill

Figure 27 illustrates the way in which per unit kill probability varies with the ratio (R_L/σ). To achieve the minimum acceptable value for P_K(=.8), R_L/σ must increase from its current value of unity to 1.8. To achieve the objective value (=.9), R_L/σ must increase to 2.125.

Figure 28 illustrates the effect of alternative targeting policies. It will be noted that it currently takes 3^+ missiles to achieve the expected kill that accures to a fleet for which P_K = .8, and it takes five missiles to achieve the equivalent effect of P_K = .9.

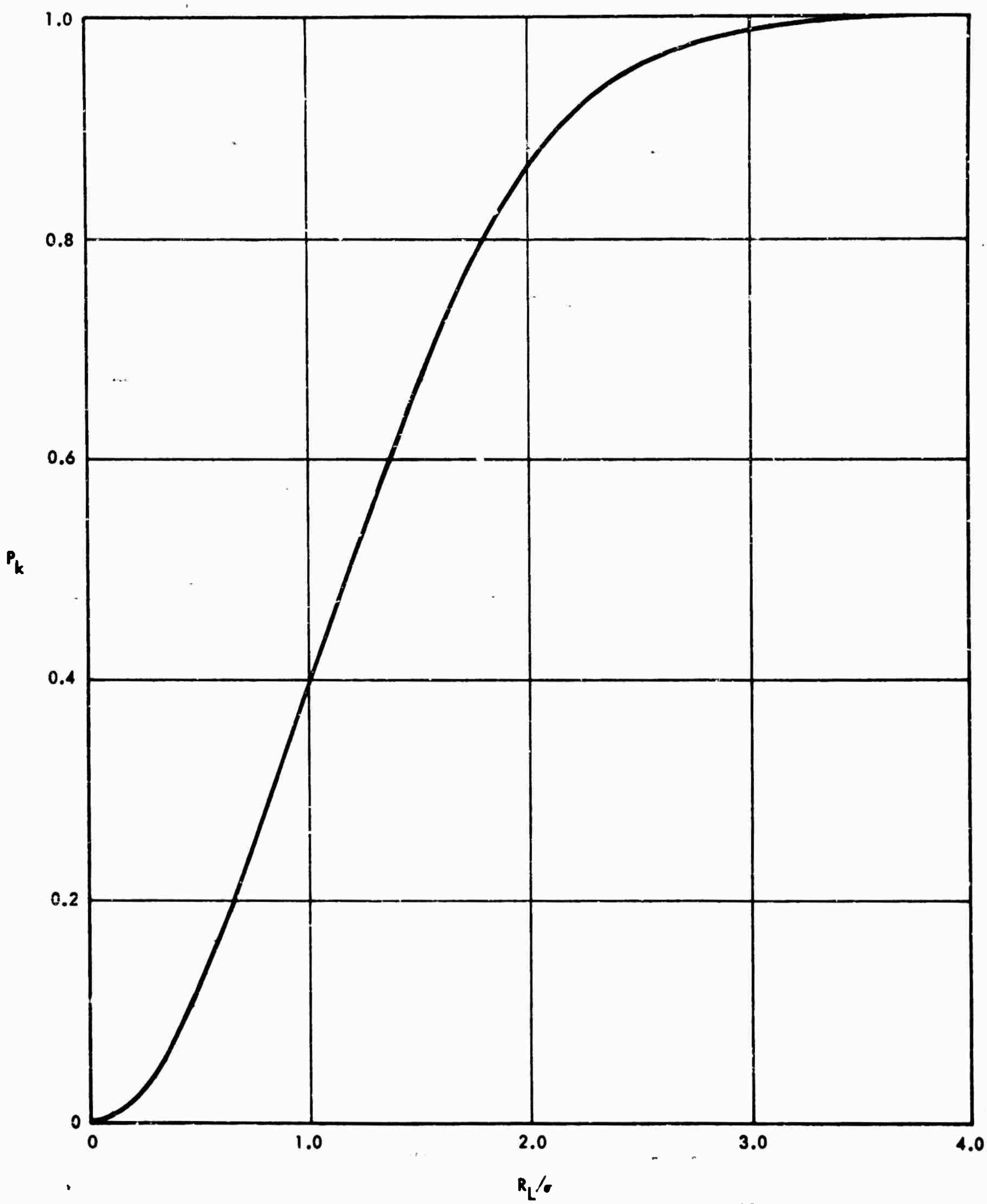


FIGURE 27. VARIATION OF UNIT KILL PROBABILITY

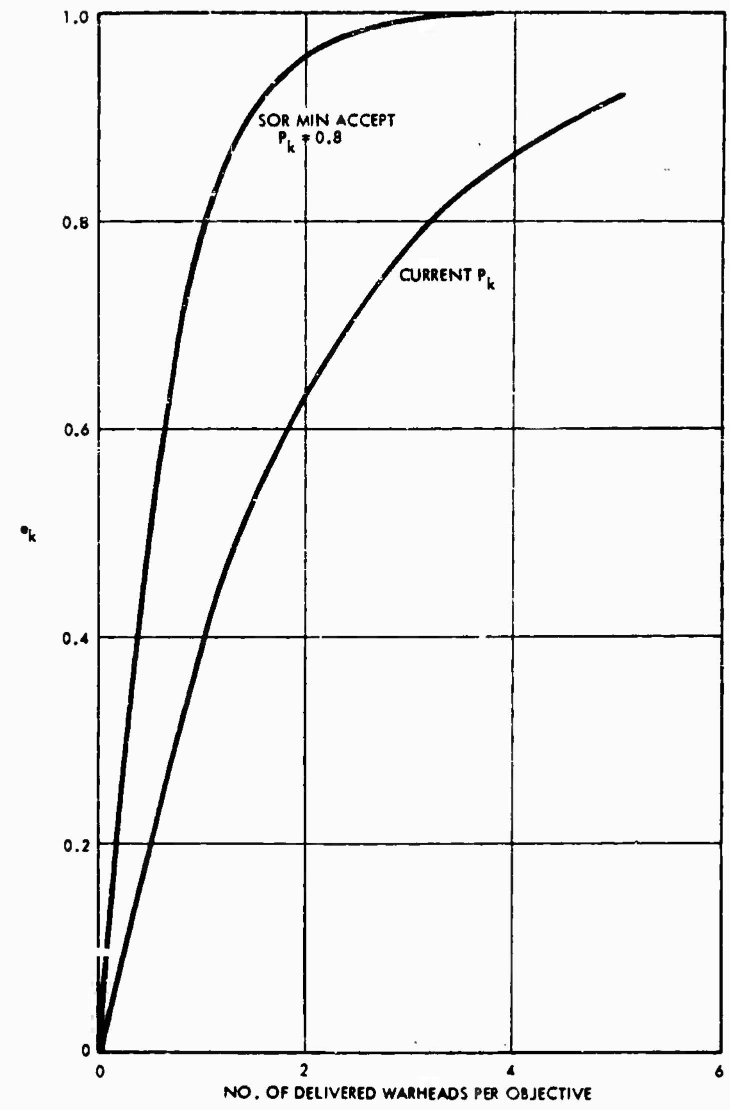


FIGURE 28. VARIATION OF EXPECTED KILL AS A FUNCTION OF THE NUMBER OF DELIVERED WARHEADS

The decision to be made here is whether it is less costly to buy more missile or to double the guidance accuracy.

3.0 Recalculation of E Based on Potential System Improvements

Table XIV summarizes the readiness figures which can accrue from revised checkout periodicity.

TABLE XIV. EFFECT OF REVISED CHECKOUT FREQUENCIES ON SUBSYSTEM AVAILABILITY

Subsystem	T_s (days)	$A_i [\infty]$
A	60	0.9475
B	3	0.8924
C	3	0.830
D	3-5	0.882
E	30	0.982
F	10	0.982
G	-	0.978
H	-	0.981
TCTO	(combine with U/M)	1.00
<CDEF>	3-5	.684

If TCTO work is combined with unscheduled maintenance and/or scheduled check-out, and if subsystem < CDEF > is checked out every five days, then

$$A_s\ [\infty] = 0.555 \tag{140}$$

If all subsystems are checked independently at their optimum periodicity,

$$A_s\ [\infty] = 0.572 \tag{141}$$

If the guidance dispersion is cut in half by redesign, etc., then

$$P_k = 0.865 \tag{142}$$

which exceeds the minimum acceptable SOR value.

Recalculation of E based upon Equations (140) and (142) yields the results listed in Table XV. Further improvement can be obtained by targeting more missiles per objective.

TABLE XV. EXPECTED KILL FOR VARIOUS SYSTEM CHANGES

Proposed Alteration	E
No alteration	0.5[illegible]
Checkout frequency optimized	1.2[illegible]
Guidance accuracy improved	1.252
Both optimum frequency and guidance improvement	2.105

Development of an algorithm for determination of the optimum policy for weapon system improvement/deployment is beyond the scope of this document, but it is clear that schedules, cost, technical feasibility of the proposed changes expected weapon system life and a host of other factors must enter at this point in order to arrive at correct management decisions.

APPENDIXES

APPENDIX I

of

EXAMPLE B

Weapon System Capability; Availability Models and Parameter Estimation

This appendix, which is a paper delivered to the Aerospace Reliability and Maintainability Conference, May 6-8, 1963, Washington D.C., describes the analytical techniques which have been used to develop the models for availability given in the body of this report.

WEAPON SYSTEM CAPABILITY : AVAILABILITY MODELS AND PARAMETER ESTIMATION

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Introduction

Background

The design and development of weapon systems has traditionally crowded the "state of the art" in materials, devices, and physical principles. In recent times, designers have been faced simultaneously with increasingly novel demands and more and more limited amounts of test data. Performance requirements almost invariably include severe reaction and response time limits which cannot be met without a tight integration of personnel, procedures, and hardware. Furthermore, modern weapon systems are rapidly becoming "one shot" devices providing little opportunity for obtaining operational usage data, either in kind or in quantity. A culmination of these factors is most clearly evident in the current ballistic missile programs. Not only have economics and time schedules frequently combined to frustrate the requirements of testing and evaluation of the weapon systems, but the sheer complexity of the systems have mitigated against effective systems management. Accordingly, what was once considered merely desirable, is now mandatory--an integrated methodology for weapon system management which will both pinpoint problem areas and which will provide a numerical measure of weapon system adequacy using the barest minimum of data. The subject matter of this paper represents an effort to contribute to the unraveling of this problem both by analytical technique and example results.

Total Measure of a System

There is no generally accepted definition of the total measure of system performance. If, however, we exclude the question of cost as being outside the immediate purview of this paper, we may define a measure which is a suitable description of many military systems as "the probability that a complex of equipment, personnel, and procedures will successfully respond to and accomplish the intent of its design when an execution directive is received at a random point in time." For a ballistic missile system, this verbal definition may usually be reduced to a function of conditional probabilities,

$$P_e = f\left[P_{ar}, P_c, P_{cd}, P_v, P_f, P_g, P_{mpd}, P_p, P_{wh}, P_k\right] \tag{1}$$

where the symbols denote random availability P_{ar}, communication reliability P_c, countdown reliability P_{cd}, vulnerability P_v, flight reliability P_f, guidance accuracy P_g, propellant depletion P_{mpd}, penetration probability P_p, warhead reliability P_{wh}, and kill probability P_k.

Scope of this Paper

It is the intent of the author to limit this paper to a consideration of the first factor of this equation (P_{ar}); not only because it is in the area of maintainability that one may expect to achieve significant improvements in an already operational system, but also because this factor of the equation is the most difficult to assess for ballistic missile systems. The crux of the difficulty is the fact that one cannot demonstrate the readiness of a ballistic missile system to the same degree that can be done for aircraft. Accordingly, a technique is required whereby the actual alert status of the weapon system may be inferred as opposed to demonstrated. This paper will show that such a method of inference exists; will delineate the factors to be considered; and will provide an example of the use of the techniques involved.

Availability

Definition of Availability [1, 3, 6, 9, 10]

Availability may be defined as the probability that a missile and its launch complex will be nonfailed and capable of entering countdown when an execution directive is received at a random point in time after initial installation and checkout. It may be calculated in either of two ways,

$$P_{ar} = \frac{\text{achieved time nonfailed and assigned to alert}}{\text{achieved total time in use}} \tag{2}$$

or

$$P_{ar} = \frac{\text{expected time nonfailed and assigned to alert}}{\text{expected total time in use}} \tag{3}$$

In the first calculation we deal with demonstrated fact. In the second calculation we make a prediction from current data. It is the latter calculation with which we shall deal here.

The Factors of Availability

Operation of a weapon system during peacetime may be resolved into three basic activities

- alert
- checkout and calibration/scheduled maintenance/training exercises
- repair (unscheduled maintenance)

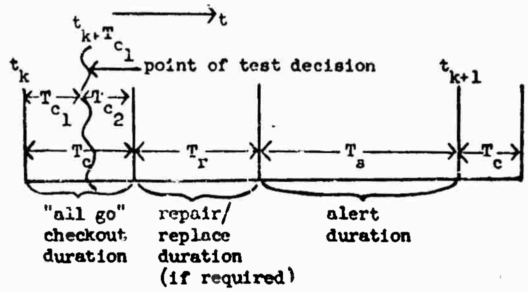


Figure 1. Time Line Sequence for a Purely Discrete Monitoring Policy

The way in which the basic activities are related to each other is defined by the management/maintenance policy for the system. Each of these activities is traditionally considered to be the resultant of the interaction of three factors

- personnel
- procedures
- hardware

Role of the Management/Maintenance Policy

These three factors and the measures thereof are meaningfully related by the system management/maintenance policy of which there are two fundamental types. In the first instance, illustrated in figure one, a system may be subjected to a discrete monitoring policy. That is, the equipment is checked "periodically" to determine if it is functional. The duration T_c of an "all go" checkout may be constant or may be a random variable. If one or more repairs are necessary, a time T_r is required to repair/replace and recheck the system. This duration is usually a random variable. Subsequent to repair, or to T_c if no repair is required, the system is assigned to an alert status for a duration T_s during which it is assumed to be nonfailed and is held in readiness to perform its design function if called upon to do so. The duration of T_s is quite frequently a fixed value.

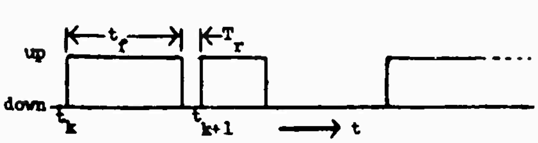


Figure 2. Time Line Sequence for a Continuous Monitoring Policy

The second type of maintenance policy is illustrated in figure two. Here the system is monitored continuously for failures so that the system is "up" or alert for times t_f, where t_f is the time to system failure after a repair. (This is a random variable). The system is "down", i.e. in repair, diagnosis, or awaiting repair for a time T_r whenever a failure occurs. In general, T_r is a random variable.

For most complex systems a real maintenance policy will consist of a mixture of these two fundamental policies.

Failure Distribution

The hardware factor of availability evidences itself as a failure distribution. For complex systems this failure distribution will be exponential, but the failure rate may differ depending upon whether the equipment is in checkout or in alert.

The Concept of a Test

It is convenient and realistic to treat personnel and procedures together. Measureable parameters which describe their interactions with hardware may be defined through the concept of a "test".

If we regard a complex system as a single unit, then a checkout or a continuous monitor may be regarded as a "test". By a test we mean, then, the examination of a set of functional characteristics of a equipment. If an observed characteristic does not meet the requirements imposed by the test, the equipment is rejected, otherwise it is passed.

The test will have four basic properties. First, it will "pass" or "reject" an equipment at a specific point in time. That is, it is assumed that the test decision occurs at a well defined point in time. Second, it will on occasion "false alarm" a nonfailed characteristic of the equipment, i.e. it will call a good system bad. Third, the test will sometimes pass a failed characteristic, that is, it will not always reject a failure which it presumably is designed to detect. A test which does this too frequently is one of poor "quality". The quality of a test we shall define as the probability of detecting a failed system, given that the system is failed on or before the time that the point of test decision is reached. The fourth and last property of a test is "coverage". By coverage we shall mean that not all the possible equipment functional characteristics are examined by the test. It is assumed that the failure of such a characteristic cannot cause the test to reject the equipment, since its effect on the equipment is indeterminate from the test. However, we shall further assume that all equipments which have failed in this manner will be eventually rejected by either a false alarm or by the detection of a failure of an observed characteristic of the equipment.

Repair

When an equipment is rejected by a test it is repaired. In the case of military systems this will generally reduce to the sequence of events

- . diagnose
- . remove
- . replace
- . recheck

It has been noted in the literature that the repair distribution for complex military systems tends to be log-normal in form.[8] This is a particularly difficult distribution to handle analytically and in this paper it will be approximated by means of the sum of exponentials.

Discrete Monitoring Policy

Analytical Definition of a Test

We shall commence the analytical portion of this paper by formalizing the above concepts of a test for a discrete monitoring policy. Having accomplished this we shall use the notion of a transition matrix to relate the test to the remainder of the maintenance policy and the equipment failure distribution. Subsequently, we shall formalize the definition of availability for a discretely monitored system. Finally, we shall turn to the problem of a continuously monitored system.

The basic time line sequence of events for a discrete monitoring policy is indicated in figure one. The period T_c is associated with the concept of a "test" which partitions equipment at the "point of test decision" indicated at $t = t_k + T_{c_1}$. For simple equipments this will generally be a real point within T_c, but for complex equipments such a point will very likely be a convenient mathematical fiction. It is assumed that at the point of test decision the test acts instantaneously to partition failed equipments from nonfailed equipments as indicated by the partition of figure three. Consider the action of the test at this point in time on a total of $N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6$ equipments all of one kind. (Alternatively, one equipment may be tested N times in succession and may be either good or bad at each test.) Let $N_1 + N_2$ be the true number of nonfailed equipments. Let $N_3 + N_4$ equipments have one or more failures which are inherently detectable; that is, they contain failures among the essential characteristics which are examined by the test, but they will not necessarily all be detected because of "noise" or because some of the failures may be marginal. In addition let there be $N_5 + N_6$ equipments which contain no failures among the characteristics which the test examines, but which do contain one or more failures among the characteristics which are not examined. We may now make the following definitions if the total number of equipments being examined is very large

$$P\left[G; t_k + T_{c_1}\right] \doteq \frac{N_1 + N_2}{\sum_{i=1}^{6} N_i} = \text{probability of no failures of any kind at } t = t_k + T_{c_1}$$

$$P\left[B_d; t_k + T_{c_1}\right] \doteq \frac{N_3 + N_4}{\sum_{i=1}^{6} N_i} = \text{probability of one or more failures of the "detectable in principle" type at } t = t_k + T_{c_1}$$

$$P\left[B_u; t_k + T_{c_1}\right] \doteq \frac{N_5 + N_6}{\sum_{i=1}^{6} N_i} = \text{probability of one or more failures that are "inherently undetectable" at } t = t_k + T_{c_1}$$

These partitions of the true facts concerning the states of the equipments at the instant of test decision are shown in the left hand boxes of Figure 3. The right hand boxes of Figure 3 show the partition of the equipments which result as a consequence of test decisions. If α is the probability of calling any nonfailed characteristic bad, given that it is examined by the test, then we may write

$$\alpha \doteq \frac{N_2}{N_1 + N_2} = \text{false alarm probability}$$

and

$$\alpha' \doteq \frac{N_6}{N_5 + N_6} = \text{false alarm probability}$$

There is no physical reason to presume that $\alpha \neq \alpha'$, hence we shall write without further justification

$$\alpha = \alpha'$$

We further define a parameter β;

$$1 - \beta \doteq \frac{N_4}{N_3 + N_4} = \text{probability of catching an "inherently detectable" failure}$$

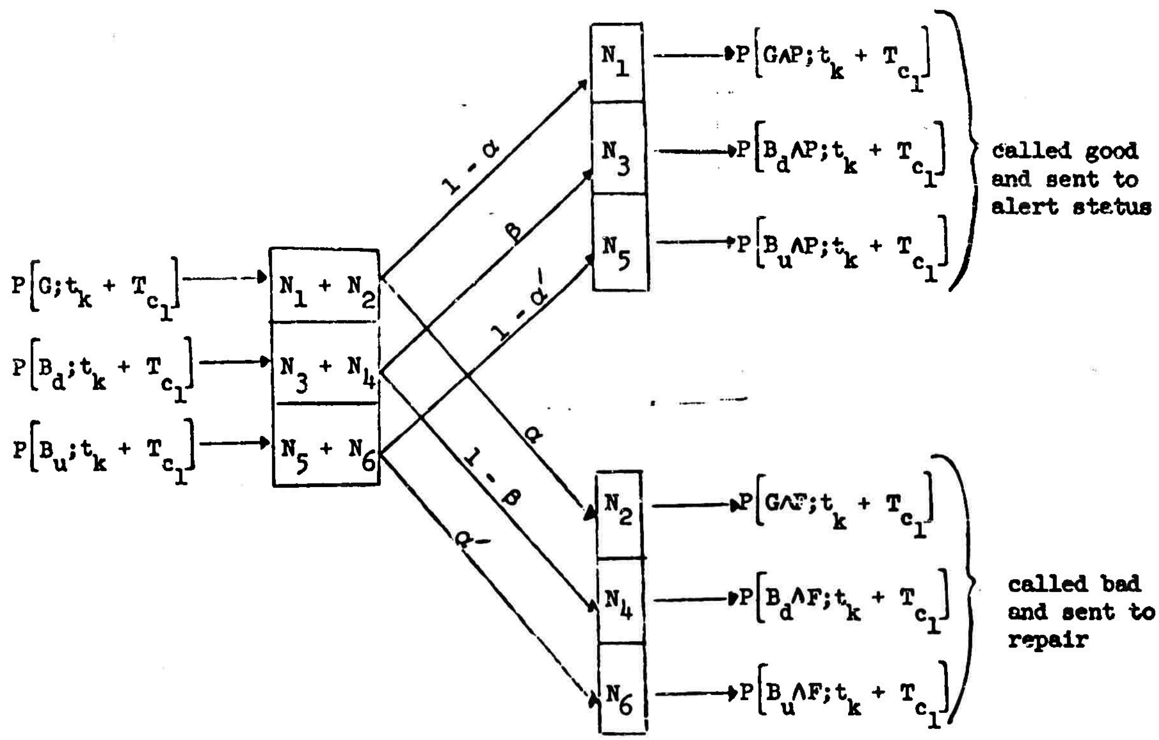
These six equations may be solved simultaneously for N_1, N_2, N_3, N_4, N_5, and N_6.

We shall make use of the following notational definitions in that which follows:

$$P\left[G \wedge P; t_k + T_{c_1}\right] \doteq \frac{N_1}{\sum N_i} = \text{probability of passing, given that the equipment is good at } t_k + T_{c_1}$$

$$P\left[B_d \wedge P; t_k + T_{c_1}\right] \doteq \frac{N_3}{\sum N_i} = \text{probability of passing, given that the equipment contains an inherently detectable failure at } t = t_k + T_{c_1}$$

$$P\left[B_u \wedge P; t_k + T_{c_1}\right] \doteq \frac{N_5}{\sum N_i} = \text{probability of passing, given that the equipment contains only inherently undetectable failures at } t = t_k + T_{c_1}$$



$$\alpha = \frac{N_2}{N_1 + N_2}$$

$$\alpha' = \frac{N_6}{N_5 + N_6}$$

$$1 - \beta = \frac{N_4}{N_3 + N_4}$$

FIGURE 3.

ACTION OF A CHECKOUT (TEST) ON A SYSTEM THAT IS DISCRETELY MONITORED AT THE "POINT OF TEST DECISION, " $t = t_k + T_{c_1}$

$$P\left[G \wedge F; t_k + T_{c_1}\right] = \frac{N_2}{\sum N_1} = \text{probability of flunking, given that the equipment is good at } t = t_k + T_{c_1}$$

$$P\left[B_d \wedge F; t_k + T_{c_1}\right] = \frac{N_4}{\sum N_1} = \text{probability of flunking, given that the equipment contains a failure which is "inherently detectable" at } t = t_k + T_{c_1}$$

$$P\left[B_u \wedge F; t_k + T_{c_1}\right] = \frac{N_6}{\sum N_1} = \text{probability of flunking, given that the equipment contains only "inherently undetectable" failures at } t = t_k + T_{c_1}$$

Using these definitions, the following matrix equation for the test results may be written,

$$\begin{bmatrix} P\left[G \wedge P; t_k+T_{c_1}\right] \\ P\left[B_d \wedge P; t_k+T_{c_1}\right] \\ P\left[B_u \wedge P; t_k+T_{c_1}\right] \\ P\left[G \wedge F; t_k+T_{c_1}\right] \\ P\left[B_d \wedge F; t_k+T_{c_1}\right] \\ P\left[B_u \wedge F; t_k+T_{c_1}\right] \end{bmatrix} = \begin{bmatrix} 1-\alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1-\alpha \\ \alpha & 0 & 0 \\ 0 & 1-\beta & 0 \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} P\left[G; t_k+T_{c_1}\right] \\ P\left[B_d; t_k+T_{c_1}\right] \\ P\left[B_u; t_k+T_{c_1}\right] \end{bmatrix} \quad (4)$$

State Vectors and Transition Matrices[7]

A particularly orderly way in which to approach the description of a system which can be in any one of a finite number of possible conditions or "states" is through the use of the concept of a state vector and transition matrices. Before proceeding to the main developments of this paper we shall first treat a simplified situation in terms of state vectors and matrix equations in order to familiarize the reader with the ideas involved.

Let it be supposed that a system is to be described at time $t = kT$. The system is either "good," i.e., nonfailed, or it is "bad," i.e., failed. There is a probability associated with being in either condition which we will refer to as the "state" of the system and which we write as the column matrix (vector)

$$\bar{P}\left[kT\right] \triangleq \begin{bmatrix} P\left[G;\ kT\right] \\ P\left[B;\ kT\right] \end{bmatrix} \quad (5)$$

This state vector is related to the state of the system at $t = (k + 1)\,T$ by a transition matrix A;

$$\begin{bmatrix} P\left[G;\ (k+1)\,T\right] \\ P\left[B;\ (k+1)\,T\right] \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P\left[G;\ kT\right] \\ P\left[B;\ kT\right] \end{bmatrix} \quad (6)$$

where the elements a_{ij} of the matrix A are called transition probabilities. They describe the way in which the state vector at $t = (k + 1)\,T$ is related to the state vector at $t = kT$. If p is the probability of being nonfailed at time $t = (k + 1)\,T$; given that the equipment is nonfailed at $t = kT$ and if μ is the probability of being nonfailed at $t = (k + 1)\,T$; given that the equipment is failed at $t = kT$, then

$$\begin{bmatrix} P\left[G;\ (k+1)\,T\right] \\ P\left[B;\ (k+1)\,T\right] \end{bmatrix} = \begin{bmatrix} p & \mu \\ 1-p & 1-\mu \end{bmatrix} \begin{bmatrix} P\left[G;\ kT\right] \\ P\left[B;\ kT\right] \end{bmatrix} \quad (7)$$

Note that the sum of the terms of each column of A is unity. If p and μ are constants when T is a constant it may be shown that the state of the system at $t = kT$ is related to the state of the system at $t = 0$ by

$$\begin{bmatrix} P\left[G;\ kT\right] \\ P\left[B;\ kT\right] \end{bmatrix} = \begin{bmatrix} p & \mu \\ 1-p & 1-\mu \end{bmatrix}^k \begin{bmatrix} P\left[G;\ 0\right] \\ P\left[B;\ 0\right] \end{bmatrix} \quad (8)$$

It is desirable to have a simple way of obtaining A^k so that the state at $t = kT$ may be related to an arbitrary initial state at $t = 0$. This may be readily accomplished using the Cayley-Hamilton relationship. This relationship states that, if A is a n x n matrix, i.e., has n rows and n columns, then A^k is given by

$$A^k = \beta_0 I + \beta_1 A + \beta_2 A^2 + \dots + \beta_{n-1} A^{n-1} \quad (9)$$

The β_i are determined by solving the n linear equations

$$\omega_i^{\,k} = \beta_0 + \beta_1 \omega_i + \beta_2 \omega_i^{\,2} + \dots + \beta_{n-1} \omega_i^{\,n-1} \quad (10)$$
$$i = 0, 1, 2, \dots n-1$$

where the ω_i are the so-called "eigenvalues" of A found from the characteristic equation

$$\left| A - \omega I \right| = 0 \quad (11)$$

$$\left| A - \omega I \right| = \begin{vmatrix} p-\omega & \mu \\ 1-p & 1-\mu-\omega \end{vmatrix} = \begin{vmatrix} a_{11}-\omega & a_{12} \\ 1-a_{11} & 1-a_{12}-\omega \end{vmatrix} = 0 \quad (12)$$

$$(a_{11}-\omega)\,(1-a_{12}-\omega) - a_{12}\,(1-a_{11}) = 0 \quad (13)$$

Hence

$$\omega_0 = 1;\ \omega_1 = a_{11} - a_{12} = p - \mu \tag{14}$$

and

$$1 = \beta_0 + \beta_1 \tag{15}$$

$$\omega_1{}^k = \beta_0 + \beta_1 \omega_1 \tag{16}$$

Therefore

$$\beta_0 = 1 - \beta_1 \tag{17}$$

$$\beta_1 = \frac{1 - \omega_1{}^k}{1 - \omega_1} = \frac{1 - (p - \mu)^k}{1 - p + \mu} \tag{18}$$

and from Equation (9)

$$A^k = \begin{bmatrix} \dfrac{a_{12} + (1 - a_{11})\,\omega_1{}^k}{1 - \omega_1} & \dfrac{a_{12}(1 - \omega_1{}^k)}{1 - \omega_1} \\ \\ \dfrac{(1 - a_{11})}{1 - \omega_1}\,(1 - \omega_1{}^k) & \dfrac{1 - a_{11} + a_{12}\omega_1{}^k}{1 - \omega_1} \end{bmatrix} \tag{19}$$

In the limit as $k \longrightarrow \infty$, the steady state vector is found to be

$$\begin{bmatrix} P[0;\ \infty] \\ \\ P[B;\ \infty] \end{bmatrix} = \begin{bmatrix} \dfrac{\mu}{1 - p + \mu} & \dfrac{\mu}{1 - p + \mu} \\ \\ \dfrac{1 - p}{1 - p + \mu} & \dfrac{1 - p}{1 - p + \mu} \end{bmatrix} \begin{bmatrix} P[G;0] \\ \\ P[B;0] \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\mu}{1 - p + \mu} \\ \\ \dfrac{1 - p}{1 - p + \mu} \end{bmatrix} \tag{20}$$

In other words, the steady state condition is independent of the initial conditions.

The fact that the terminal probabilities of the state vector (20) are independent of the initial states may be put to good use. For example, whenever the one step transition matrix is made up of constant terms, as in (7), we may determine the terminal states by equating the state vectors at equivalent successive points in time. According to (19), once steady state has been reached;

$$\lim_{k \to \infty} \begin{bmatrix} P[G;\ kT] \\ \\ P[B;\ kT] \end{bmatrix} = \lim_{k \to \infty} \begin{bmatrix} P[0;\ (k + j)\ T] \\ \\ P[B;\ (k + j)\ T] \end{bmatrix}; \tag{21}$$

$$j = 0, 1, 2, \ldots.$$

If we substitute (21) into (7) and transpose the left hand side to the right hand side;

$$\lim_{k \to \infty} \begin{bmatrix} p - 1 & \mu \\ \\ 1 - p & -\mu \end{bmatrix} \begin{bmatrix} P[G;\ kT] \\ \\ P[B;\ kT] \end{bmatrix} = 0 \tag{22}$$

Now, since

$$P[0;\ kT] + P[B;\ kT] = 1 \tag{23}$$

We may solve the matrix equation as follows. Expanding either row of (22), for example the first row;

$$(p - 1)\ P[0;\ kT \longrightarrow \infty] + \mu P[B;\ kT \longrightarrow \infty] = 0 \tag{24}$$

and from (23) in conjunction with equation (24);

$$P[G;\ kT \longrightarrow \infty] = \frac{\mu}{1 - p + \mu} \tag{25}$$

and of course

$$P[B;\ kT \longrightarrow \infty] = 1 - \frac{\mu}{1 - p + \mu} = \frac{1 - p}{1 - p + \mu} \tag{26}$$

These are precisely the results stated in (20).

Application of Transition Matrices

Let us now apply the concept of a transition matrix to a simple, but realistic situation. Consider a single equipment group which is tested and rejected (or passed) as a unit. We shall assume that the durations of scheduled alert T_s, checkout times T_{c_1} and T_{c_2} are constant and that the probability that a nonfailed equipment will survive a time τ is a function of τ only (exponential failure distribution). The time sequence of events is shown in figure one for the kth maintenance cycle. We shall assume that each maintenance cycle starts with a checkout. We shall further assume that all failures are detectable in principle, hence the system is describable as being in either of two states, "good" or "bad". The state of the system at time $t = t_k + T_{c_1}$ just prior to test decision is related to the state of the system at $t = t_k$ by the matrix equation

$$\begin{bmatrix} P[G;\ t_k + T_{c_1}] \\ P[B;\ t_k + T_{c_1}] \end{bmatrix} = \begin{bmatrix} P_{c_1} & 0 \\ 1 - P_{c_1} & 1 \end{bmatrix} \begin{bmatrix} P[G;\ t_k] \\ P[B;\ t_k] \end{bmatrix} \tag{27}$$

where P_{c_1} is the probability that the equipment survives T_{c_1}, given that it was good when it entered checkout. The test decision partitions the system with the transition probabilities indicated in figure three. Hence, at time $t = t_k + T_{c_1}$ immediately after the test decision is made

$$\begin{bmatrix} P[G \wedge P;\ t_k + T_{c_1}] \\ P[B \wedge P;\ t_k + T_{c_1}] \\ P[G \wedge F;\ t_k + T_{c_1}] \\ P[B \wedge F;\ t_k + T_{c_1}] \end{bmatrix} = \begin{bmatrix} 1 - \alpha & 0 \\ 0 & \beta \\ \alpha & 0 \\ 0 & 1-\beta \end{bmatrix} \begin{bmatrix} P[G;\ t_k + T_{c_1}] \\ P[B;\ t_k + T_{c_1}] \end{bmatrix} \tag{28}$$

The state of the system at entrance to standby depends upon whether the system was nonfailed and passed the test or whether it failed the test and was sent to repair. Considering only the possibility of a perfect repair we have at $t = t_{s_k}$ (entrance to scheduled alert)

$$\begin{bmatrix} P[G;\ t_{s_k}] \\ P[B;\ t_{s_k}] \end{bmatrix} = \begin{bmatrix} P_{c_2} & 0 & 1 & 1 \\ 1 - P_{c_2} & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P[G \wedge P; t_k + T_{c_1}] \\ P[B \wedge P; t_k + T_{c_1}] \\ P[G \wedge F; t_k + T_{c_1}] \\ P[B \wedge F; t_k + T_{c_1}] \end{bmatrix} \tag{29}$$

where P_{c_2} is the constant probability of surviving the remainder of checkout after the test decision is made, given that the system was passed and was in fact nonfailed. Note that if the system is good and passing at $t = t_k + T_{c_1}$ no T_r occurs and consequently the a_{11} term of the transition matrix is P_{c_2} not $P_{c_2}P_r$.

The state of the system at entrance to the succeeding checkout interval is

$$\begin{bmatrix} P[G;\ t_{k+1}] \\ P[B;\ t_{k+1}] \end{bmatrix} = \begin{bmatrix} P_s & 0 \\ 1 - P_s & 1 \end{bmatrix} \begin{bmatrix} P[G;\ t_{s_k}] \\ P[B;\ t_{s_k}] \end{bmatrix} \tag{30}$$

Where P_s is the constant probability that the equipment survives T_s, given that it enters T_s good. Therefore the state of the system just prior to the next test decision is related to the state of the system at the previous test decision by the product of the four transition matrices (27), (28), (29), and (30).

$$\begin{bmatrix} P[G;\ t_{k+1} + T_{c_1}] \\ P[B;\ t_{k+1} + T_{c_1}] \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ 1 - \alpha_1 & 1 - \alpha_2 \end{bmatrix} \begin{bmatrix} P[G;\ t_k + T_{c_1}] \\ P[B;\ t_k + T_{c_1}] \end{bmatrix}$$

$$\alpha_1 = P_s P_c (1 - \alpha) + P_s P_{c_1} \alpha$$

$$\alpha_2 = (1 - \beta)\, P_s\, P_{c_1} \tag{32}$$

This equation is the single step transition between successive test decisions.

From (14), the eigenvalues of this two by two matrix of constant transition probabilities are

$$\omega_0 = 1$$

$$\omega_1 = a_{11} - a_{12} = P_s P_{c_1} \left[\alpha + P_{c_2}(1 - \alpha) - (1 - \beta)\right] \tag{33}$$

From (19), the initial state of the system at $t = T_{c_1}$ is related to the state at $t = t_k$ just prior to test decision by:

$$\begin{bmatrix} P[G;\ t_k + T_{c_1}] \\ P[B;\ t_k + T_{c_1}] \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ 1 - \alpha_1 & 1 - \alpha_2 \end{bmatrix} \begin{bmatrix} P[G;\ T_{c_1}] \\ P[B;\ T_{c_1}] \end{bmatrix}$$

$$\alpha_1 = 1 - \frac{1 - \omega_1^{\,k}}{1 - \omega_1}(1 - a_{11})$$

$$\alpha_2 = \frac{1 - \omega_1^{\,k}}{1 - \omega_1} a_{12} \tag{34}$$

and it follows that in the steady state

$$\begin{bmatrix} P\left[G;\ t_k + T_{c_1} \longrightarrow \infty\right] \\ \\ P\left[B;\ t_k + T_{c_1} \longrightarrow \infty\right] \end{bmatrix} = \begin{bmatrix} \dfrac{P_s P_{c_1}(1-\beta)}{1-P_s P_{c_1}\left[\alpha-(1-\beta)+P_{c_2}(1-\alpha)\right]} \\ \\ 1 - \dfrac{P_s P_{c_1}(1-\beta)}{1-P_s P_{c_1}\left[\alpha-(1-\beta)+P_{c_2}(1-\alpha)\right]} \end{bmatrix} \stackrel{\Delta}{=} \bar{P}\left[t_k + T_{c_1} \longrightarrow \infty\right] \qquad (35)$$

Because we have specifically assumed constant transition matrices in this example, we could have obtained the result (35) by the alternative procedure of equating the state vectors at like points in time as noted in (21).

Note that all other states in the kth period, or in the steady state, are determinable from equations (34) and (35) respectively by one step transitions. For example, in the steady state as $t_k \longrightarrow \infty$, the state of the system at entrance to standby is obtainable from the product of equations (28), (29) and (35);

$$\begin{bmatrix} P\left[G;\ t_{s_k} \rightarrow \infty\right] \\ \\ P\left[B;\ t_{s_k} \rightarrow \infty\right] \end{bmatrix} = \begin{bmatrix} P_{c_2} & 0 & 1 & 1 \\ \\ 1 - P_{c_2} & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1-\alpha & 0 \\ 0 & \beta \\ \alpha & 0 \\ 0 & 1-\beta \end{bmatrix} \begin{bmatrix} P\left[G;\ t_k + T_{c_1} \rightarrow \infty\right] \\ \\ P\left[B;\ t_k + T_{c_1} \rightarrow \infty\right] \end{bmatrix} = \begin{bmatrix} \dfrac{1-\beta}{1-P_s P_{c_1}\left[\alpha-(1-\beta)+P_{c_2}\ (1-\alpha)\right]} \\ \\ \dfrac{1-\beta}{1-P_s P_{c_1}\left[\alpha-(1-\beta)+P_{c_2}\ (1-\alpha)\right]} \end{bmatrix} \qquad (36)$$

Availability of a Discretely Monitored System

We are now in a position to define the availability of a discretely monitored system. Consider the definition (3) and the basic cycle of figure one. Defining zero time as being at initial installation and checkout, this basic cycle repeats itself "n" and a fraction number of times in some interval of time "t". According to the definition (3) we must calculate the ratio of the expected amount of nonfailed time to the total time "t". Ignoring the fractional cycle left over, the expected amount of nonfailed time is given by summing up to "n" the product of being nonfailed at entrance to T_s and the expected time nonfailed in T_s. That is,

$$\text{Expected good time duration in "n" cycles} = \sum_{k=1}^{m} P\left[G;\ t_{s_k}\right] E\left[T_{s_k}\right] \qquad (37)$$

$P\left[G;\ t_{s_k}\right]$ = probability of being good at entrance to T_s

$E\left[T_{s_k}\right]$ = expected time nonfailed in T_{s_k}; given that equipment was nonfailed at entrance to T_{s_k}

$$= \int_0^{T_{s_k}} \left(1 - F\left[t\right]\right) dt \qquad (38)$$

where $F\left[t\right]$ is the equipment failure distribution. The total duration of "n" cycles is given by summing up the durations of T_s, T_c, and T_r if it occurs. That is,

$$\text{Expected duration of "n" cycles} = \sum_{k=1}^{m} \left(E\left[T_{s_k}\right] + E\left[T_{c_k}\right] + P\left[F;\ t_k + T_{c_1}\right] E\left[T_{r_k}\right]\right) \qquad (39)$$

$E\left[T_{s_k}\right]$ = expected duration of kth alert interval

$E\left[T_{c_k}\right]$ = expected duration of kth checkout

$E\left[T_{r_k}\right]$ = expected duration of kth repair/replace cycle if it occurs

$P\left[F;\ t_k + T_{c_1}\right]$ = probability of being rejected at the test decision point

Therefore, in general for "n" complete cycles,

$$P_{ar}\left[t_n\right] = \frac{\sum_{k=1}^{n} P\left[G;\ t_{c_k}\right] E\left[T_{s_k}\right]}{\sum_{k=1}^{n} \left(E\left[T_{s_k}\right] + E\left[T_{c_k}\right] + P\left[F;\ t_k + T_{c_1}\right] E\left[T_{r_k}\right]\right)} \quad (40)$$

As an example of the application of this formula, let us briefly consider the following simple variation of the basic cycle of figure one,

- equipment out of action during T_c and T_r
- T_c is a constant
- T_r is a constant
- T_s is a constant
- $F\left[t\right] = 1 - e^{-\lambda_s t}$
- repair is by remove and replace

$$P_{ar}\left[n\right] = \frac{\sum_{k=1}^{n} P\left[0;\ t_{s_k}\right] E\left[T_s\right]}{\sum_{k=1}^{n} \left\{T_s + T_c + P\left[F;\ t_k + T_{c_1}\right] E\left[T_r\right]\right\}} \quad (41)$$

where

$$P\left[F;\ t_k + T_{c_1}\right] = P\left[0 \wedge F;\ t_k + T_{c_1}\right] + P\left[B \wedge F;\ t_k + T_{c_1}\right]$$

$$= P\left[0;\ t_k + T_{c_1}\right]\alpha + P\left[B;\ t_k + T_{c_1}\right](1-\beta) \quad (42)$$

$$P\left[0;\ t_{s_k}\right] = P\left[0;\ t_k + T_{c_1}\right]\left[(1-\alpha) P_{c_2} + \alpha\right] + P\left[B;\ t_k + T_{c_1}\right](1-\beta) \quad (43)$$

and from (38)

$$E\left[T_s\right] = \frac{1 - e^{-\lambda_s T_s}}{\lambda_s} \quad (44)$$

We shall simply denote $E\left[T_r\right]$ by T_r and postpone a discussion of the distribution of the T_r until a later section. Expanding and collecting terms,

$$P_{ar}\left[n\right] = \frac{\left\{(n+1)\ b_0 + b_1 \frac{1 - \omega_1^{n+1}}{1 - \omega_1}\right\} \frac{1 - e^{-\lambda_s T_s}}{\lambda_s}}{(n+1)\ (T_s + T_c) + \left\{(n+1)\ a_0 + a_1 \frac{1 - \omega_1^{n+1}}{1 - \omega_1}\right\} T_r} \quad (45)$$

where

$$a_0 = \frac{(1-\beta)\left[1 - P_s P_c (1-\alpha)\right]}{1 - \omega_1},\ b_1 \quad (46a)$$

$$a_1 = \left[1 - a_{11} P\left[0;T_{c_1}\right] - a_{12} P\left[B;T_{c_1}\right]\right] \frac{(1 - \beta - \alpha)}{1 - \omega_1} \quad (46b)$$

$$b_0 = \frac{1-\beta}{1-\omega_1} \quad (47a)$$

$$b_1 = -\left[1 - a_{11} P\left[0;T_{c_1}\right] - a_{12} P\left[B;\ T_{c_1}\right]\right] \times \left[1 - \beta - \alpha - (1-\alpha) P_{c_2}\right] \frac{1}{1-\omega_1} \quad (47b)$$

In the limit as $n \to \infty$

$$\lim_{n\to\infty} P_{ar}\left[n\right] = \frac{b_0\ E\left[T_s\right]}{T_s + T_c + a_0 T_r}$$

$$= \frac{\left\{\left(1 - e^{-\lambda_s T_s}\right)/\lambda_s\right\} \frac{1-\beta}{1 - P_s P_{c_1}\left[\alpha + P_{c_2}(1-\alpha) - (1-\beta)\right]}}{T_s + T_c + \frac{(1-\beta)\left[1 - P_s P_c (1-\alpha)\right] T_r}{1 - P_s P_{c_1}\left[\alpha + P_{c_2}(1-\alpha) - (1-\beta)\right]}} \quad (48)$$

<u>Transient State Probabilities</u>

Note that the transient in $P_{ar}\left[n\right]$ is proportional to ω_1^n. To assess the effect of the transient, consider a modified form for availability which we shall define as

$$P'_{ar}\left[k\right] \triangleq \frac{P\left[0;\ t_{s_k}\right] E\left[T_s\right]}{T_s} \quad (49)$$

Then we have for the condition that $P\left[B;\ T_{c_1}\right] = 1$;

$$P'_{ar}\left[k\right] \wedge (1-\beta) \frac{(1 - \omega_1^{k+1})}{1 - \omega_1} \frac{E\left[T_s\right]}{T_s} \quad (50)$$

$$\omega_1 = P_a P_{c_1}\left[\alpha + P_{c_2}\,(1-\alpha) - (1-\beta)\right] \qquad (51)$$

But we may set

$$\omega_1 \omega_1^{\,k} = \omega_1 \exp\left[-\frac{kT_s}{\tau}\right] \qquad (52)$$

where

$$\tau = -\frac{T_s}{\ln \omega_1} = \frac{T_s}{\ln\left(\dfrac{1}{P_a P_{c_1}\left[\alpha + P_{c_2}(1-\alpha)-(1-\beta)\right]}\right)} \qquad (53)$$

This expression has the dimensions of time. It is a measure of the expected time required for the state probabilities to reach their asymptotic values, given that the system initially enters checkout in a failed condition.

__Imperfect Repair and Undetectable Failures__

The above results may be readily generalized to account for the possibility of imperfect repair and an inherently undetectable failure rate. If we define

$P_{d_{T_s}}$ = The probability that none of those characteristics of the system which are tested will fail during T_s, given that they are nonfailed at entrance to that interval.

$P_{u_{T_s}}$ = The probability that none of those characteristics of the system which are not tested will fail during T_s, given that they are nonfailed at entrance to that interval.

$P_{d_{T_{c_1}}}$ = The probability that none of those characteristics of the system which are tested will fail during checkout before the test decision is made.

$P_{u_{T_{c_1}}}$ = The probability that none of those characteristics of the system which are not tested will fail during checkout before the test decision is made.

$P_{d_{T_{c_2}}}$ = The probability that none of those characteristics of the system which are tested will fail during T_{c_2}, given that they are nonfailed at the test decision and are passed.

$P_{u_{T_{c_2}}}$ = The probability that none of those characteristics of the system which are not tested will fail during T_{c_2}, given that they are nonfailed at the test decision and the equipment is passed.

μ_1 = The probability that a replaced unit is nonfailed.

μ_2 = The probability that a replaced unit is failed in an inherently detectable manner.

$1-\mu_1-\mu_2$ = The probability that a replaced unit is failed in an inherently undetectable manner.

λ_s = Failure rate of the inherently detectable characteristics during T_s.

λ_u = Failure rate of the inherently undetectable characteristics during T_s.

Given the above definitions it can be shown that for the same cycle and constraints discussed earlier

$$P_{tr}[\infty] = \frac{\dfrac{\mu_1(1-\ldots)\{1-(1-\ldots)\ldots\}}{\{1-\ldots P_d[T_s+T_c](1-\ldots)\}\{1+P_u[T_s+T_{c_1}](1-\ldots)(1-\ldots)-\ldots[T_s+T_c](1-\ldots)\}}\cdot\dfrac{1-e^{-(\lambda_s+\lambda_u)T_s}}{\lambda_u+\lambda_s}}{T_s+T_c+\dfrac{(1-\ldots)\{1-(1-\alpha)\ldots[T_s+T_c]\}\ldots}{1+P_d[T_s+T_{c_2}](1-\mu_2)(1-\ldots)-\ldots[T_s+T_c](1-\ldots)}} \qquad (5\ldots)$$

Continuous Monitoring Policy for Repair by Remove and Replace

Definitions and Assumptions

Let it be assumed that failures, false alarms, failure detection, restoration of false alarms to service, and repair of failures are Poisson distributed. We define,

λ_d = Failure rate associated with the characteristics of the equipment which are monitored in a test. A failure of a monitored characteristic is "inherently detectable in principle."

λ_u = Failure rate associated with the characteristics of the equipment which are not monitored during testing. A failure of such a characteristic is "inherently undetectable in principle" since it is always unobserved by the test.

α_c = False alarm rate associated with the monitored characteristics.

e = Rate of detection of failed characteristics of the "inherently detectable in principle" class.

μ_{11} = Rate of restoration of equipment to the nonfailed state. (It is specifically assumed that repair is by remove-and-replace so that the state of the equipment leaving repair is independent of the state during or entering repair.)

μ_{12} = Rate of restoration of equipment to service with one or more failures of the inherently detectable in principle class.

μ_{13} = Rate of restoration of equipment to service with one or more failures of the inherently undetectable in principle class, but no inherently detectable failures.

Further assume that

- All equipment is monitored continuously.
- Failures of the inherently undetectable in principle class can be caught only by false alarming one or more of the nonfailed, observed characteristics.
- An equipment is either nonfailed, failed detectably, or failed undetectably, and, if failed undetectably, may subsequently fail detectably.
- The equipment cannot fail during repair.

We define the following notation

$P_{ug}[t]$ = Probability of being "up" (assigned to service) and "good" (nonfailed) at time t.

$P_{ub_d}[t]$ = Probability of being up and bad, (failed) but detectable in principle at time t.

$P_{ub_u}[t]$ = Probability of being up and bad, and not detectable in principle at time t.

$P_{dg}[t]$ = Probability of being "down" (assigned to repair), but "good" (nonfailed) at time t.

$P_{db_d}[t]$ = Probability of being down with a detectable class of failure at time t.

$P_{db_u}[t]$ = Probability of being down with an undetectable class of failure at time t.

The Basic Difference/Differential Equations Of Transition

In view of the assumption of exponential holding times for the failure distribution, et al., the following difference equations may be written:

$$P_{ug}[t+\Delta t] = P_{ug}[t]\left\{1-(\lambda_d+\lambda_u+\alpha_c)\Delta t\right\} + P_{dg}[t]\mu_{11}\Delta t + P_{db_d}[t]\mu_{11}\Delta t + P_{db_u}[t]\mu_{11}\Delta t \tag{55a}$$

$$P_{ub_d}[t+\Delta t] = P_{ug}[t]\lambda_d\Delta t + P_{ub_d}[t](1-e\Delta t) + P_{dg}[t]\mu_{12}\Delta t + P_{db_d}[t]\mu_{12}\Delta t + P_{db_u}[t]\mu_{12}\Delta t + P_{ub_u}[t]\lambda_d\Delta t \tag{55b}$$

$$P_{ub_u}[t+\Delta t] = P_{ug}[t]\lambda_u\Delta t + P_{ub_u}[t]\left\{1-(\alpha_c+\lambda_d)\Delta t\right\}$$

$$+ P_{dg}[t]\mu_{13}\Delta t + P_{db_u}[t]\mu_{13}\Delta t + P_{db_d}[t]\mu_{13}\Delta t \qquad (55c)$$

$$P_{dg}[t+\Delta t] = P_{ug}[t]\alpha_c\Delta t + P_{dg}[t]\left(1-\sum_{i=1}^{3}\mu_{1i}\Delta t\right) \qquad (55d)$$

$$P_{db_d}[t+\Delta t] = P_{ub_d}[t]e\Delta t + P_{db_d}[t]\left(1-\sum_{i=1}^{3}\mu_{1i}\Delta t\right) \qquad (55e)$$

$$P_{db_u}[t+\Delta t] = P_{ub_u}[t]\alpha_c\Delta t + P_{db_u}[t]\left(1-\sum_{i=1}^{3}\mu_{1i}\Delta t\right) \qquad (55f)$$

All other transitions are of higher order in Δt and hence may be neglected. Noting that

$$\lim_{\Delta t\to 0}\frac{P[t+\Delta t]-P[t]}{\Delta t} = \frac{dP[t]}{dt} \triangleq \dot{P}[t] \qquad (56)$$

We may write the above set of difference equations as a set of differential equations by employing (56). In vector form

$$\begin{bmatrix} \dot{P}_{ug}[t] \\ \dot{P}_{ub_d}[t] \\ \dot{P}_{ub_u}[t] \\ \dot{P}_{dg}[t] \\ \dot{P}_{db_d}[t] \\ \dot{P}_{db_u}[t] \end{bmatrix} = \begin{bmatrix} -(\lambda_d+\lambda_u+\alpha_c) & 0 & 0 & \mu_{11} & \mu_{11} & \mu_{11} \\ \lambda_d & -e & \lambda_d & \mu_{12} & \mu_{12} & \mu_{12} \\ \lambda_u & 0 & -(\alpha_c+\lambda_d) & \mu_{13} & \mu_{13} & \mu_{13} \\ \alpha_c & 0 & 0 & -\sum_{i=1}^{3}\mu_{1i} & 0 & 0 \\ 0 & e & 0 & 0 & -\sum_{i=1}^{3}\mu_{1i} & 0 \\ 0 & 0 & \alpha_c & 0 & 0 & -\sum_{i=1}^{3}\mu_{1i} \end{bmatrix} \begin{bmatrix} P_{ug}[t] \\ P_{ub_d}[t] \\ P_{ub_u}[t] \\ P_{dg}[t] \\ P_{db_d}[t] \\ P_{db_u}[t] \end{bmatrix} \qquad (57)$$

<u>Steady State Solution</u>

The steady state solution for each component of the state vector may be obtained directly by noting that

$$\lim_{t\to\infty}\dot{P}[t] = 0 \qquad (58)$$

and $$\sum_{i,j} P_{ij}[t] = 1 \quad \text{for all } t. \qquad (59)$$

Also from the definition (3), it may be shown that

$$P_{ar}[t] = \frac{1}{T}\int_0^T P_{ug}[t]\,dt \qquad (60)$$

Hence $$P_{ar}[\infty] = P_{ug}[\infty] \qquad (61)$$

which for the equation set (57) is,

$$P_{ar}[\infty] = \frac{\mu_{11}\,e\,(\lambda_d+\alpha_c)}{(\alpha_c+\lambda_d+\lambda_u)\left\{e(\alpha_c+\lambda_d+\mu_{11}+\mu_{13})+\alpha_c\mu_{12}+\lambda_d\sum_{i=1}^{3}\mu_{1i}\right\}} \qquad (62)$$

The transient solutions to (57) may also be readily obtained but are of essentially academic interest except for the determination of the distribution of up-to-down and down-to-up times.

<u>Mixed Maintenance Policies</u>

In general, if a system consists of k subunits, each of which has a probability $P_{ug_i}[t]$ of being nonfailed and assigned to alert, the availability of the system as a whole will be given by

$$P_{ar}[T] = \frac{1}{T}\int_0^T \prod_{i=1}^{k} P_{ug_i}[t]\,dt \qquad (63)$$

If the maintenance policies of each of the subunits are independent in the sense that the maintenance policy of the ith subunit is unrelated to that of the jth subunit, e.g. scheduled or unscheduled down time for the ith subunit does not imply that down time is required of the jth subunit, and if

$$\lim_{t\to\infty} P_{ug_i}[t] \text{ exists} \qquad (64)$$

one may write

$$\lim_{T\to\infty} P_{ar}[T] = \lim_{T\to\infty}\frac{1}{T}\int_0^T \prod_{i=1}^{k} P_{ug_i}[t]\,dt = \prod_{i=1}^{k} P_{ug_i}[\infty]$$

$$= \prod_{i=1}^{k} P_{ar_i}[\infty] \qquad (65)$$

In particular, if a system consists of two subunits, the first of which is continuously monitored and the second of which is periodically monitored, the availability of the system as a whole is given by the product of equations (54) and (62). In general, if the maintenance policies are not strictly periodic, i.e. inspections and check-outs are not held to fixed calendar dates, relation (65) will hold. Note that this will always be the case when one subunit utilizes a periodic policy (strictly periodic or otherwise) and any number of other subunits are continuously and independently monitored.

Estimation of Parameters

Statement of the Problem

The preceding developments contain a number of factors such as λ_s, λ_u, μ_1, μ_2, α and β which are ordinarily not directly observable quantities. For example, the only way in which it may be determined by direct observation that a system is nonfailed is to attempt to operate it; but the very act of operating the system imposes stresses that may cause system failure. If one is attempting to determine the quiescent failure rate λ_s, how can we differentiate between failures due to turn on stress and running stress from those arising from the stresses of the quiescent mode of operation? The obvious answer is: by failure analysis. This answer is usually not at all satisfactory once the system has left the engineering development stage and become operational. The limitations of existing field data reporting systems, the expense of retaining the required technically qualified personnel, the logistic backlog of unrepaired items - - these and a host of related factors make extensive failure analysis quite an impractical undertaking. Fortunately, there is an adequate alternative - - statistical inference. The method of statistical inference may be understood by resorting to oversimplified examples.

Periodically Monitored System[10]

Referring to figure four, consider a periodically monitored system which is subjected to two modes of operation during each of which it exhibits an exponential failure distribution. During T_s it is in a quiescent mode of operation where it exhibits a failure rate λ_s. During T_c it is operated to determine whether it has failed. The failure rate during this time is λ_c. Assume:

- the point of test decision corresponds to the end point of T_c
- there is no significant turn on stress
- the test is perfect
- T_s is a constant
- T_c is a constant

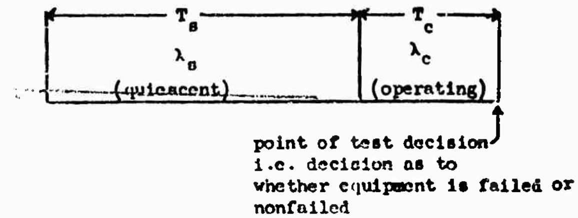


Figure 4.

Let there be two groups of initially nonfailed equipment numbering M_1 and M_2 total items respectively. Let the M_1 equipments remain in the quiescent mode for a time T_{s_1} and the M_2 equipments remain in the quiescent mode for a different time T_{s_2}. Each group is then subjected to a test of fixed length T_c, at the end of which it is observed that k_1 have failed in the first group and k_2 have failed in the second group. The probability that exactly k_1 failures will be observed in M_1 trials is

$$P\left[M_1, k_1\right] = \binom{M_1}{k_1}\left(P_{s_1} P_c\right)^{M_1 - k_1}\left(1 - P_{s_1} P_c\right)^{k_1} \tag{66}$$

where

$$P_{s_1} = e^{-\lambda_s T_{s_1}} \tag{67}$$

$$P_c = e^{-\lambda_c T_c} \tag{68}$$

For convenience, define

$$E_1 \triangleq \ln P\left[M_1, k_1\right] \tag{69}$$

then the maximum likelihood estimate $\hat{\lambda}_s$ for the true failure rate λ_s is given by solving the equation set

$$\frac{\partial E_i}{\partial \lambda_s} = 0; \; i = 1, 2 \tag{70}$$

By straightforward manipulation,

$$\hat{\lambda}_s = \frac{1}{T_{s_1} - T_{s_2}} \ln \left(\frac{1 - \frac{k_2}{M_2}}{1 - \frac{k_1}{M_1}} \right) \tag{71}$$

$$\hat{\lambda}_c = \frac{1}{T_c} \ln \frac{1}{1 - \frac{k_1}{M_1}} - \hat{\lambda}_s \frac{T_{s_1}}{T_c} \qquad (72)$$

Continuously Monitored System [10]

Estimates of the parameters of a continuously monitored system are equally easy to obtain. Consider, for example, a simple, continuously monitored system that can occupy the three states,

- nonfailed and assigned to alert $(U \wedge G)$
- nonfailed but false alarmed $(D \wedge G)$
- down in repair/replace $(D \wedge G) \vee (D \wedge B)$

Where the mean rates of transition between these states are λ, α, and μ as indicated in figure five.

$$\begin{tikzcd}
U \wedge G \arrow[r, "\lambda"] \arrow[d, "\alpha"] & D \wedge B \arrow[r, "\mu"] & U \wedge G \\
D \wedge G \arrow[d, "\mu"] & & \\
U \wedge G & &
\end{tikzcd}$$

Figure 5.

λ is the real failure rate, α is the false alarm rate and μ is the repair and/or replacement rate for both the $D \wedge B$ and $D \wedge G$ states. An important point to be noticed is that the average time (expected time) for a transition from $U \wedge G$ to $D \wedge B$ equals $\frac{1}{(\alpha + \lambda)}$. Similarly, the average transition time from $U \wedge G$ to $D \wedge G$ equals $\frac{1}{(\alpha + \lambda)}$. The basic reason for this phenomenon is that the causes of false alarm and the causes of real failures are acting concurrently.

The expected time from $U \wedge G$ to down (apparent failure) equals the probability that an apparent failure is actually a false alarm $\frac{\alpha}{(\alpha + \lambda)}$ multiplied by the expected (average) time to false alarm $\frac{1}{(\alpha + \lambda)}$ plus the probability that an apparent failure is actually a real failure $\frac{\lambda}{(\alpha + \lambda)}$ multiplied by the expected time to real failure $\frac{1}{(\alpha + \lambda)}$ That is

$$\frac{\alpha}{(\alpha + \lambda)} \left(\frac{1}{\alpha + \lambda}\right) + \frac{\lambda}{\alpha + \lambda} \left(\frac{1}{\alpha + \lambda}\right) = \frac{1}{\alpha + \lambda} \qquad (73)$$

Similarly the expected repair time equals

$$\frac{\alpha}{\alpha + \lambda} \left(\frac{1}{\mu}\right) + \frac{\lambda}{\alpha + \lambda} \left(\frac{1}{\mu}\right) = \frac{1}{\mu} \qquad (74)$$

Now suppose up time, down time and a failure analysis is obtained for M systems, Let u_i equal the time to apparent failure for the i^{th} system, let d_i equal the time for repair or replacement of the i^{th} system and let $f(i) = 1$ if the i^{th} system experienced a real failure, and let $f(i) = 0$ if the i^{th} system experienced a false alarm.

Since the expected value of u_i, $E(u_i) = \frac{1}{(\alpha + \lambda)}$ it follows that

$$E\left(\frac{1}{M} \sum_{i=1}^{M} u_i\right) = \frac{1}{(\alpha + \lambda)} \qquad (75)$$

Therefore an estimate of $\frac{1}{(\alpha + \lambda)}$ is obtained by computing

$$\frac{1}{M} \sum_{i=1}^{M} u_i \qquad (76)$$

That is

$$\left[\widehat{\frac{1}{(\alpha + \lambda)}}\right] = \frac{1}{M} \sum_{i=1}^{M} u_i \qquad (77)$$

An estimate of $\frac{\lambda}{(\alpha + \lambda)}$ (the proportion of apparent failures which actually were real failures) can be obtained by computing

$$\frac{1}{M} \sum_{i=1}^{M} f(i) \qquad (78)$$

That is

$$\left[\widehat{\frac{\lambda}{(\alpha + \lambda)}}\right] = \frac{1}{M} \sum_{i=1}^{M} f(i) \qquad (79)$$

Then an estimate of λ, $\hat{\lambda}$ is obtained by computing

$$\frac{\left[\widehat{\frac{\lambda}{(\alpha + \lambda)}}\right]}{\left[\widehat{\frac{1}{(\alpha + \lambda)}}\right]} = \hat{\lambda} = \frac{\frac{1}{M} \sum_{i=1}^{M} f(i)}{\frac{1}{M} \sum_{i=1}^{M} u_i} = \frac{\sum_{i=1}^{M} f(i)}{\sum_{i=1}^{M} u_i} \qquad (80)$$

Similarly

$$\hat{\alpha} = \frac{1 - \frac{1}{M} \sum_{i=1}^{M} f(i)}{\frac{1}{M} \sum_{i=1}^{M} u_i} = \frac{M - \sum_{i=1}^{M} f(i)}{\sum_{i=1}^{M} u_i} \qquad (81)$$

Since the expected value of d_i,

$$E(d_i) = \frac{1}{\mu}, \quad E\left(\frac{1}{M} \sum_{i=1}^{M} d_i\right) = \frac{1}{\mu} \qquad (82)$$

therefore the estimate of μ is given by

$$\hat{\mu} = \frac{M}{\sum_{i=1}^{M} d_i} \qquad (83)$$

Thus, the estimate of each of the parameters of this model λ, α and μ are obtained.

It will be noted that we have assumed the need for failure analysis in the case of a continuously monitored system. We can partially circumvent this necessity by fitting the theoretical distributions of up-to-down and down-to-up times to actual field data. For example the apparent failure distribution for a continuous monitoring policy may be obtained from Equation set (57) by setting the repair rates equal to zero and selecting the initial conditions such that the system is initially up. Because the factors e and α_c are involved, the result will not be the true system failure distribution, but rather, the distribution of up-to-down times. We have from (57),

This equation inverts to the general form,

$$P_{uu}[t] = A\, e^{-(\lambda_d+\lambda_u+\alpha_c)t} + B\, e^{-et} + C\, e^{-(\alpha_c+\lambda_d)t} \qquad (89)$$

where

$$A \equiv 0 \qquad (90)$$

$$B = -\frac{(\alpha_c - e)\, P_{ub_d}[o] + \lambda_d}{\alpha_c + \lambda_d - e} \qquad (91)$$

$$B + C = 1 \qquad (92)$$

This is the probability of remaining up, given that the equipment is initially up. Note that, although each of the parameters e, α_c, λ_u, and λ_d are associated with exponentials, the resultant of their interaction will not, in general, be an exponential since the sum of exponentials is not representable by a single exponential term. There are four possible exceptions. (con't page 15)

$$\begin{bmatrix} \dot{P}_{ug}[t] \\ \dot{P}_{ub_d}[t] \\ \dot{P}_{ub_u}[t] \end{bmatrix} = \begin{bmatrix} -(\lambda_d+\lambda_u+\alpha_c) & 0 & 0 \\ \lambda_d & -e & \lambda_d \\ \lambda_u & 0 & -(\alpha_c+\lambda_d) \end{bmatrix} \begin{bmatrix} P_{ug}[t] \\ P_{ub_d}[t] \\ P_{ub_u}[t] \end{bmatrix} \qquad (84)$$

and

$$P_{ug}[o] + P_{ub_d}[o] + P_{ub_u}[o] = 1 \qquad (85)$$

by assumption.

Taking the Laplace transform[4] of (84)

$$\begin{bmatrix} P_{ug}[o] \\ P_{ub_d}[o] \\ P_{ub_u}[o] \end{bmatrix} = \begin{bmatrix} -(s+\lambda_d+\lambda_u+\alpha_c) & 0 & 0 \\ \lambda_d & -(s+e) & \lambda_d \\ \lambda_u & 0 & -(s+\lambda_d+\alpha_c) \end{bmatrix} \begin{bmatrix} P_{ug}[s] \\ P_{ub_d}[s] \\ P_{ub_u}[s] \end{bmatrix} \qquad (86)$$

where the probability $P_{uu}[t]$ of remaining up, given that the system is initially up, is

$$P_{uu}[t] = P_{ug}[t] + P_{ub_d}[t] + P_{ub_u}[t] \qquad (87)$$

Hence;

$$P_{uu}[s] = \frac{-(s+e)(s+\alpha_c+\lambda_d)P_{ug}[o] + (s+\lambda_d+\lambda_u+\alpha_c)\left\{(s+\alpha_c+\lambda_d)P_{ub_d}[o] + \lambda_d P_{ub_u}[o] + \lambda_d P_{ug}[o]\right\} - (s+e)\left\{(s+\lambda_d+\lambda_u+\alpha_c)P_{ub_u}[o] + \lambda_u P_{ug}[o]\right\}}{(s+\lambda_d+\lambda_u+\alpha_c)(s+e)(s+\alpha_c+\lambda_d)} \qquad (88)$$

If,

- $e >> \alpha_c + \lambda_d$
- $\alpha_c + \lambda_d >> e$
- ~~$e \equiv \alpha_c + \lambda_d$~~
- $P_{ub_d}[o] = \frac{\lambda_d}{e - \alpha_c}$ (93)

then the net distribution is a single exponential. The second and third possibilities are extremely unlikely to occur in practice. The fourth possibility will not be observable from field data, provided that the system has reached steady state. Once steady state is achieved we will observe

$$P_{ub_d}[o] = P_{ub_d}[\infty]$$

$$= \frac{1}{1 + e\dfrac{(\lambda_d + \mu_{11} + \mu_{13} + \alpha_c)}{\lambda_d \sum_{i=1}^{3} \mu_{1i} + \mu_{12}\alpha_c}} \quad (94)$$

which will quite clearly not result in a single exponential term for $P_{uu}[t]$.

The net effective repair distribution may be obtained from equation set (57) by assuming that all the equipment is initially down, that no additional equipment enters the down state after $t = 0$, and then solving the equation set for the time to return to the up state. In this case equation set (57) reduces to

$$\begin{bmatrix} \dot{P}_{dg}[t] \\ \dot{P}_{db_d}[t] \\ \dot{P}_{db_u}[t] \end{bmatrix} = \begin{bmatrix} -\sum_{i=1}^{3}\mu_{1i} & 0 & 0 \\ 0 & -\sum_{i=1}^{3}\mu_{1i} & 0 \\ 0 & 0 & -\sum_{i=1}^{3}\mu_{1i} \end{bmatrix} \begin{bmatrix} P_{dg}[t] \\ P_{db_d}[t] \\ P_{db_u}[t] \end{bmatrix}$$

(95)

where

$$P_{dg}[0] + P_{db_d}[0] + P_{db_u}[0] = 1 \quad (96)$$

and the probability $P_{dd}[t]$ of remaining down, given that the system is initially down, is

$$P_{dd}[t] = P_{dg}[t] + P_{db_d}[t] + P_{db_u}[t] \quad (97)$$

Proceeding exactly as outlined above we have

$$P_{dd}[s] = \frac{1}{s + \sum_{i=1}^{3}\mu_{1i}} \quad (98)$$

Therefore

$$P_{dd}[t] = e^{-\left(\sum_{i=1}^{3}\mu_{1i}\, t\right)} \quad (99)$$

This is the probability of remaining down given that the equipment is initially down.

Examples

Scope

We shall very briefly illustrate three applications of the above developments.

- A fit of the theoretical up-to-down distribution to the field data of a continuously monitored system.
- A fit of the theoretical down-to-up distribution to the field data of a continuously monitored system.
- Calculation of β, α, and λ_s for a discretely monitored system.

Fitting the Apparent Failure Distribution of a Continuously Monitored System

Figure six illustrates a sample of data for the reported up-to-down times of an ICBM System. The data is given in arbitrary time dimensions for reasons of security. A Chi squared test of the data indicated that it is exponential at the 90% acceptance level. The theoretical distribution (equation (89) indicates that exponentiality of reported up-to-down times is consistent with the assumed model if, and only if, one of the four conditions of (93) hold. Subsequent shop action on reported failures indicated that 20% of the rejects were in fact good (false alarmed), hence we are forced to accept the first of conditions (93), i.e.

$$\alpha_c + \lambda_d << e$$

Hence the arithmetic mean of the data of figure six (4.1 in the arbitrary time units) is the maximum likelihood estimate of $1/(\alpha_c + \lambda_d)$.

Further, since 20% of the rejects were in fact good, we estimate that

$$\hat{\alpha}_c = \frac{(\text{number of false alarms})}{(\text{total number of rejects})} (\widehat{\lambda_d + \alpha_c}) \quad (100)$$

$$= 0.2 \times \frac{1}{4.1} \simeq 0.05 \text{ time units}$$

and we estimate that

$$\hat{\lambda}_d = \left(1 - \frac{(\text{number of false alarms})}{(\text{total number of rejects})}\right)(\widehat{\lambda_d + \alpha_c}) \quad (101)$$

$$= (1-0.2)\,\frac{1}{4.1} = 0.2 \text{ time units}$$

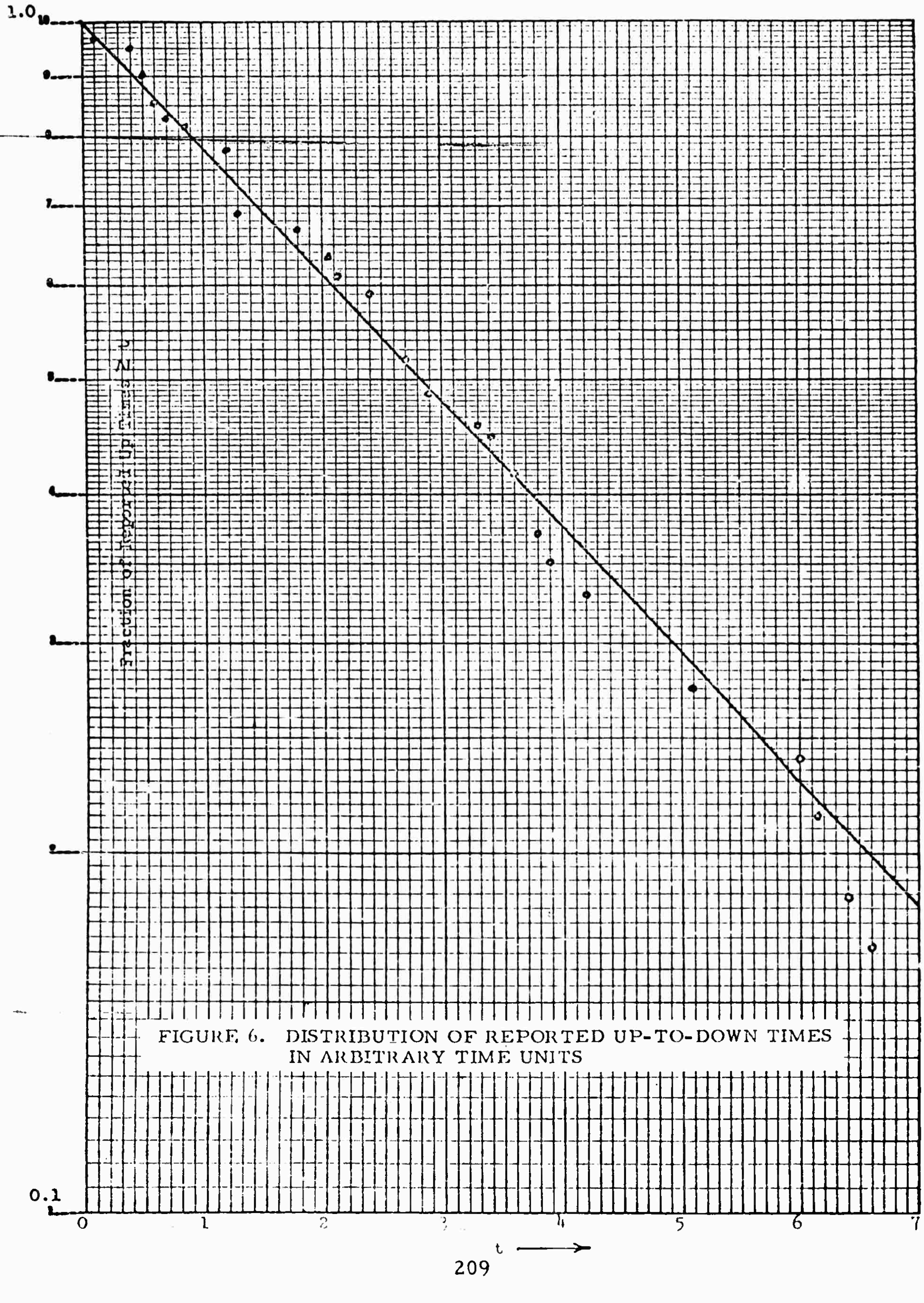


FIGURE 6. DISTRIBUTION OF REPORTED UP-TO-DOWN TIMES IN ARBITRARY TIME UNITS

The Down-To-Up Distribution

Figure seven illustrates a sample of data for the reported down-to-up times of an ICBM System. The dimensions of the data are arbitrary for reasons of security. The previously assumed model indicated that the distribution would be exponential if the underlying repair (remove/replace) distribution was exponential (99). As indicated in figure seven, the field data suggests that the underlying distribution is log normal. This tendency has been noted elsewhere in the literature.[8] Since this type of distribution cannot be handled analytically, recourse to the "black box" technique of Morse[11] is necessary. Without regard for the actual structure of the remove/replace/repair process we may postulate an "n" state exponential process that duplicates the behavior of the observed data. In the present instance the data may be fitted nicely by assuming two internal states. A failure is assigned to the first down state with probability "a" and to the second down state with probability "1-a". Return to the up state from the first down state occurs at rate μ_1, and from the second down state with rate μ_2. Under these assumptions, the down-to-up distribution can be shown to be

$$P_{du}\left[t\right] = a\, e^{-\mu_1 t} + (1 - a)\, e^{-\mu_2 t} \qquad (102)$$

The curved line in figure seven shows the results of assuming a two state process for the observed sample of data. It will be noted that either the two state exponential process or the log normal curve are equally representative of the observed data.

Parameter Estimation by Inference Methods for a Discretely Monitored System

At the time of this writing the results of estimating parameters for discretely monitored systems from field data by inference methods is not complete. However, the potential usefulness of the method has been investigated in detail by the Monte Carlo technique on an IBM 7090 computer.[10] As an example of the results which have been achieved, consider the controlled system exercise indicated in figure eight. Assume that each time that a (sub) system leaves repair it is subjected to a sequence of three checkouts of duration $T_c^{(1)}$, $T_c^{(2)}$, and $T_c^{(3)}$ with the test decision occuring at the end of each checkout. Further assume

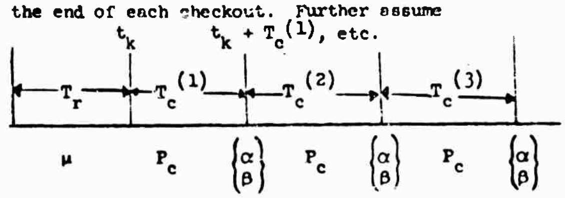


Figure 8

that regardless of whether the system fails or passes at the first and second decision it is subjected to the subsequent checkout without recourse to a diagnosis/remove/replace cycle. There are eight possible outcomes of this test sequence: (PPP), (PPF), (PFP),(PFF), (FPP), (FPF), (FFP), and (FFF) where P denotes "pass" and F denotes "flunk". The probability of any one of these outcomes is readily calculated, for example

$$\text{prob}\left[PPP\right] = (1-\mu P_c)\,\beta^3 + \mu P_c(1-P_c)(1-\alpha)\beta^2 + \mu P_c^2(1-P_c)(1-\alpha)^2\,\beta + \mu P_c^3(1-\alpha)^3 \qquad (103)$$

If this sequence is repeated for M systems, all of one kind, and N [PPP] are the number of systems with outcomes PPP etc, then it can be shown that the maximum likelihood estimates of β, α, P_c, and μ are given by

$$\hat{\beta} = 1 - \frac{N\left[PFF\right] - N\left[FPF\right]}{N\left[PFP\right] + N\left[PFF\right] - N\left[FFP\right] - N\left[FPF\right]} \qquad (104)$$

$$\hat{\alpha} = \frac{N\left[FPF\right] - N\left[FFP\right]}{N\left[PPF\right] + N\left[FPF\right] - N\left[PFP\right] - N\left[FFP\right]} \qquad (105)$$

$$\hat{P}_c = \frac{N\left[PPF\right] + N\left[FPF\right] - N\left[PFP\right] - N\left[FFP\right]}{N\left[PFP\right] + N\left[PFF\right] - N\left[FPF\right] - N\left[FPP\right]} \qquad (106)$$

$$\hat{\mu} = \frac{\dfrac{N\left[FPP\right] + N\left[FPF\right] + N\left[FFP\right] + N\left[FFF\right]}{M} - 1 + \hat{\beta}}{\hat{P}_c\,(\hat{\alpha} - 1 + \hat{\beta})} \qquad (107)$$

Having estimates for the above parameters, an estimate for λ_s may be obtained by considering a second set of K systems coming out of repair, all of which enter standby and remain in standby for the same length of time (T_s) and which are checked out just once. Let R be the number of these K systems which fail the checkout; then the expected value of (R/K),

$$E(R/K) = 1 - \beta + \mu P_c e^{-\lambda_s T_s}(\alpha - 1 + \beta). \qquad (108)$$

Using the estimates of β, μ, P_c, and α which have been obtained, λ_s can be estimated by using the above equation; this results in

$$\hat{\lambda}_s = -\frac{1}{T_s} \ln\left(\frac{(R/K) - 1 + \hat{\beta}}{\hat{\mu}\hat{P}_c\,(\hat{\alpha} - 1 + \hat{\beta})}\right) \qquad (109)$$

Monte Carlo runs of the sequence of figure eight were performed many times using the "true" values,

$$\begin{aligned} K &= M = 500 \\ \alpha &= 0.1 \\ \beta &= 0.1 \\ \mu &= 0.8 \\ P_c &= 0.75 \\ \lambda_s &= 0.0021 \text{ failures/day} \end{aligned} \qquad (110)$$

A typical run yielded (111)

$R = 284$	$N\left[PPP\right] = 135$	$N\left[FPP\right] = 14$
$T_s = 7$ days	$N\left[PPF\right] = 50$	$N\left[FPF\right] = 29$
	$N\left[PFP\right] = 18$	$N\left[FFP\right] = 20$
	$N\left[PFF\right] = 76$	$N\left[FFF\right] = 158$

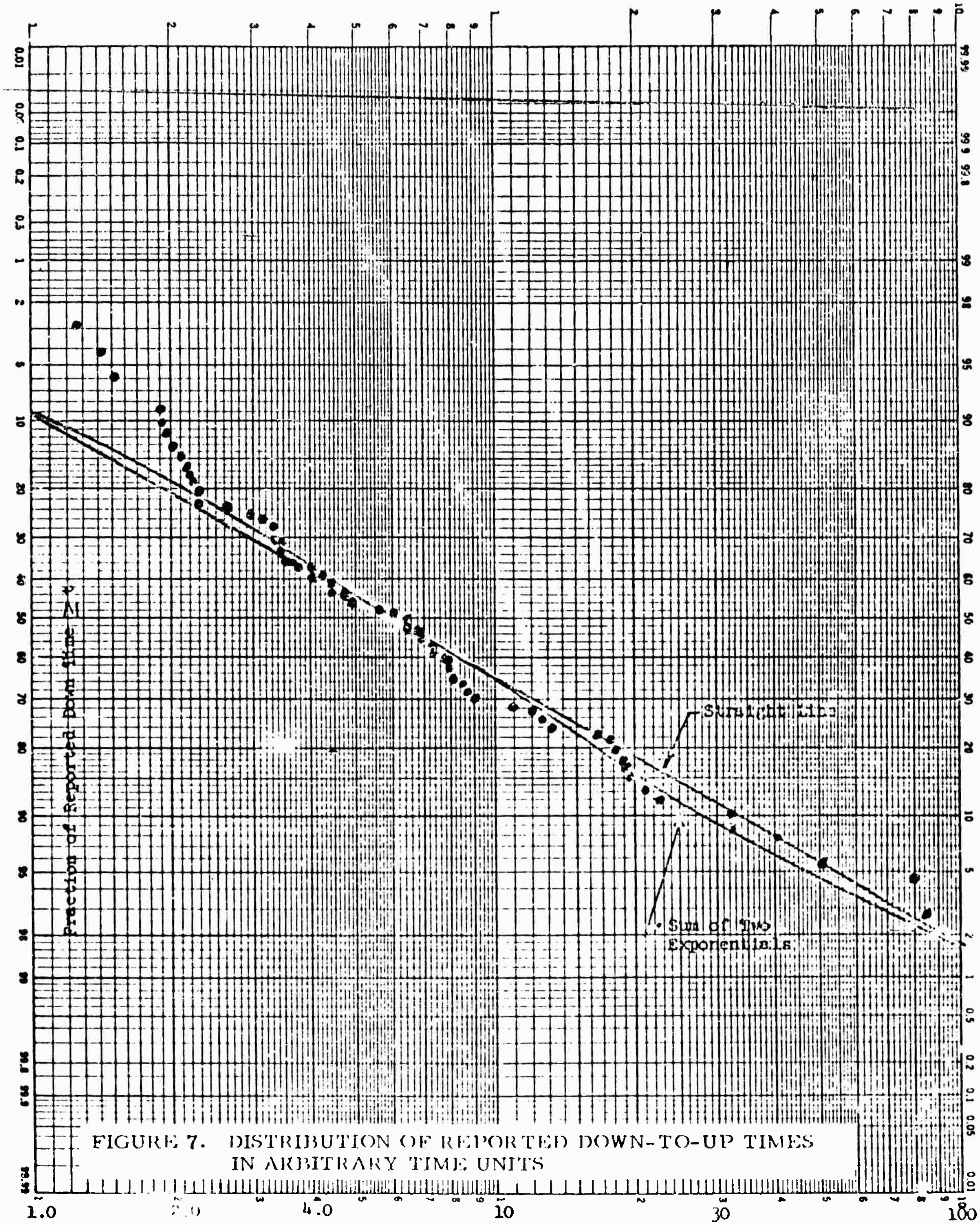


FIGURE 7. DISTRIBUTION OF REPORTED DOWN-TO-UP TIMES IN ARBITRARY TIME UNITS

From which it was estimated that

$$\hat{\beta} = 0.08$$
$$\hat{\alpha} = 0.22$$
$$\hat{P}_c = 0.80 \qquad (112)$$
$$\hat{\mu} = 0.85$$
$$\hat{\lambda}_s = 0.0019 \text{ failures/day}$$

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APPENDIX II

of

EXAMPLE B

The Probability of Launch When Two Attempts are Permissible

INTRODUCTION

It is assumed that the sites are sufficiently provisioned that two tactical launch attempts may be made in the environment which exists post first attack, given that the site is assigned to alert. One launch attempt may be made if the site is initially off alert. It is the purpose of this appendix to derive an expression for the probability of successful launch under these conditions.

DERIVATION

We shall treat a simplified situation wherein it is assumed that:

- All latent failures are detected by a <u>tactical</u> launch attempt.
- No aborts arise from false alarms.
- Repair of an aborted launch attempt is perfect, occurs at a net rate μ_c, and has the density function

$$p_4(t_r) = \mu_c e^{-\mu_c t_r} \tag{B-1}$$

- The conditional density distribution for the duration of launch attempts, irrespective of the way the attempt terminates; given that no latent failures exist, is given by

$$p(t_{CD}) = L e^{-L(t_{CD}-\theta)} U[t_{CD}-\theta] \tag{B-2}$$

- The conditional density distribution for the duration of launch attempts, given that latent failures exist, is given by

$$p[t_{CD}] = \delta[t_{CD}-\theta] \tag{B-3}$$

- The density distribution for the probability of system failure given that the system is initially nonfailed, is given by

$$p[t_f] = \lambda_L e^{-\lambda_L t_f} \tag{B-4}$$

where λ_L is the total system failure rate.

There are three initial system states

- Up and nonfailed
- up and failed (unknown latent failure)
- down (in repair)

We shall further assume that the only permissible state transitions are defined by Figure B-1. State 1 is a launch attempt commencing from the truly ready state. It terminates either in a launch (State L) or an abort (State 4). State 2 is a launch attempt commencing from the apparent ready, but a truly failed state. It terminates with probability one in the repair State 4. State 3 accounts for those missiles which are not assigned up when the execution directive is received. Only one repair is permitted from this state. State 5 is a launch attempt entered from repair. Exit from State 5 to State 6 terminates the multiple launch attempt sequence in a permanent down state.

The conditional probability of terminating a launch attempt with a launch; given that the missile/site is nonfailed at the time of initiation of any launch attempt is given from Equations (B-2) and (B-4) by;

$$P_1[t_L/U\wedge G] = \int_0^{t_f} e^{-\lambda_L t_{CD}} L e^{-L(t_{CD}-\theta)} U[t_{CD}-\theta]\, d\, t_{CD} \tag{B-5}$$

$$= P_{CD}[\infty] \left\{ 1 - e^{-(\lambda_L+L)(t_f-\theta)} \right\} U[t_L-\theta]$$

where

$$P_{CD}[\infty] \triangleq \int_0^{\infty} e^{-\lambda_L t_{CD}} L e^{-L(t_{CD}-\theta)} U[t_{CD}-\theta]\, d\, t_{CD} \tag{B-6}$$

$$= \frac{L}{L+\lambda_L} e^{-\lambda_L \theta}$$

and

$$p_1[t_L/U\wedge G] = (\lambda_L + L) P_{CD}[\infty] e^{-(\lambda_L+L)(t_L-\theta)} U[t_L-\theta] \tag{B-7}$$

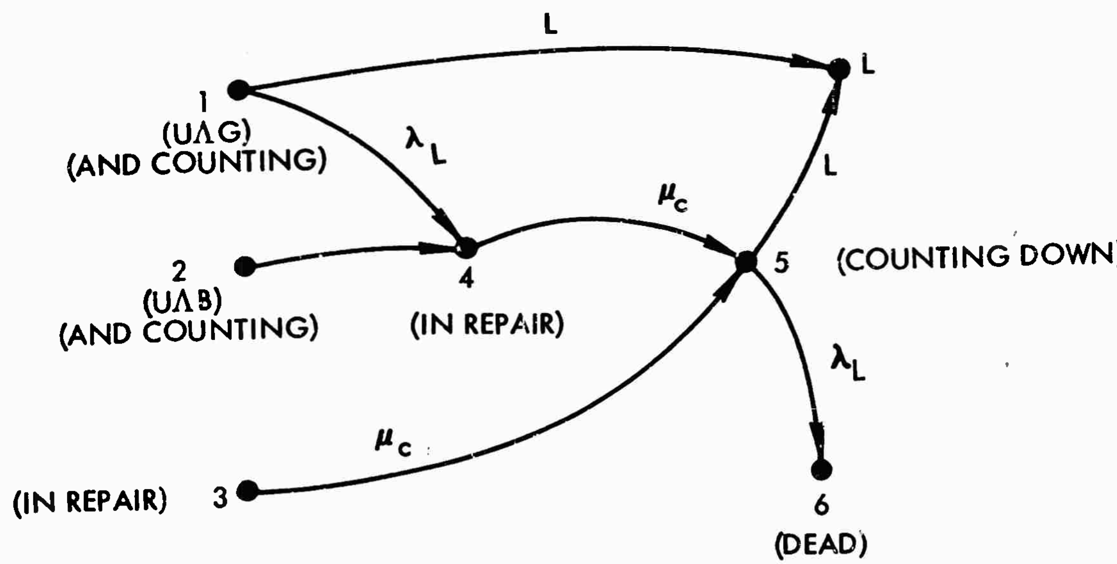


FIGURE B-1 PERMISSIBLE STATE TRANSITIONS IN MULTIPLE LAUNCH ATTEMPT SEQUENCE

The conditional probability of terminating a launch attempt with an abort; given that the missile/site is nonfailed at the time of initiation of any launch attempt is given from Equations (B-2) and (B-4) by;

$$P_2[t_f/U \wedge G] = \int_0^{t_f} (1 - e^{-\lambda_L t_{CD}}) \; L e^{-L(t_{CD}-\theta)} U[t_{CD}-\theta] \, d\, t_{CD} \tag{B-8}$$

$$= \left\{1 - e^{-L(t_f-\theta)} - P_{CD}[\infty](1 - e^{-(L+\lambda_L)(t_f-\theta)})\right\} U[t_f-\theta]$$

and

$$p_2[t_f/U \wedge G] = \left\{ L e^{-L(t_f-\theta)} - P_{CD}[\infty]\,(L+\lambda_L)\, e^{-(L+\lambda_L)(t_f-\theta)} \right\} U[t_f-\theta] \tag{B-9}$$

$$= 0 \text{ elsewhere}$$

The conditional probability of terminating a launch attempt with an abort; given that the missile/site contains a latent failure at entrance to countdown is given from Equation (B-3);

$$P_3[t_f/U \wedge B] = U[t_f-\theta] \tag{B-10}$$

and

$$p_3[t_f/U \wedge B] = \delta[t_f-\theta] \tag{B-11}$$

The conditional probability of launch in two or less attempts; given that the missile/site enters from the truly ready state is given by

$$P_L\cancel{[t/U \wedge G]} = P[t_L \leq t] + P[t_f + t_r + t_L \leq t] \tag{B-12}$$

The conditional probability of launch in two or less attempts; given that the missile/site enters from the up condition with a latent failure is given by;

$$P_L[t/U \wedge B] = P[\theta + t_r + t_L \leq t] \tag{B-13}$$

The conditional probability of launch in two or less attempts; given that the missile/site is in repair at the time of receipt of the execution directive is given by,

$$P_L[t/\bar{u}] = P[t_r + t_L \le t] \tag{B-14}$$

We ~~may express true readiness~~ A [∞]) as the product of apparent readiness $P_u[\infty]$ and the probability that the system is nonfailed; given that it is assigned to alert, namely $P_{g/u}[\infty]$. Then,

$$P_{g/u}[\infty] = \frac{A[\infty]}{P_u[\infty]} \tag{B-15}$$

and we may then write the total probability of launching in two or less attempts as;

$$\begin{aligned} P_L[t] = \; & A[\infty] \left\{ P[t_L \le t] + P[t_f + t_r + t_L \le t] \right\} \\ & + P_u[\infty] \left(1 - \frac{A[\infty]}{P_u[\infty]}\right) P[\theta + t_r + t_L \le t] \\ & + (1 - P_u[\infty]) \, P[t_r + t_L \le t] \end{aligned} \tag{B-16}$$

This expression is readily evaluated by resorting to Laplace transforms. We have that

$$\begin{aligned} P_L[s] = \; & A[\infty] \frac{p_1[s]}{s} + A[\infty] \frac{p_2[s]\, p_4[s]\, p_1[s]}{s} \\ & + P_u[\infty] \left(1 - \frac{A[\infty]}{P_u[\infty]}\right) \frac{p_3[s]\, p_4[s]\, p_1[s]}{s} \\ & + (1 - P_u[\infty]) \frac{p_4[s]\, p_1[s]}{s} \end{aligned} \tag{B-17}$$

where;

$$p_1[s] = \frac{(\lambda_L + L)\, P_{CD}[\infty]\, e^{-s\theta}}{s + \lambda_L + L}$$

$$p_2[s] = \frac{L\,e^{-s\theta}}{s+L} - \frac{P_{CD}[\infty]\,(\lambda_L+L)\,e^{-s\theta}}{s+\lambda_L+L} \tag{B-18}$$

$$p_3[s] = e^{-s\theta} \tag{B-19}$$

$$p_4[s] = \frac{\mu_c}{s+\mu_c} \tag{B-20}$$

Carrying out the multiplications indicated in Equation (B-17) and performing the inverse transformation we arrive at the final expression,

$$\begin{aligned}
P_L[t] = &\Big\{ A[\infty]\,P_{CD}[\infty]\,\big(1 - e^{-(\lambda_L+L)(t-\theta)}\big) \\
&+ (1 - P_u[\infty])P_{CD}[\infty]\,\Big(1 - \frac{\mu_c}{\mu_c-L-\lambda_L}\,e^{-(\lambda_L+L)(t-\theta)} \\
&+ \frac{L+\lambda_L}{\mu_c-L-\lambda_L}\,e^{-\mu_c(t-\theta)}\Big)\Big\}\,U[t-\theta] \\
&+ A[\infty]\,P_{CD}[\infty]\Big\{1 - \frac{L(L+\lambda_L)}{(\mu_c-L)(\mu_c-L-\lambda_L)}\,e^{-\mu_c(t-2\theta)} \\
&+ \frac{\mu_c L}{\lambda_L(\mu_c-L-\lambda_L)}\,e^{-(L+\lambda_L)(t-2\theta)} - \frac{\mu_c(L+\lambda_L)}{\lambda_L(\mu_c-L)}\,e^{-L(t-2\theta)}\Big\}\,U[t-2\theta] \\
&- A[\infty]\,P_{CD}^2[\infty]\Big\{1 - \frac{(L+\lambda_L)^2}{(\mu_c-L-\lambda_L)^2}\,e^{-\mu_c(t-2\theta)} \\
&- \frac{(t-2\theta)\,\mu_c\,(L+\lambda_L)}{(\mu_c-L-\lambda_L)}\,e^{-(L+\lambda_L)(t-2\theta)} \\
&- \frac{\mu_c(\mu_c-2L-2\lambda_L)}{(\mu_c-L-\lambda_L)^2}\,e^{-(L+\lambda_L)(t-2\theta)}\Big\}\,U[t-2\theta] \\
&+ (P_u[\infty] - A[\infty])\,P_{CD}[\infty]\Big\{1 + \frac{(L+\lambda_L)}{\mu_c-L-\lambda_L}\,e^{-\mu_c(t-2\theta)}
\end{aligned} \tag{B-21}$$

$$- \frac{\mu_c}{\mu_c - L - \lambda_L} e^{-(L+\lambda_L)(t-2\theta)} \Big\} U[t-2\theta]$$

Note that as $t \rightarrow \infty$;

$$P_L[\infty] = A[\infty] P_{CD}[\infty] + P_{CD}[\infty] (1 - A[\infty] P_{CD}[\infty]) \qquad \text{(B-22)}$$

It is unfortunate that neither the transient solution nor the steady state value can be expressed as the simple product of readiness and reliability. Due note should be taken of this result since it indicates that the formal mathematical structure adopted by Task Group II is too restrictive under certain circumstances.

APPENDIX III

of

EXAMPLE B

Derivation of the Expressions for the Expected Change of Status Delay in a Several Unit System

INTRODUCTION

In a system composed of several subunits all of which enter scheduled check-out at the same time, the system as a whole is not reassigned to alert status until the last subunit is checked out and/or repaired. Since each subunit is itself a separate system, such a policy implies a delay in the change of status of one or more subunits. During this delay period these subunits may fail since they are presumably being stressed at their normal standby stress levels while waiting reassignment to alert. Therefore, this delay time should be accounted for. It is the purpose of this appendix to show how this delay time is computed for the subsystem <CDEF> .

DERIVATION

Table C-I is a truth table indicating all the possible down state combinations which can arise from the <CDEF> subsystem. Zero in the table denotes a go checkout for the individual subunits C, D, E, or F. A one in the table denotes a no-go checkout. In accordance with the data summary (Table II of the text) we obtain the total system checkout time, including repair, for each possible combination of subunit states as indicated. To determine which logical combinations of the subunits give rise to which total times, let us denote the fact that subunit i entered repair by means of its letter designator i, and the fact that it has a go checkout by $\bar{i}$.

TABLE C-I

System Down Time by Subunit State as a Result of Checkout and/or Repair

C	D	E	F	Total System Down Time
0	0	0	0	T_c
0	0	0	1	$T_c + T_r^F$
0	0	1	0	$T_{c_1}^E + T_r^E$
0	0	1	1	$T_{c_1}^E + T_r^E$
0	1	0	0	$T_{c_1}^D + T_r^D$
0	1	0	1	$T_{c_1}^D + T_r^D$
0	1	1	0	$T_{c_1}^E + T_r^E$
0	1	1	1	$T_{c_1}^E + T_r^E$
1	0	0	0	T_c
1	0	0	1	$T_c + T_r^F$
1	0	1	0	$T_{c_1}^E + T_r^E$
1	0	1	1	$T_{c_1}^E + T_r^E$
1	1	0	0	$T_{c_1}^D + T_r^D$
1	1	0	1	$T_{c_1}^D + T_r^D$
1	1	1	0	$T_{c_1}^E + T_r^E$
1	1	1	1	$T_{c_1}^E + T_r^E$

Then the duration T_c, for example, occurs whenever the following logical proposition is true;

$$X = \bar{C}\,\bar{D}\,\bar{E}\,\bar{F} + \bar{C}\,\bar{D}\,\bar{E}\,F + C\,\bar{D}\,\bar{E}\,\bar{F} + C\,\bar{D}\,\bar{E}\,F$$

But this may be readily simplified by the rules of Boolean algebra to yield,

$$X = \bar{D}\,\bar{E}$$

The probability that X is true is given by

$$P[X] = (1 - \overline{P_D[F]})\,(1 - \overline{P_E[F]})$$

where $\overline{P_i[F]}$ is the probability of failing the test for the ith subunit.

Therefore, the contribution to total system down time $e[T_c]$ arising from T_c is

$$e[T_c] = (1 - \overline{P_D[F]})\,(1 - \overline{P_E[F]})\;T_c$$

The expected system down time $\bar{t}_d$ is the sum of the individual contributions to expected down time shown in Table C-2.

$$\bar{t}_d = \bar{D}\,\bar{E}\,T_c + E(T_{c_1}^{\;E} + T_r^{\;E}) + D\,\bar{E}\,(T_{c_1}^{\;D} + T_r^{\;D}) + \bar{D}\,\bar{E}\,F\,T_r^{\;F}$$

TABLE C-II

Contributions to Expected System Down Time

Down Time Duration	Probability of Occurance
T_c	$(1 - \overline{P_D[F]}\ (1 - \overline{P_E[F]})$
$T_{c_1}{}^E + T_r{}^E$	$\overline{P_E[F]}$
$T_{c_1}{}^D + T_r{}^D$	$\overline{P_D[F]}\ (1 - \overline{P_E[F]})$
$T_r{}^F$	$(1 - \overline{P_D[F]})\ (1 - \overline{P_E[F]})\ \overline{P_F[F]}$

The expected status delay time for each subunit may likewise be found by returning to Table C-I. For example, consider subunit C. The delay time $T_{r_1}{}^C$ which occurs when C has a go checkout is derived from the first eight entries of this table by subtracting the C go checkout duration from the total system down times as shown below in Table C-III.

TABLE C-III

Logical Table for Derivation of C Status Delay Time for a C go Checkout

D	E	F	$T_{r_1}^{C}$
0	0	0	0
0	0	1	T_r^F
0	1	0	$T_{c_1}^{E} + T_r^{E} - T_c$
0	1	1	$T_{c_1}^{E} + T_r^{E} - T_c$
1	0	0	$T_{c_1}^{D} + T_r^{D} - T_c$
1	0	1	$T_{c_1}^{D} + T_r^{D} - T_c$
1	1	0	$T_{c_1}^{E} + T_r^{E} - T_c$
1	1	1	$T_{c_1}^{E} + T_r^{E} - T_c$

Then;

$$T_{r_1}^{C} = \overline{D}\,\overline{E}\,\overline{F}\,(0) + \overline{D}\,\overline{E}\,F\,T_r^{F} + E(T_{c_1}^{E} + T_r^{E} - T_c)$$

$$+ D\,\overline{E}(T_{c_1}^{D} + T_r^{D} - T_c)$$

In this case, the table is completely symmetrical so that for a no go on subunit C,

$$T_{r_2}^{C} = \overline{D}\,\overline{E}\,\overline{F}\,(T_c - T_{c_1}^{C} - T_{r_1}^{C}) + \overline{D}\,\overline{E}\,F\,(T_c + T_r^{F} - T_{c_1}^{C} - T_{r_1}^{C})$$

$$+ E\,(T_{c_1}^{E} + T_r^{E} - T_{c_1}^{C} - T_{r_1}^{C}) + D\,\overline{E}\,(T_{c_1}^{D} + T_r^{D} - T_{c_1}^{C} - T_{r_1}^{C})$$

Similarly, for the remaining subsystems,

$$T_{r_1}{}^{D} = \overline{E}\,\overline{F}\,(0) + \overline{E}\,F\,T_r{}^{F} + E\,(T_{c_1}{}^{E} + T_r{}^{E} - T_c)$$

$$T_{r_2}{}^{D} = \overline{E}\,(0) + E\,(T_{c_1}{}^{E} + T_r{}^{E} - T_{c_1}{}^{D} - T_r{}^{D})$$

$$T_{r_1}{}^{E} = \overline{D}\,\overline{F}\,(0) + \overline{D}\,F\,T_r{}^{F} + D\,(T_{c_1}{}^{D} + T_r{}^{D} - T_c)$$

$$T_{r_2}{}^{E} = 0$$

$$T_{r_1}{}^{F} = \overline{D}\,\overline{E}\,(0) + E\,(T_{c_1}{}^{E} + T_r{}^{E} - T_c) + D\,\overline{E}\,(T_{c_1}{}^{D} + T_r{}^{D} - T_c)$$

$$T_{r_2}{}^{F} = \overline{D}\,\overline{E}\,(0) + E\,(T_{c_1}{}^{E} + T_r{}^{E} - T_c - T_r{}^{F}) + D\,\overline{E}\,(T_{c_1}{}^{D} + T_r{}^{D} - T_c - T_r{}^{F})$$

The probabilities associated with the $T_{r_i}{}^{j}$ are typically illustrated by $P_{d_{r_1}}{}^{D}$,

$$P_{d_{r_1}}{}^{D} = \overline{E}\,\overline{F} + \overline{E}\,F\,e^{-\lambda_{d_s}{}^{D}\,T_r{}^{F}} + E\,e^{-\lambda_{d_s}{}^{D}\,(T_{c_1}{}^{E} + T_r{}^{E} - T_c)}$$

where

$$E = \overline{P_E[F]}$$

$$\overline{E} = 1 - E$$

$$F = \overline{P_F[F]}$$

$$\overline{F} = 1 - F$$

APPENDIX IV

of

EXAMPLE B

The Detailed Model for a Remove and Replace Maintenance Cycle

INTRODUCTION

It was tacitly assumed in the treatment given in the example of this memorandum that the total down time in repair and the effectiveness of repair could be treated in a lumped fashion. This view is most useful when using gross field data. During Cat. II, however, it is desirable to consider the details of the remove and replace sequence since the data obtainable is apt to be somewhat different. We consider, then, the possibly repetitive sequence of test failure, remove and replace, recheck.

DERIVATION

Figure D-1 illustrates the time line for the sequence which occurs when two remove and replace actions arise as a result of entering checkout in the first place. We assume the same basic notation used previously except that primes are used to avoid confusion.

The probability of failing to pass the checkout on the nth attempt F_n, is in general given by the recursive relation,

$$F_n = [(\mu_1' + \mu_3')\left\{(1 - \beta') - (1 - \beta' - \alpha')\, P_{d_{c_1}}\right\} + \mu_2'\,(1 - \beta')]F_{n-1} \tag{D-1}$$

$$F_n = X\, F_{n-1} \quad ; \quad F_1 \triangleq \overline{P[F]}$$

The probability of being good and passing $P_n[G \wedge P]$, bad detectable and passing $P_n[B_d \wedge P]$, or bad undetectable and passing $P_n[B_u \wedge P]$ on the nth checkout is given by,

$$\begin{bmatrix} P_n[G \wedge P] \\ P_n[B_d \wedge P] \\ P_n[B_u \wedge P] \end{bmatrix} = \begin{bmatrix} \mu_1'\, P_{d_{c_1}}\, (1 - \alpha')\, P_{u_{c_1}} \\ (\mu_1' + \mu_3')\,(1 - P_{d_{c_1}})\, \beta' + \mu_2'\, \beta' \\ P_{d_{c_1}}\,(1 - \alpha')\left\{\mu_3'\, \mu_1'\,(1 - P_{u_{c_1}})\right\} \end{bmatrix} [F_{n-1}] \tag{D-2}$$

The probabilities of entering standby in various conditions are,

$$\begin{bmatrix} \mu_{1_n} \\ \mu_{2_n} \\ \mu_{3_n} \end{bmatrix} = \begin{bmatrix} P_{u_{c_2}} P_{u_{c_1}} & 0 & 0 \\ 1 - P_{d_{c_2}} & 1 & 1 - P_{d_{c_2}} \\ P_{d_{c_2}} (1 - P_{u_{c_2}}) & 0 & P_{d_{c_2}} \end{bmatrix} \begin{bmatrix} P_n[G \wedge P] \\ P_n[B_d \wedge P] \\ P_n[B_u \wedge P] \end{bmatrix} \tag{D-3}$$

The probability of failing checkout is given from (D-1) by

$$\begin{aligned} \bar{F} &= \sum_{n=1}^{\infty} F_n \\ &= \frac{\overline{P[F]}}{\left\{\beta' + P_{d_{c_1}} (1 - \alpha' - \beta')\right\} (\mu_1' + \mu_3') + \mu_2' \beta'} \end{aligned} \tag{D-4}$$

Hence, a probabilities of exiting repair; given that repair is initiated, in the various possible states are from (D-2), (D-3), and (D-4).

$$\mu_1 = \frac{\mu_1' P_{d_c} P_{u_c} (1 - \alpha')}{\left\{\beta' + P_{d_{c_1}} (1 - \alpha' - \beta')\right\} (\mu_1' + \mu_3') + \mu_2' \beta'}$$

$$\mu_2 = \frac{(\mu_1' + \mu_3') \left\{\beta' + P_{d_{c_1}} (1 - \alpha' - \beta') - P_{d_c} (1 - \alpha')\right\} + \mu_2' \beta'}{\left\{\beta' + P_{d_{c_1}} (1 - \alpha' - \beta')\right\} (\mu_1' + \mu_3') + \mu_2' \beta'}$$

$$\mu_3 = \frac{\left\{\mu_1' (1 - P_{u_c}) + \mu_3'\right\} P_{d_c} (1 - \alpha')}{\left\{\beta' + P_{d_{c_1}} (1 - \alpha' - \beta')\right\} (\mu_1' + \mu_3') + \mu_2' \beta'}$$

The expected down time is given by,

$$\bar{t}_d = (1 - \overline{P[F]})\,T_c + \overline{P[F]} \sum_{n=0}^{\infty} X^n(1 - X) \left\{T_c + (n + 1)\,(T_r + T_{c_1})\right\}$$

$$= T_c + \frac{\overline{P[F]}}{1-X}\,(T_r + T_{c_1})$$

where $\overline{P[F]}$ is the probability of entering repair and X is the probability of re-entry to repair as defined by (D-1).

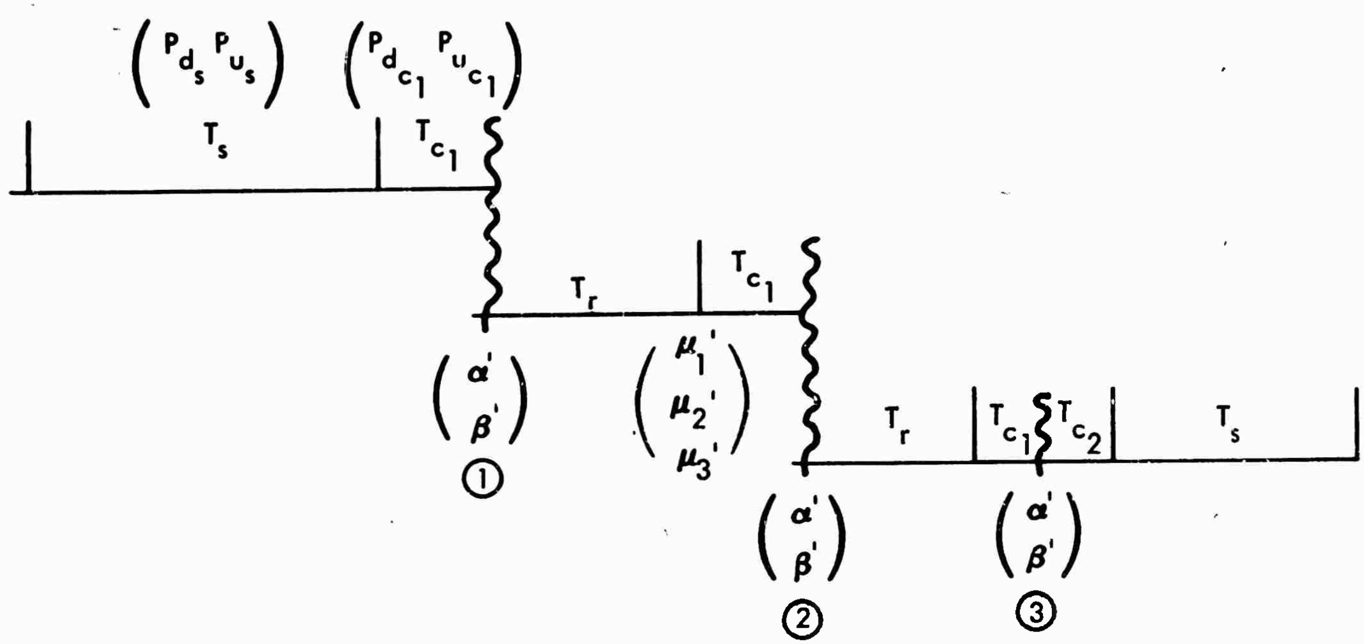


FIGURE D-1 CHECKOUT SEQUENCE WHICH OCCURS WHEN TWO REMOVE AND REPLACE ACTIONS OCCUR

EXAMPLE C

RADAR SURVEILLANCE SYSTEM

TABLE OF CONTENTS

ILLUSTRATIONS

TABLES

I. INTRODUCTION AND SUMMARY

In this example, the Effectiveness of a radar surveillance and threat evaluation system is determined. The system configuration proposed during the Program Definition phase is analyzed and evaluated in accordance with the proposed model,

$$E = \overline{A}'[D]\overline{C}.$$

The example discusses in detail the sub-models employed in determining the elements of each vector and matrix. It also illustrates procedures by which the number of system states to be considered may be minimized.

Following the evaluation of the first system configuration, another is proposed which is intended to improve the overall effectiveness. Since the changes made reflect only the use of redundancy in various functions, the basic equipment characteristics, i.e., reliability, maintainability, are not changed. The sub-models for $\overline{A}'$, $[D]$, and $\overline{C}$, however, are modified to account for the new configuration, and the effectiveness of this system is determined.

II. EFFECTIVENESS ESTIMATION

1.0 Mission Definition

In this example, the stated function of the system is to provide, within a specified time, a warning of an enemy airborne pre-emptive attack. Specifically, the system shall:

a. Detect airborne objects in the surveillance sector at a range of not less than 3000 nautical miles.

b. Identify the objects, and determine, within 30 minutes whether or not they constitute a threat.

c. Convey results of classification to decision making point.

2.0 System Description

2.1 General Configuration

It must be expected that the system configuration will change as it evolves through its life cycle. There will be definite hardware changes reflecting updating programs and advances in the state-of-the-art. Even the original concept of the system will tend to change in response to changes in the world geopolitical climate. In this example, the following system is postulated during the Program Definition Phase. It is referred to as System Configuration No. 1.

a. Three radar equipments, each of which shall provide surveillance of a specified sector; switching arrangements will permit any radar to provide surveillance of any sector.

b. A data link function for each radar equipment to transfer radar data to a computational center.

c. A computer function to store input data and to predict impact areas (the single computer shall serve all radar equipments).

d. Three communication functions, each of which shall convey data from its associated radar to its data processor.

e. Three data processors and three displays to present data from associated radar to decision maker.

f. Necessary prime power to support each of the three subsystems.

2.2 Block Diagram

A functional block diagram representing the system described above is shown in Figure 1.

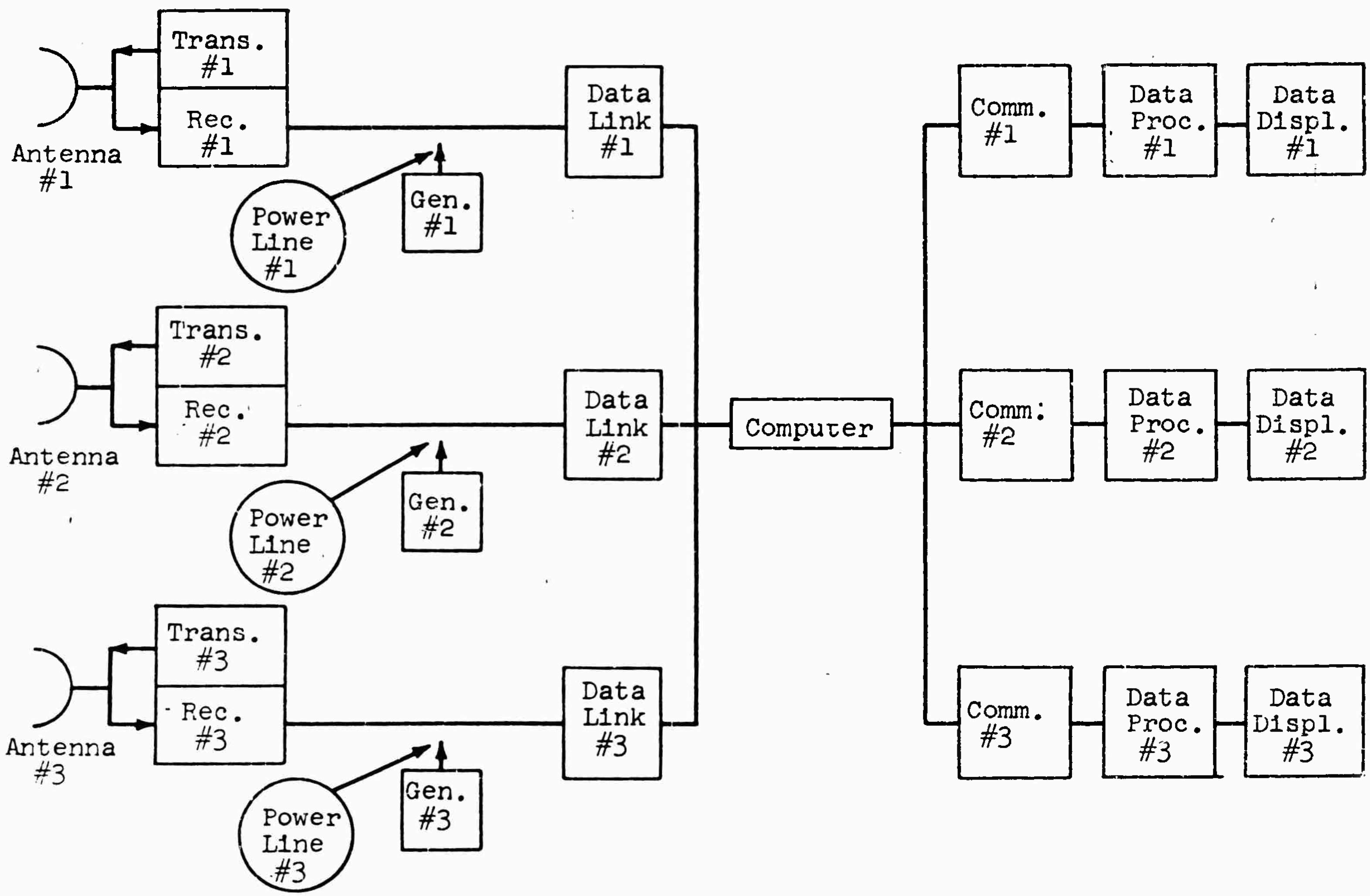


FIGURE 1. BLOCK DIAGRAM SYSTEM CONFIGURATION NO. 1

2.3 Mission Profile

The equipment is operated continuously until failure.

2.4 Delineation of Mission Outcome

In the example cited, each of the three radar subsystems provides surveillance of a specified sector. As noted earlier, switching makes it possible for any radar to provide surveillance of any sector. Assume that the probability that an enemy attack will come from each sector is known (or can be estimated). It is apparent, then, that even if one or two sectors are not under surveillance because of failures in the radars, for example, there still exists a probability--admittedly reduced--that the required warning of an attack will be given. In order to account for such possibilities, an evaluation of the capabilities of the system in various system states must be made.

3.0 Specification of Figure of Merit

The fundamental figure of merit for this system will be taken as "the probability that the system will provide a 30 minute warning, given an enemy airborne pre-emptive attack at a random point in time."

4.0 Identification of Accountable Factors

The potentially important factors of a system may be quite extensive. However, in the current example only the following assumptions concerning major subsystems (i.e., Radar Unit - Antenna, Receiver, and Transmitter; Data Link; Computer; Communications; Data Processor; Data Display; Generator; and Power Line) are made:

a. Operational Conditions

(1) Climatic Environment: The site will be located in an Arctic environment. Conditions in all equipment spaces, however, will be maintained at normal room environment.

(2) Atmospheric Phenomena: Aurora - System shall be capable of target detection in presence of aurora borealis. Wind - Wind loading up to 100 mph. Icing - Ice coating up to 2 inches on antennas.

(3) Enemy Actions and Counter Measures: Equipment shall have provisions for selectable tuning change to prevent jamming. Provisions shall be made in the computer for target discrimination from decoys.

(4) Usage: System is to be operated continuously. No operating personnel shall be required at the antenna site.

b. Mathematical Assumptions

Times between failures are exponentially distributed.

c. Maintenance Concept

(1) General: Adequate maintenance facilities and personnel shall be provided so that any required corrective action can be accomplished at the site.

(2) Personnel: A total maintenance force of ten men shall be stationed at the site. Three 8-hour shifts per day shall be maintained. The classifications and required skill levels are shown below.

Rank/Rating	Specialty	Number
Captain		1
T-9	Electronics	1
T-9	Electrical	1
T-7	Radar	2
T-5	Radar	4

(3) Test Equipment and Tools: All test equipment and tools needed to permit the required maintenance at the site shall be provided. Facilities for emergency repair of the test equipment shall be provided.

(4) Spare Parts and Components: Adequate spare parts and components shall be provided to permit independent operation of the site for a period of ten weeks. In cases where system failure is corrected by replacement of units, repair of the replaced unit shall not be required at the site.

d. Capability Factors

The definition of states of capability must account for factors which define the performance of each component of the system. Examples are given below:

(1) Radar

Transmitter power output
Frequency stability
Frequency range
Antenna gain (beam width)
Receiver signal/noise ratio
Switching times
Anti-jamming features
Pulse repetition frequency
Pulse shape

(2) Computer

Memory capacity
Computational speed
Programming requirements
Switching times
Input/output formats
Word length

(3) Communications

Transmitter power outputs
Receiver sensitivities

(4) Data processors

Input/output format requirements

- Memory size
- Computational speed
- Word lengths

(5) Data displays

Type of presentation

- Visibility
- Readability
- Retentivity
- Ease of interpretation

(6) Power generation

Capacity
Regulation

Voltage
Frequency

Efficiency
Ease of switching

(7) Power distribution

Conductor sizes
Power losses
Protection requirements
Installation requirements
Insulation requirements

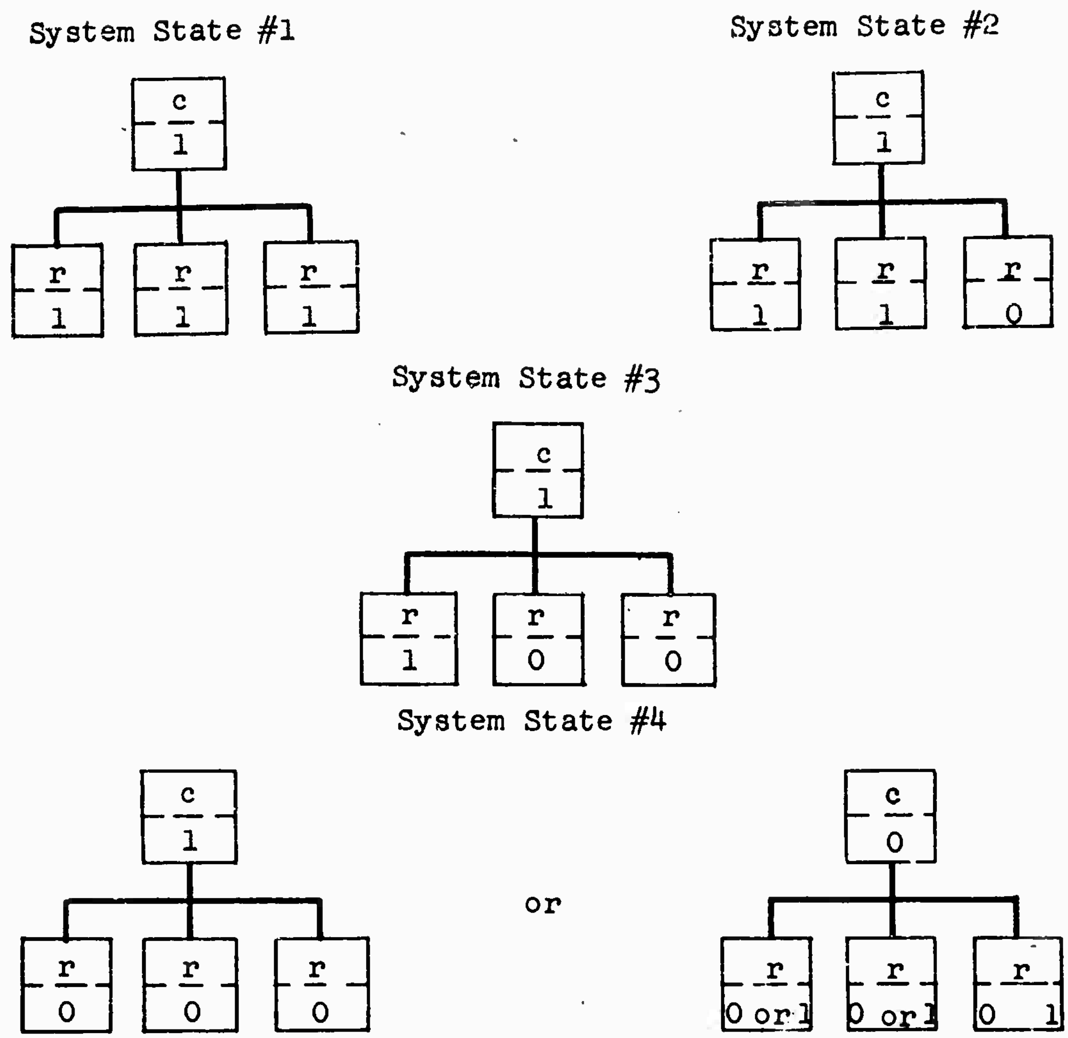
4.1 Identification of Data Constraints

Model construction should be conducted in the full knowledge of the constraints which may be imposed by data availability. For example, if it is known that only a limited sample of life tests are to be conducted, the effect of the small sample size on the output of the proposed analyses should be investigated. Or again, if piece part data is the only data that will be available until late in the program, the model construction must reflect this fact. In the present example, we shall assume that during the Program Definition phase, dependence must be placed on generally accepted prediction procedures. In later phases, results of tests of the actual hardware subsystems may be employed.

5.0 Model Construction

5.1 Delineation of System States

For System Configuration No. 1 the probabilities of mission success under various conditions must be described. If all combinations of success and failure for every individual subsystem are considered, the number of possible system states is extremely high. However, by considering collectively all of the subsystems (except for the computer) in each surveillance path, an appreciable reduction in the number of significant states is made. A further simplification is possible if all states in which no system capability exists are treated collectively as a single state. Figure 2 illustrates the significant states to be considered in evaluating Configuration 1 under these assumptions.



NOTE:

Letters in upper portion of block indicate equipment or equipments of interest; number in lower portion of block represents condition of equipment or equipments. "c" represents computer; "r" represents all serial equipments required to provide surveillance of one sector, i.e., generator, power lines, receiver, transmitter, data link, communications, data processor and data display. "1" indicates that all equipment(s) in the block are operable. "0" indicates that one or more equipments in the block has failed.

FIGURE 2. SYSTEM STATE DIAGRAMS

5.2 System Model

For this system evaluation, the basic model to be employed is:

$$E = \bar{A}' \cdot D\ \bar{C} \tag{1}$$

where

E = probability that the system will provide a 30 minute warning, given an enemy airborne pre-emptive attack at any random point in time;

A = availability vector ; $\bar{A}'$ **is its transpose**

= probability that at any random point in time, the system will be in state i, where i can be any integer from 1 to n, inclusive, n = number of system states to be considered;

$[D]$ = dependability matrix

= probability of transition from system state i to system state j during the required operating period (0.5 hours), given state i at the beginning of this period;

$\bar{C}$ = capability vector

= probability that the system can successfully perform the required functions, given that the system is in state j during the period of interest.

5.3 Availability

The element A_i of the availability vector $\bar{A}$ is a function of the availabilities of the various equipments a_j. Symbolically,

$$A_i = f\left[a_1, a_2, \ldots, a_j, \ldots, a_r\right] \tag{2}$$

The specific functional relationship is dependent upon the system configuration and the number of possible system states.

For the system configuration being considered, the models for A_i are shown in Table I.

The a_j are computed using the generally accepted expression for the availability of a continuously observed system;

$$a_j = \frac{t_{f_j}}{t_{f_j} + t_{d_j}} \tag{3}$$

where

t_{f_j} = mean time between failures for the j^{th} subsystem.

t_{d_j} = mean down time for the j^{th} subsystem.

5.4 Dependability

The probability of transition from one state to another during the actual time of mission performance must be

TABLE I

Models for A_i - System Configuration No. 1

$$A_1 = a_c a_r^3$$

$$A_2 = 3a_c a_r^2 (1 - a_r)$$

$$A_3 = 3a_c a_r (1 - a_r)^2$$

$$A_4 = a_c (1 - a_r)^3 + (1 - a_c)$$

Where

a_c = probability that the computer is operable (available).

a_r = probability that all components in each sector surveillance and display subsystem (excepting the computer) are operable (available).

Note that $A_1 + A_2 + A_3 + A_4 = 1.0$, indicating that the defined states represent all possible states.

accounted for. Use of matrix notation for this purpose facilitates consideration of all possible state transitions.

For system Configuration No. 1, it will be recalled that there are four possible system states. If during the mission, transition were possible from any one state to any other, 16 possible results would exist, including 4 situations in which no transition occurs, viz.:

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \qquad (4)$$

In the actual matrix to be employed, the probability of each indicated transition occuring will be entered. For example, the D_{11} element will be the reliability figure which is the one normally discussed in simple reliability analyses; i.e., the probability of no failure during the mission, given that the system was completely within specification at time zero.

In this example, maintenance is assumed to be ineffective during the actual mission; therefore, no transition from a lower to a higher state will be possible, and each element below the diagonal line will be zero. The remaining elements in the matrix are evaluated by use of the following equations:

$$
\begin{aligned}
D_{11} &= R_c R_r^{\,3} \\
D_{12} &= 3R_c R_r^{\,2}(1 - R_r) \\
D_{13} &= 3R_c R_r\,(1 - R_r)^2 \\
D_{14} &= R_c(1 - R_r)^3 + (1 - R_c) \\
D_{22} &= R_c R_r^{\,2} \\
D_{23} &= 2R_c R_r(1 - R_r) \\
D_{24} &= R_c\,(1 - R_r)^2 + (1 - R_c) = 1 - R_c R_r(2 - R_r) \\
D_{33} &= R_c R_r \\
D_{34} &= R_c\,(1 - R_r) + (1 - R_c) \\
D_{44} &= 1
\end{aligned}
\tag{5}
$$

where

R_c = computer reliability

R_r = reliability of all other equipments in each sector surveillance and display subsystem.

In some situations, it is possible that the mission reliability need not be evaluated. This might be the case, for example, if the actual length of the mission is very short compared to the mean times between failures for the systems. In such cases, each element of the main diagonal approaches unity while all others approach zero. If this approximation is acceptable, the "identity matrix" can be employed and will considerably simplify computations. In essence, this permits the mission reliability factor to be represented by unit. This approach will be illustrated in a later example for a different system configuration.

5.5 Capability

Although a system may be available and functioning as designed during the mission, the system can still fail to accomplish its design purpose due to a variety of factors. In the case of a surveillance system such factors would include:

(1) Signal masking due to background thermal noise.

(2) Range and doppler velocity limitations due to radar pulse repetition rate.

(3) Angle discrimination due to finite antenna beam width.

These factors are conveniently lumped together in a vector which is called the "design capability" factor of the effectiveness equation. In the present illustration, a simplified example will be used to show how this calculation is undertaken. Specifically, we shall consider the effect of background thermal noise on the ability of a radar to detect and accurately track a potential threat.

The distribution of the amplitude in volts (E_n) of the system noise is Gaussian so that the probability density of noise amplitude is given by:

$$p(E_n) = \frac{E_n}{2\sigma_n^2} e^{-\frac{E_n^2}{2\sigma_n^2}}; \quad -\infty \leq E_n \leq +\infty \tag{6}$$

The density distribution of noise power is obtained by recognizing that noise power (P_n) is proportional to the square of noise voltage (E_n) and then applying a transformation of variable to (6). That is:

$$P_n = \frac{E_n^2}{a} \tag{7}$$

$$\therefore E_n = (aP_n)^{\frac{1}{2}}$$

And

$$p(E_n)\, dE_n = p[E_n(P_n)] \left| \frac{E_n}{P_n} \right| dP_n \tag{8}$$

With attention to the change of limits on (6);

$$0 \leq P_n \leq +\infty \tag{9}$$

Thus

$$p\left[P_n\right] = \frac{a}{2\sigma_n^2} e^{-\frac{aP_n}{2\sigma_n^2}} \tag{10}$$

The signal power (S) returned from a potential threat is given by:

$$S = \frac{CJ}{r^4} \tag{11}$$

Where

r = radial distance to threat,

J = reflectivity of threat, and

C = function of antenna gain, transmitter power.

In general, the time which it takes to perform the threat evaluation will depend upon the ratio S/P_n, shorter evaluation times being associated with larger values of this ratio since this reduces the required signal tracking and smoothing times.

There will be some value ξ below which there is a vanishingly small probability of threat evaluation within the prescribed time; thus the probability of detecting a threat is given by

$$P_d = \int_0^{S/\xi} p\left[P_n\right] dP_n \tag{12}$$

Therefore the probability of detection becomes

$$P_d = 1 - e^{-\frac{a}{2\,{}_n^2}(S/\,\cdot\,)} = 1 - e^{-\frac{a}{2\,{}_n^2}\frac{CJ}{r^4}} \qquad (13)$$

Each system state i will have a particular value of P_{d_i}.

In addition, there will be other performance factors as listed in paragraph 4.1.d above, associated with each system state which will influence the probability of detection and track. Thus, in general, we can write a capability vector,

$$\bar{C} = \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix} \qquad (14)$$

where

$$C_i = K_i P_{d_i}$$

6.0 Data Acquisition

6.1 Data Sources - Reliability

In the program definition phase, data is generally not available from actual tests of the system under consideration. In this case, use is usually made of available generic data sources. A tabluation of reports and papers which treat reliability prediction is included in the section of this document entitled "Data Sources".

Determination of failure rates for given stress conditions are included in some of the data sources. In employing these data, care must be taken to utilize only those data sources which closely duplicate the expected environment of the system under development. Wherever such data is unobtainable, available data must be modified by appropriate proportional stress factors.

In later development phases, reliability data is often available from contractor tests. During the early acquisition phase, such data obtained from the contractors' bench tests may be used to supplement generic failure rate information. During the operational phase additional data may be obtained from standard Air Force reporting forms.

6.2 Data Sources - Maintainability

Some predictive models exist for estimating mean-down-times (or components thereof). Three examples of such models are presented for specific operational conditions in the documents referenced below:

Airborne Systems	"Maintainability Prediction: Theoretical Basis and Practical Approach" (Revised) ARINC Research Corporation
Ground Systems	"RADC-TDR-63-85, Vol. II - Maintainability Engineering" Radio Corporation of America RCA Service Company
Shipboard Systems	"A Maintainability Prediction Procedure for Designers of Shipboard Electronic Equipment and Systems" Federal Electric Corporation

In general, these procedures concentrate on the predictions of "active repair time." This is defined as the length of time required to complete the repair, given that one or more technician is actively engaged in repairing the equipment. The procedures consider such factors as system construction, accessibility, diagnostic devices and test equipment availability. The treatments accorded such other factors as administrative delays, skill level of maintenance personnel, availability of spares, and queuing resulting from an insufficient number of maintenance personnel vary considerably.

7.0 Parameter Estimation

7.1 Estimating Basic Equipment Characteristics

Depending upon the program phase, the estimation procedures used to determine t_f will vary appreciably. In the operational phase, for example, the value of t_f might be computed from field data by

$$t_f = \frac{t_o}{f} \tag{15}$$

where

t_o = total observed operating times

f = total observed number of failures.

In earlier phases, as noted earlier, the value of t_f might be synthesized from generic failure rates on individual parts or components comprising the system. For a single equipment involving no redundancy in which it may be assumed that all parts or components exhibit constant failure rates, t_f would then be given by:

$$t_f = \frac{1}{\sum_{i=1}^{n} \lambda_i} \tag{16}$$

Where

$_i$ = failure rate of the i^{th} component

n = number of components in the equipment.

As in the case of t_f, the down time models will differ in various program phases. Again, in the operational phase,

t_d might be calculated from field data:

$$t_d = \frac{T_d}{n} \tag{17}$$

where

T_d = total time during which equipment was down (not operable) during a specified period.

n = number of separate maintenance actions during the specified period.

In earlier phases, predictions based upon system configuration and the support situation will be necessary.

Because of the complexities involved in estimating t_f and t_d, it is not reasonable to explore the details of the procedures in this example. It is assumed, therefore, that through the use of appropriate techniques, the numerical values shown in Table II were developed.

7.2 Determination of Availability

The availability of each subsystem is determined from Equation 3 utilizing the data of Table II. The results are shown in Table III.

TABLE II

Mean-Times-Between-Failures (t_f)
and Mean Repair Times (t_r) for Equipments

Equipment	t_f (hours)	t_r (hours)
Power Lines	26,280	24.0
Generator	17,520	22.0
Transmitter	800	1.0
Receiver and DTO	2,000	0.5
Data Link	43,800	1.0
Computer	250	1.0
Communication	4,000	0.5
Data Processor	4,000	0.5
Data Display	4,000	0.5

TABLE III

Availability of Individual Equipment

Equipments	a_j
Power Lines	.999087
Generator	.998745
Transmitter	.998751
Receiver and DTO	.999750
Data Link	.999977
Computer	.996016
Communication	.999875
Data Processor	.999875
Data Display	.999875

where

$$a_j = \frac{t_{f_j}}{t_{f_j} + t_{r_j}}$$

t_{f_j} = mean time between failures for the j^{th} subsystem

t_{r_j} = mean time to repair for the j^{th} subsystem

The probability of being in any system state at a random point in time is given by utilizing the results of Table III in the Equations of Table I. The components of the availability vector are therefore, the values listed in Table IV.

TABLE IV Numerical Values of A_i
System Configuration No. 1
A_1 = .983934
A_2 = .012033
A_3 = .000049
A_4 .003985

7.3 Determination of Dependability

Use of the same input data employed in the availability analysis, e.g., values of failure rates, permits determination of numerical values for each element of the dependability matrix utilizing equations 5 and the data of Table II, we have for system configuration No. 1:

$$[D] = \begin{bmatrix} 0.994088 & 0.003909 & 0.000005 & 0.001998 \\ 0 & 0.995391 & 0.002609 & 0.001999 \\ 0 & 0 & 0.996696 & 0.003304 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

7.4 Determination of Capability

We shall assume that the possibilities of an attack emanating from a particular sector, coupled with the systems' abilities in various conditions led to the assignment of the state capabilities shown in Table V.

TABLE V

Capability for System Configuration No. 1

State Number	Capability
1	1P
2	0.75P
3	0.35P
4	0P

In these tables, P is the capability, i.e., the probability of successful mission performance, when the system is fully within specification.

The probability is assumed to be given by evaluating Equation (14) in conjunction with the other significant performance factors of each system state.

In the present example, it is simply assumed that this has been done for each partial system failure delineated by the block diagram of Figure 2. Each leg of this block diagram is, then, a specifically accountable system state. The numerical values are assumed to be for each of the four system states:

$$\begin{aligned} C_1 &= P = 0.998 \\ C_2 &= 0.75P = 0.749 \\ C_3 &= 0.35P = 0.349 \\ C_4 &= 0P = 0.000 \end{aligned} \qquad (19)$$

8.0 Model Exercise

8.1 Effectiveness Evaluation

At this point, we are in a position to evaluate the expression

$$E = \overline{A}\,[D]\,\overline{C}$$

For System Configuration No. 1, the multiplication of the three terms yields an Effectiveness of 0.9880.

8.2 Modified System Configuration - Acquisition Phase

Analysis of the results obtained from evaluation of System Configuration No. 1 during the Definition phase suggested several modes for improving effectiveness. The analysis indicated that the Computer was the greatest single adverse factor influencing Effectiveness. Considering redundancy in this function and other changes noted in the following system definition indicated a probable increase in effectiveness from the value of 0.9880 estimated for Configuration No. 1 to 0.9940, showing the positive effects of redundancy. At this stage (Acquisition Phase), the system is defined to be:

(1) Three radar equipments, each of which shall provide surveillance of a selected sector. Three antennas are to be provided, which are to be switchable among radars. Each radar shall provide detection capability at the 3000 nautical range.

(2) Two data link subsystems, each of which shall be completely capable of handling all radar data.

(3) Two storage and computing subsystems each of which shall be completely capable of storing all input data and predicting impact area.

(4) Two communication subsystems each of which shall be completely capable of conveying all necessary data to the decision point.

(5) Two data processor and display subsystems, each of which shall be completely capable of processing and displaying all required data.

(6) Four independent power generating devices. Any pair of which, when operating at full capacity, shall be capable of supplying the total power requirement. In normal operations, three generators shall be on-line, each operating at two-thirds of full load capability.

(7) Power lines capable of transferring power with no more than 0.5% power loss at maximum load.

Similar analyses during the Acquisition Phase coupled with more definite information on reliability, the difficulties inherent in the logistic support problem, and the importance of target threat evaluation led to further changes. The system in the operational phase (System Configuration No. 2) consists of:

(1) Three radar equipments, each of which shall provide surveillance of a selected sector. Any of the radar equipments shall be capable of operating with any of the three antennas. Switching shall be possible in less than three minutes for the transmitters and in less than 1.5 minutes for the receivers; a spare transmitter shall be provided which can be switched into any of the three equipments in less than three minutes.

(2) Two data link subsystems, each of which shall be completely capable of handling all radar data.

(3) Two storage and computing subsystems each of which shall be completely capable of storing all input data and predicting impact area.

(4) Three communications subsystems, any one of which shall be completely capable of conveying all necessary data to the decision point.

(5) Two data processor subsystems, either of which shall be completely capable of processing all required data.

Three data display subsystems, any one of which shall be completely capable of displaying all required data.

(6) Six independent power generating devices, any four of which, when operating at full capacity, shall be capable of supplying the total power requirement. In normal operations, five generators shall be on-line, each operating at 80% of full load capacity. Power lines capable of transferring power with no more than 0.5% power loss at maximum load.

8.3 Modified System Configuration - Operational Phase

The operational configuration defined above is now evaluated following the same procedures used in evaluating configuration 1. Figure 3 presents the functional block diagram for system configuration 2.

The complexity of the system has now been increased by the redundant equipments. If all combinations of success and failure for every equipment in System Configuration No. 2 are considered, the number of possible system states is approximately 10^8. For this example--and in general--however, consideration of the system's capabilities for various combinations of subsystem failures permits an appreciable reduction in the number of significant states. As an example, consider System Configuration No. 2 when only one transmitter is operable. The system capability is no different in this case whether one, two, or three receivers are operable, since only one can be employed. Therefore,

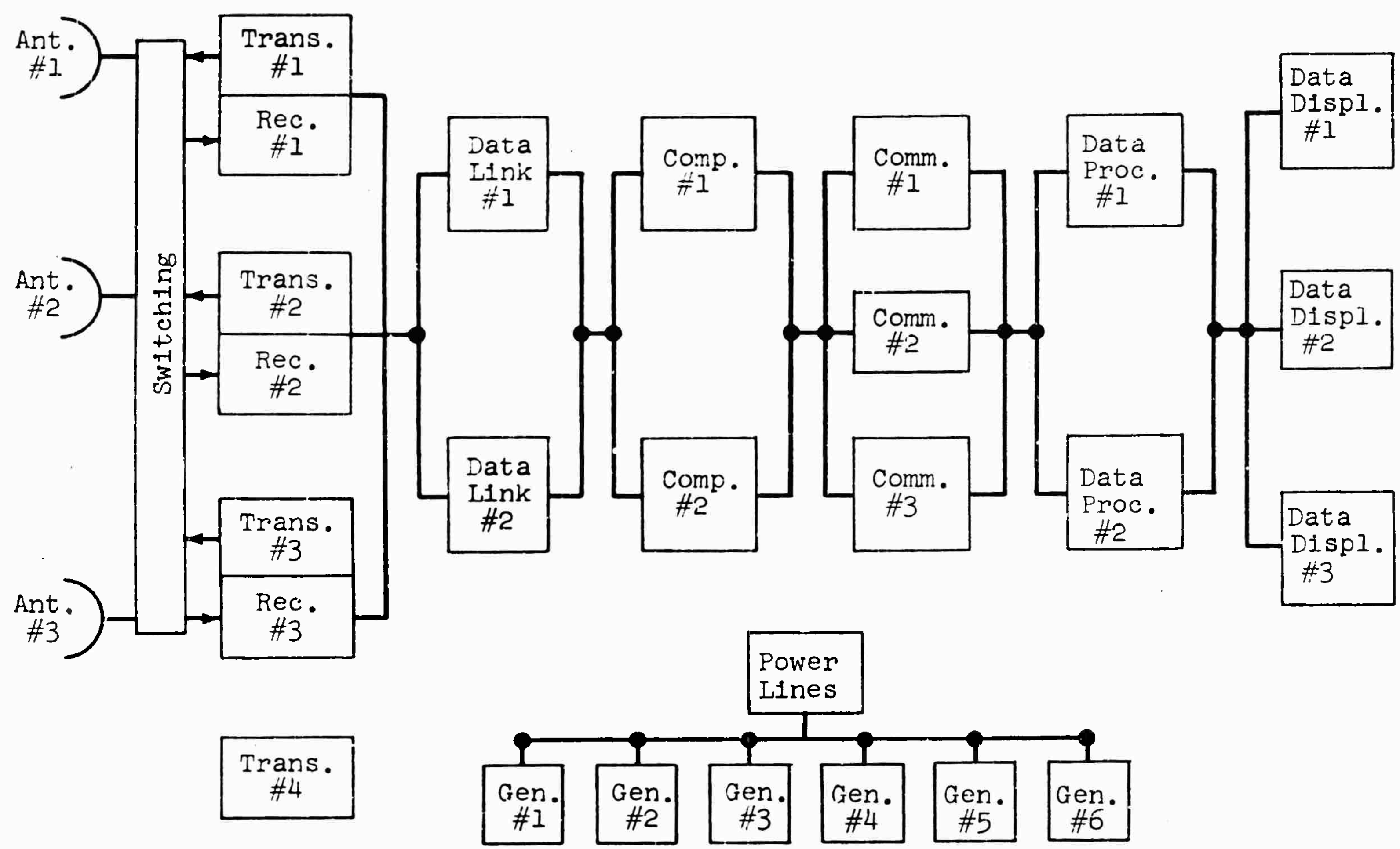


FIGURE 3. BLOCK DIAGRAM SYSTEM CONFIGURATION NO. 2

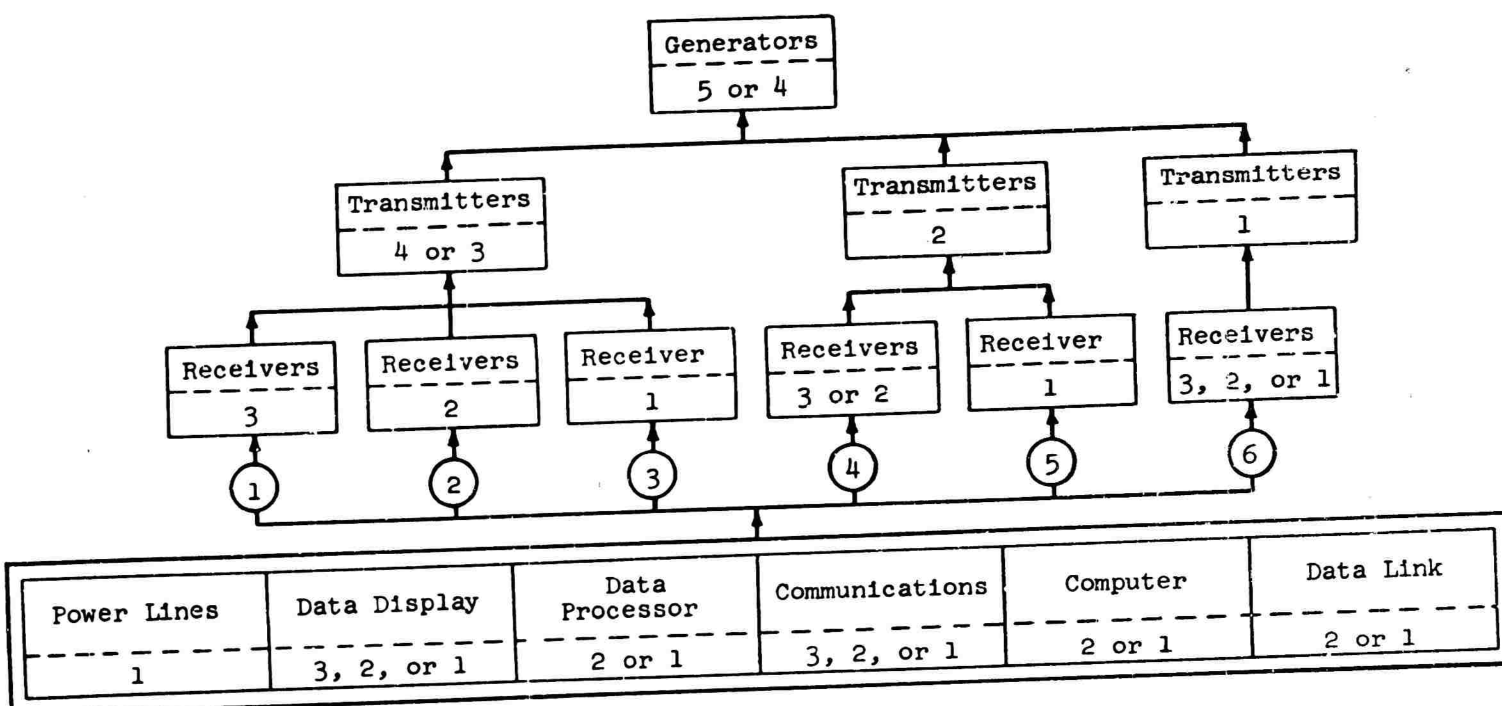
only two states of receiver operation need be considered, viz., none operable and one or more operable. This assumes, of course, that the time to return the system to alert does not depend upon the number of failed subsystems. This may be a bad assumption.

A further simplification is possible if all states in which no system capability exists are treated collectively as a single state.

Figure 4 illustrates the significant states to be considered in evaluating this system under these assumptions.

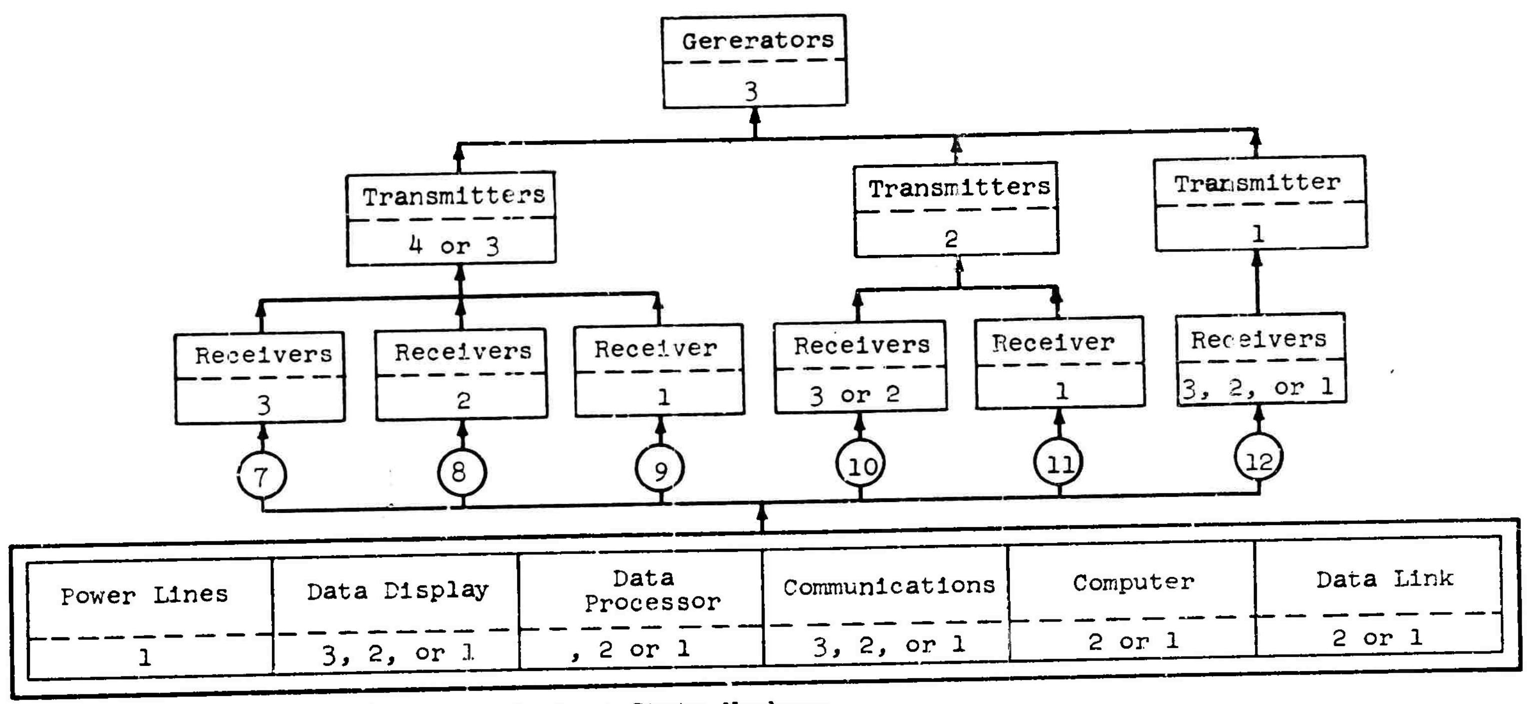
8.3.1 Availability Determination (Configuration No. 2)

The first step in this evaluation is the determination of the state availabilities (A_i's). The model for each state is developed by considering the equipment states. Whether the system is in a particular state depends upon the number of equipments in each of the subsystems which are operable or failed. For system configuration No. 2 to be in state 10, for example, exactly three generators; two transmitters; two or three receivers; one or two data links; one or two computers; one, two, or three communication sets; one or two data processors; one, two, or three data



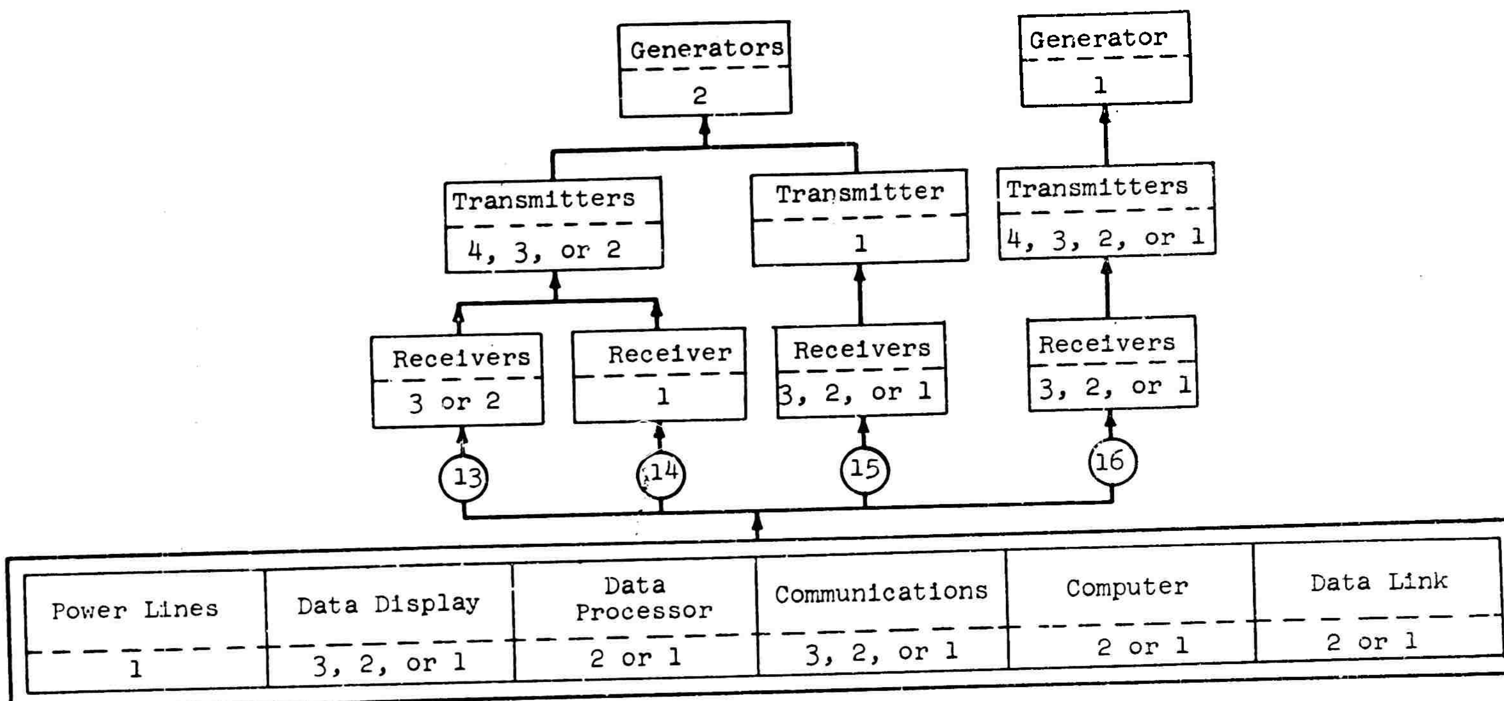
Notes:
1. Circled numbers are System State Numbers.
2. Lower portion of each subsystem block indicates the number of each subsystem which is functioning.

FIGURE 4. SYSTEM STATE DIAGRAMS SYSTEM CONFIGURATION NO. 2



Notes: 1. Circled numbers are Systems State Numbers.
2. Lower portion of each subsystem block indicates the number of each subsystem which is functioning.

FIGURE 4. (Continued) SYSTEM STATE DIAGRAMS SYSTEM CONFIGURATION NO. 2



Notes: 1. Circled numbers are System State Numbers.
2. Lower portion of each subsystem block indicates the number of each subsystem which is functioning.

FIGURE 4 (Concluded) SYSTEM STATE DIAGRAMS SYSTEM CONFIGURATION NO. 2

displays; and the power lines must be operable. If any one of the conditions is not met, the system will be in a different state.

Thus, the equation which expresses the probability of being in state 10 is

$$A_{10} = a_G^{(3)}\ a_T^{(2)}\ a_R^{(3,2)}\ a_O \tag{20}$$

where

$a_G^{(3)}$ = probability that 3 generators will be operable (available)

$a_T^{(2)}$ = probability that 2 transmitters will be operable (available)

$a_R^{(3,2)}$ = probability that either 3 or 2 receivers will be operable

a_O = probability that 1 or 2 data links; 1 or 2 computers; 1, 2, or 3 communication sets; 1 or 2 data processors; 1, 2, or 3 data displays; and the power lines will be operable (available)

A similar analysis for each state leads to the models shown in Table VI.

The sub-models for evaluating the terms in the right hand side of the equations in Table VI are shown in Table VII; as are the numerical results obtained when the equipment availabilities from Table III are introduced.

TABLE VI

Models for A_i - System Configuration No. 2

$$A_1 = a_O a_G^{(4,5)} a_T^{(4,3)} a_R^{(3)}$$

$$A_2 = a_O a_G^{(4,5)} a_T^{(4,3)} a_R^{(2)}$$

$$A_3 = a_O a_G^{(4,5)} a_T^{(4,3)} a_R^{(1)}$$

$$A_4 = a_O a_G^{(4,5)} a_T^{(2)} a_R^{(3,2)}$$

$$A_5 = a_O a_G^{(4,5)} a_T^{(2)} a_R^{(1)}$$

$$A_6 = a_O a_G^{(4,5)} a_T^{(1)} a_R^{(3,2,1)}$$

$$A_7 = a_O a_G^{(3)} a_T^{(4,3)} a_R^{(3)}$$

$$A_8 = a_O a_G^{(3)} a_T^{(4,3)} a_R^{(2)}$$

$$A_9 = a_O a_G^{(3)} a_T^{(4,3)} a_R^{(1)}$$

$$A_{10} = a_O a_G^{(3)} a_T^{(2)} a_R^{(3,2)}$$

$$A_{11} = a_O a_G^{(3)} a_T^{(2)} a_R^{(1)}$$

$$A_{12} = a_O a_G^{(3)} a_T^{(1)} a_R^{(3,2,1)}$$

$$A_{13} = a_O a_G^{(2)} a_T^{(4,3,2)} a_R^{(3,2)}$$

$$A_{14} = a_O a_G^{(2)} a_T^{(4,3,2)} a_R^{(1)}$$

$$A_{15} = a_O a_G^{(2)} a_T^{(1)} a_R^{(3,2,1)}$$

$$A_{16} = a_O a_G^{(1)} a_T^{(4,3,2,1)} a_R^{(3,2,1)}$$

$$A_{17} = 1 - \sum_{i=1}^{16} A_i$$

where a denotes availability; a_G - generator; a_T - transmitter; a_R - receiver; and a_O - power, data link, computer, communications, data processor, and data display.

Superscripts indicate number of equipments of this type which must be operable. Example:

$a_G^{(4,5)}$ = probability that exactly 4 or 5 generators are operable (available).

TABLE VII

Sub-Models for Evaluating Terms in Equations of Table VI

GENERATOR

$a_G(5)=a_G^5=0.993741$

$a_G(4)=5a_G^4(1-a_G)=0.006244$

$a_G(3)=10a_G^3(1-a_G)^2=1.57\times10^{-5}$

$a_G(2)=10a_G^2(1-a_G)^3=1.97 \times 10^{-8}$

$a_G(1)=5a_G(1-a_G)^4=1.24 \times 10^{-11}$

$a_G(4,5)=a_G(5)+a_G(4)=0.999984$

TRANSMITTER

$a_T(4)=(a_T)^4=0.995013$

$a_T(3)=4(a_T)^3(1-a_T)=0.004977$

$a_T(2)=6(a_T)^2(1-a_T)^2=9.34\times10^{-6}$

$a_T(1)=4a_T(1-a_T)^3=7.78\times10^{-9}$

$a_T(4,3)=a_T(4)+a_T(3)=0.999991$

$a_T(4,3,2)=a_T(4,3)+a_T(2)=0.999993$

$a_T(4,3,2,1)=a_T(4,3,2)+a_T(1)=0.999993+$

RECEIVER

$a_R(3)=a_R^3=0.999250$

$a_R(2)=3a_R^2(1-a_R)=7.50\times10^{-4}$

$a_R(1)=3a_R(1-a_R)^2=1.87\times10^{-7}$

$a_R(3,2)=a_R(3)+a_R(2)=0.9999996$

$a_R(3,2,1)=a_R(3,2)+a_R(1)=0.9999998$

REST OF SYSTEM

$$a_O = \{1-(1-a_D)^2\}\{1-(1-a_C)^2\}\{1-(1-a_{CO})^3\}\{1-(1-a_P)^2\}$$
$$\{1-(1-a_S)^3\} \quad a_L = 0.999071$$

where a_D denotes data link; a_C - computer; a_{CO} - communications; a_P - data processor; a_S - display; and a_L - power lines.

The probability of being in any system state at a random point in time is given by utilizing the results of Table VII in the equations of Table VI. The components of the availability vector are then listed in Table VIII.

8.3.2 Capability Determination (Configuration No. 2)

The assignment of the state capabilities for configuration 2 followed the same assumptions as those made for Configuration 1, and are shown in Table IX.

8.3.3 Dependability Determination (Configuration No. 2)

As noted in the discussion of the dependability matrix for Configuration 1, the elements of the main diagonal approach unity while all others approach zero if the actual length of the mission is short compared to the mean times between failures for the systems. To provide an indication of the effect of approximating the Dependability Matrix with the identity matrix, the effectiveness of configuration 2 was first estimated using the dependability matrix $[D]$ equal to the identity matrix $[I]$. A value of E = 0.9970 was obtained. A second estimate was made using the matrix $[D]$ made up of its elements which

TABLE VIII

Numerical Values of A_i

System Configuration No. 2

$A_1 = 0.998297$

$A_2 = 0.749 \times 10^{-3}$

$A_3 = 0.187 \times 10^{-6}$

$A_4 = 0.933 \times 10^{-5}$

$A_5 = 0.175 \times 10^{-11}$

$A_6 = 0.776 \times 10^{-8}$

$A_7 = 0.157 \times 10^{-4}$

$A_8 = 0.118 \times 10^{-7}$

$A_9 = 0.294 \times 10^{-11}$

$A_{10} = 0.146 \times 10^{-9}$

$A_{11} = 0.274 \times 10^{-16}$

$A_{12} = 0.122 \times 10^{-12}$

$A_{13} = 0.197 \times 10^{-7}$

$A_{14} = 0.369 \times 10^{-14}$

$A_{15} = 0.153 \times 10^{-15}$

$A_{16} = 0.124 \times 10^{-10}$

$A_{17} = .00093$

TABLE IX

Numerical Values of C_i

System Configuration No. 2

$C_1 = P = 0.998$

$C_2 = .95P = 0.948$

$C_3 = .35P = 0.349$

$C_4 = .75P = 0.749$

$C_5 = .35P = 0.349$

$C_6 = .35P = 0.349$

$C_7 = .90P = 0.898$

$C_8 = .75P = 0.749$

$C_9 = .35P = 0.349$

$C_{10} = .75P = 0.749$

$C_{11} = .35P = 0.349$

$C_{12} = .35P = 0.349$

$C_{13} = .65P = 0.649$

$C_{14} = .35P = 0.349$

$C_{15} = .35P = 0.349$

$C_{16} = .25P = 0.249$

$C_{17} = 0 = 0$

were significantly different from zero. The value obtained in this case was E = 0.99696 which is equivalent to the value of 0.9970 obtained in the estimate using the matrix I

8.4 Analysis of Configuration No. 2

In analyzing Configuration No. 2 for "bottlenecks" such as the single computer of Configuration No. 1, it appears that including a switchable spare receiver in addition to the switchable spare transmitter might substantially increase effectiveness. However, when the model was exercised with this additional change, the effectiveness was only increased to 0.99707 (equivalent to 0.9971) an approximate increase of 0.0001. This improvement was not felt to warrant the expense involved in making the modification, and Configuration No. 2 was selected as the operational model.

EXAMPLE D

SPACECRAFT SYSTEM DEPENDABILITY

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EXAMPLE D

SPACECRAFT SYSTEM DEPENDABILITY

1. INTRODUCTION AND SUMMARY

In this example, attention will be focused on the prediction of the Dependability characteristics of a Spacecraft System. This example is included in order to provide a more detailed analysis in the area of reliability prediction than was generally presented in the previous examples. Again, the procedure demonstrated here should not be interpreted as constituting a standard technique. Rather, the example illustrates the criteria involved in selecting a predictive approach and the exercise of the technique selected.

This example also shows one approach to the prediction of the reliability of a structure. It is important to recognize that the structural portion of many systems are important contributors to the system's effectiveness, and must, therefore, be evaluated in terms of their influence on mission success.

Because this example is limited to the Dependability analysis, the treatment of some of the steps in the general Effectiveness evaluation procedure will be less complete than in the previous examples. The assumption is made that the Dependability matrix developed herein will be compatible with other portions of the Effectiveness analysis.

II. DEPENDABILITY EVALUATION

1.0 Mission Definition

The spacecraft system shall be capable of placing a variety of payloads, including multiple satellites, into precise orbits about the earth. It shall have the capability of restarting in space after a sufficient coast period, dependent on the specific payload and attitude orientation in space. The system shall be designed as an upper stage rocket propulsion vehicle.

2.0 System Description

The system described herein is a spacecraft for placing satellites in earth orbits.

2.1 General Configuration

The spacecraft is a liquid-propellant upper-stage rocket propulsion vehicle providing all the control elements necessary for placing a variety of payloads in precise orbits above the earth. The spacecraft has the capability of injecting multiple satellites into orbit about the earth after completion of one or more restart cycles in space. Thrust vector and roll control during powered flight supplemented by coast attitude control provide capability for obtaining precise circular orbits.

The payloads are protected with an aerodynamic shroud during the appropriate periods of flight, and upon injection into orbit are separated from the stage by an automatic pre-programmed sequence. Retro-thrust applied at separation decelerates the vehicle, preventing possible collision with the payloads.

Throughout the mission the satellite launching spacecraft telemeters extensive flight data. A tracking beacon and range safety destruct capability are also provided.

2.2 System Block Diagram

Figure 1 shows a breakdown of the spacecraft into four systems, viz, propulsion, forward section, ordnance (except range safety) and structure. The safety and arming mechanism of the range safety destruct subsystem, which is located physically in and must perform correctly integrally with the propulsion system, is considered essential to mission performance. It may be argued that the destruct arming mechanism, which is not required to function except in the case of failure should not be included in the flight reliability model. Nevertheless, this has been done in the interest of a thoroughly conservative treatment of range safety ordnance. The risk that range safety electronics will fail to actuate the destruct mechanism, if required to do so, is distributed into a separate calculation. This failure mode is always contingent upon an

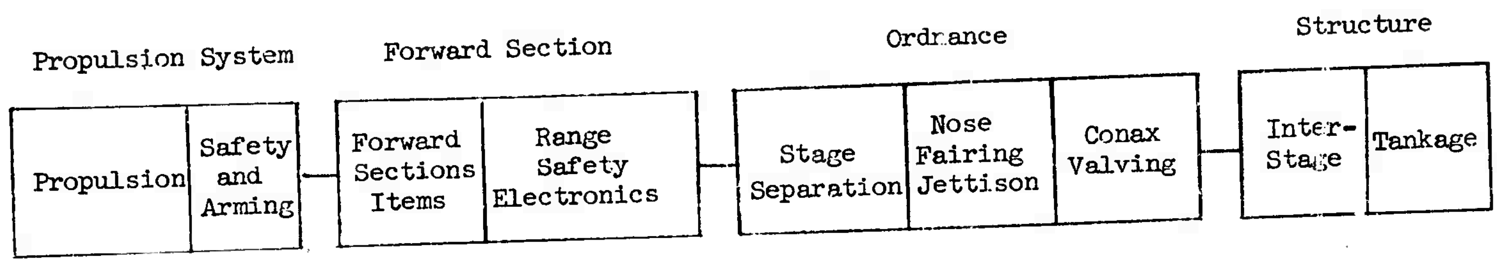

FIGURE 1.

BLOCK DIAGRAM OF SPACECRAFT

already unacceptable flight, and therefore does not affect the transition probabilities in the given formulation of the dependability matrix.

2.3 Mission Profile

In Figure 2 is shown a graphical representation of the sequence of events of interest in the dependability evaluation to be made. A tabulation of main events in a typical mission is shown in Table I. These events determine the time during which various stresses will be experienced by various components. Individual computations for these components will be made for each time period.

2.4 Delineation of Mission Outcomes

It is assumed here that in the complete Effectiveness analysis, mission outcomes have been defined such that three system states must be considered. These states represent system conditions in which one of three outcomes results:

(a) Perfect operation;

(b) Acceptable operation; or,

(c) Unacceptable operation.

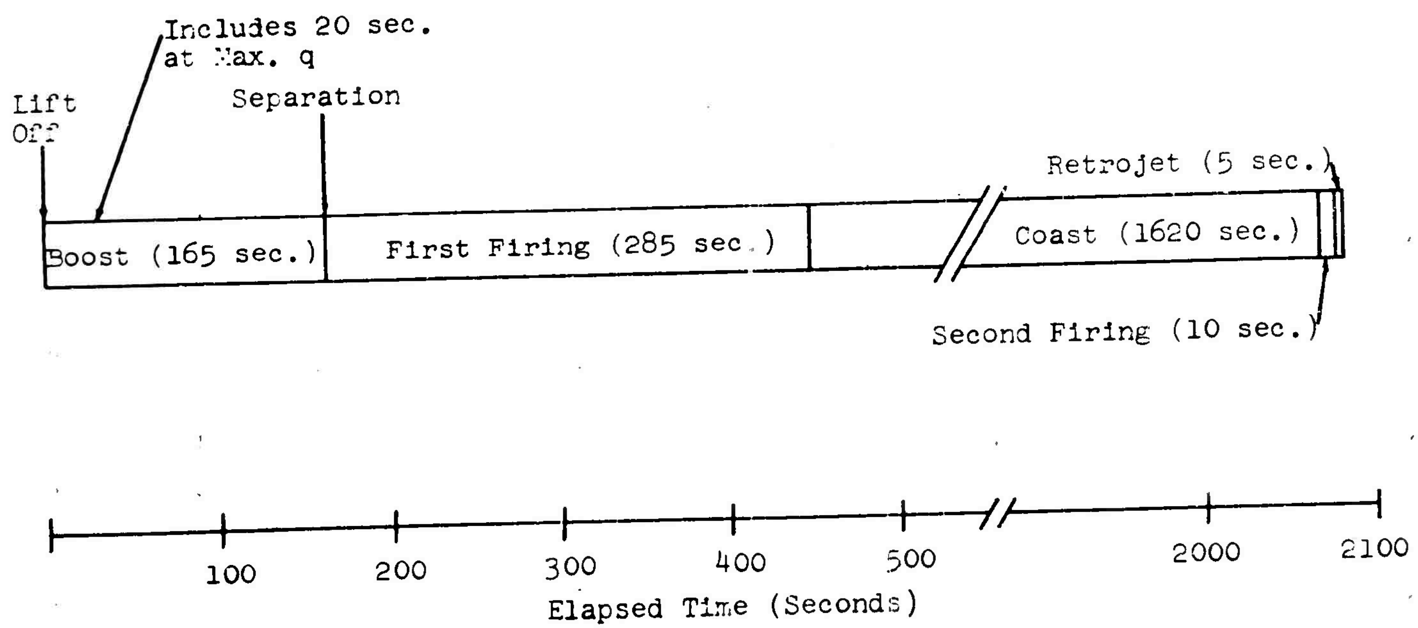

FIGURE 2. SEQUENCE OF EVENTS

TABLE I

TYPICAL IN-FLIGHT EVENTS

Sequence	Event	Sequence	Event
0.	Ride booster	13.	a. Cut off engine (nominal command) b. Enable coast attitude control c. Cut off engine (accelerometer backup) d. Cage accelerometer
1.	a. Cut-off booster engine b. Start Ablestar programmer	14.	Stop hydraulic pump
2.	a. Fire Ablestar engine b. Uncage gyros c. Enable Ablestar steering d. Uncage accelerometer	15.	Stop pitch program
		16.	Start hydraulic pump
3.	Blow booster separation bolts	17.	Actuate restart acceleration jets
4.	Remove separation bolt power	18.	a. Re-start engine b. Uncage accelerometer
5.	Start pitch program		
6.	Start command guidance	19.	Disable coast attitude control
7.	Ignite nose fairing bolt squibs	20.	a. Enable thrust chamber pressure switch b. Enable oxidizer probe cutoff
8.	Remove bolt squib power	21.	a. Cut off engine (accelerometer) b. Enable coast attitude control
9.	Ignite nose fairing actuator squibs		
10.	Remove actuator squib power	22.	Cut off engine (backup)
11.	Enable accelerometer cutoff	23.	Stop hydraulic pump
12.	Stop command guidance	24.	a. Apply payload ordnance power (separate payload) b. Start retrojets
		25.	Vent fuel tank

3.0 Specification of Figure of Merit

Again, it is assumed that the overall Effectiveness analysis has dictated that the Dependability analysis provide the probabilities that the system will complete the mission in each of the three states, given the state of the system at the beginning of the mission.

4.0 Identification of Accountable Factors

Determination of the elements of the Dependability matrix will depend upon several factors. These are discussed below.

4.1 Time-Between-Failures Distributions

With the exception of the safety and arming assembly, all components in the propulsion system and in the forward section exhibit constant failure rates.

The reliabilities of the ordnance and structure portions of the system are assumed to be independent of time and will be estimated by appropriate methods.

4.2 Stresses

Since the stresses experienced by system components differ at various points in the mission, their individual effects must be considered.

4.3 Maintenance Policy

Insofar as Dependability is concerned, the significant observation concerning maintenance is that no repairs can be accomplished during the mission.

5.0 Model Construction

5.1 Delineation of System States

The three mission outcomes delineated in Section 2.4 imply that three physical system states must be considered. These states may be defined in terms of the conditions of specific components of the system.

State 1, which results in "perfect" operation, requires that all system components function properly.

State 2, results from failure of certain non-essential components which cause degraded, but acceptable system operation.

State 3 represents unacceptable operation resulting from failure of one or more essential components.

In this example, all Ordnance, Structural, Range Safety, and Safety and Arming components are essential to an acceptable flight. That is, any failure in these portions of the system will result in unacceptable operation.

In the Propulsion System and Forward Section, however, the failure of certain components will result only in the loss of specific desirable but non-essential functions. For this reason, these items -- and their probabilities of failure -- must be treated separately from the essential items.

In Table II are tabulated those Propulsion System components which provide functions that are not essential for an acceptable flight. Table III shows a similar list for non-essential items in the Forward Section.

The three states may be defined, then, as:

State 1 -- All components operating properly;

State 2 -- All essential components operating properly; and,

State 3 -- One or more essential components not operating properly.

5.2 Operational Considerations and Equipment Usage

During the mission of the spacecraft, several different environmental conditions are experienced. In Table IV these conditions are qualitatively described and the duration of each stress condition noted.

Because the probability of failure is related to the stresses experienced and to the time duration of these stresses, the model must reflect this effect. This will be accomplished by considering individually the probabilities of success for each subsystem during each stress period, and then combining the results.

TABLE II PROPULSION COMPONENTS NOT ESSENTIAL TO ACCEPTABLE FLIGHT
Pressure Transducers (10)
Flowmeter
Sensing Unit (Transonics)
Oxidizer Probe
Thermistor Probe
Valves
Pitch & Yaw Tank Settling Low Thrust Roll
Oxidizer Vent Valve
Fuel Vent Valve
Quick Disconnects
Liquid Gas
Wiggins Valves

TABLE III ELECTRONIC ASSEMBLIES (FORWARD SECTION) NOT ESSENTIAL TO ACCEPTABLE FLIGHT
Telemetry Conditioners
Telemetry Transmitters
Telemetry Antenna
Telemetry Battery
Vibration Transducers (3)
Temperature Sensors (4)
Low Pass Filter

TABLE IV

SPACECRAFT ENVIRONMENTS AND DURATIONS

Environment	Time (Seconds)
Booster Duration (Smooth)	145
Booster Duration (High Vibration)	20
First Firing Duration	285
Coasting Duration	1620
Second Firing Duration	10
Total Mission Length	2080

Certain components of the system are not required to operate during all portions of the mission. This fact must be reflected in the predictions made. Specifically, the Range Safety System is of importance only until the first vehicle firing is complete. Therefore, this system will be evaluated over a period of only 450 seconds.

5.3 System Model

The fact that three states of the system are possible requires a three by three dependability matrix. The matrix will be of the standard form,

$$[D] = \begin{matrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{matrix}$$

Because no maintenance is possible during the mission, no transition from a lower to a higher state is possible. Further, since a system initially in State 3 cannot move to a lower state, the d_{33} element is equal to 1.0. The actual matrix to be evaluated, then, is:

$$[D] = \begin{matrix} d_{11} & d_{12} & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 1 \end{matrix}$$

where

d_{11} = probability that spacecraft will have no failure during mission; given that it is initially nonfailed.

d_{12} = probability that the spacecraft will have one or more non-critical failures in the mission, given that it was initially non-failed.

d_{13} = probability that the spacecraft will have one or more critical failures in the mission; given that it was initially non-failed.

d_{22} = probability that the spacecraft will not have a critical failure in the mission; given that it initially has one or more non-critical failures.

d_{23} = probability that the spacecraft will have one or more critical failures in the mission; given that it initially has one or more non-critical failures.

It is assumed in this example that the several subsystems are independent of each other. Therefore, the Product Rule is applicable. The element d_{11} can be evaluated from:

$$d_{11} = R_P R_F R_O R_S$$

where

R_P = probability of no propulsion failure

R_F = probability of no forward section failure

R_O = probability of no ordnance failure

R_S = probability of no structural failure

R_P and R_F will be computed from the exponential equation

$$R = e^{-\sum_{i=1}^{N} \lambda_i t_i}$$

where

t_i = time period of interest (total time period = $\sum_{i=1}^{N} t_i$)

λ_i = failure rate anticipated during this time period.

R_O and R_S will be determined from a peak stress analysis.

The element d_{12} represents transition from State 1 to State 2 and is represented as:

$$d_{12} = R_P'R_F'(1-R_P''R_F'')\ R_O R_S$$

where

R_P' = probability of no essential component failure in propulsion system

R_P'' = probability of no non-essential component failure in propulsion system

R_F' = probability of no essential component failure in forward section

R_F'' = probability of no non-essential component failure in forward section.

This may also be expressed as

$$d_{12} = R_P'R_F'R_O R_S - d_{11}$$

Values for R_P', R_P'', R_F' and R_F'' may also be computed from the exponential expression.

The d_{13} element is computed from:

$$d_{13} = 1 - R_P'R_F'R_O R_S$$

or

$$d_{13} = 1 - (d_{11} + d_{12})$$

The expression for d_{22} is simply:

$$d_{22} = R_P'R_F'R_O R_S$$

and for d_{23}:

$$d_{23} = 1 - R_P'R_F'R_O R_S$$
$$= 1 - d_{22}.$$

6.0 Data Acquisition

In this section, the sources of failure rate data employed in this analysis will be discussed. Additionally, because the stresses encountered by the system components will differ during various phases of the mission, adjustment factors must be employed to modify the basic failure rates. Therefore, a brief discussion of the environmental stresses will also be presented, and the factors selected for application to specific components during various mission phases will be shown.

6.1 Propulsion System Data

Failure rates for components in the propulsion system were determined from actual usage in the various hangar checkout tests of the spacecraft and its propulsion system, both at the manufacturing site and at AMR prior to launch. Failure data on identical components from both the Able and Ablestar programs was employed. These data include information from the "Able" program which preceded the current program, and from test data on six AJ10-104 propulsion systems.

6.1.1 Data from Previous Program

The failure rates in prior analyses were based on thirteen successful flights of Able-type units prior to the first Ablestar firing with an AJ10-104 propulsion system. In addition, there were four other Able units which, unfortunately, never had opportunity to perform due to malfunctions occurring in the first stage vehicles. The total flight time for these units of 1332 seconds represented an average of a little over 100 seconds operation per propulsion system. It was evident that a valid estimate could not be made with this data alone, since the time on each unit was only a little more than one-third the expected AJ10-104 firing time and the total firing time was only four and one-half times that of a single AJ10-104 propulsion system's operating time.

To obtain more operating time, data were obtained from the pre-flight rating, acceptance, and checkout tests of these prior Able-type vehicles flown. The "hot" firings for all vehicles added up to 3999 seconds. Based on ten checkout tests of AJ10-40 and AJ10-42 propulsion systems, the average checkout time of a single vehicle is 64.4 hours. Therefore, it was concluded that even the hot firing test time was not sufficiently significant for the analysis. For seventeen vehicles the total checkout time is 17 x 64.4 = 1095 hours, which was used as the time base for all failures that occurred in any test phase.

6.1.2 Data from Current Program

Since that time, eleven AJ10-104 propulsion systems have been fabricated and ground tested. Of these, six have a history of time-related test data. This time can be broken into 240

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Table VI.

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used. These failure rates are also [illegible]

TABLE V. FAILURE RATE OF PROPULSION SYSTEM COMPONENTS

(Based on Failures in Ground Tests)

Part Type	Able Type Test Time (No. of Items)(Hours)		Failures in Able Type	Ablestar Test Time (No. of Items)(Hours)		Failures in Ablestar	Total Failures per 10^3 Hours
[1]Accumulator, Bendix							2.119
Accumulator Assy., Pressure	1	1095	1	1	455	0	0.645
Actuator Assy., Servo	1	1095	4	2	321	4	4.606
Block Assy., Servo	Not Available			6	445	2	0.749
Harness Assy., Propulsion	22	1095	4	11	321	0	0.145
Hose Assy., Resistoflex	3	1095	3	7	455	1	0.618
Manifold:							
Able Unit-Manifold	2	1095	1				
Conax Valve Manifold				1	115	0	
Hydraulic Manifold				1	321	0	
Attitude Control System				4	455	1	
Propulsion System				3	445	0	
Manifold Total		2190	1		3581	1	0.347
Probe Assy., Oxidizer	2	1095	2	2	455	0	1.299
Quick Disconnect (Liquid)	5	1095	1	2 6	321) 455)	0) 5)	0.678
Quick Disconnect (Gas)	5	1095	1	3	455	0	.146
[1]Reservoir Assy.	1	1095	1		321	2	2.119
[2]Switch Assy.	1	1095	0	1	455	0	.446
Transducer, Pressure	9	1095	10	9	455	2	0.860
Pressure Lines & Tubes	299	1095	4	298	305	2	0.0176
<u>Valves</u>							
Check Valve FCV	5	1095	1	1	445	1	.337
Check Valve OCV	5	1095	1	2	455	5	.940
Check Valve (Pneumatic & Hydraulic)	5	1095	1	5 3	455 321	0	.115
Valve Assy., Fuel Control (FTCV)	1	1095	5	1	455	0	3.226
Valve, Solenoid (Subject to Propellant Fumes)	4	1095	6	4	455	5	1.774
Valve Assy. Oxidizer Control (OTCV)	1	1095	1	1	455	1	1.290
Valve Assy. Pressure Reg, Helium	1	1095	2	1	455	3	3.226
Valve Solenoid, Gas	7	1095	3	3 3	455) 455)	2) 2)	.675
Valve Assy Pressure Reg, Nitrogen	1	1095	2	1	445	0	1.299
[3]Valve Assy Tavco Relief	1	1095	0	1	445	0	.446
Valve Relief Hydraulic Pressure	1	1095	1	1	321	0	.706

[1]Bendix Accumulator is assumed to have the same failure rate as Reservoir Assembly.

[2]For Confidence of γ = .50, 0 failures in 1550 hours indicate 1 failure in 2240 hours.

[3]For Confidence of γ = .50, 0 failures in 1540 hours indicates 1 failure in 2240 hours.

TABLE VI

[1]AEROSPACE CORPORATION GENERIC FAILURE RATES		[2]MARTIN CORPORATION GENERIC FAILURE RATES	
Part Name	Failure Rate (Failures/10^6 Hours)	Part Name	Failure Rate (Failures/10^6 Hours)
Capacitor, Paper & Ceramic	.10	Accelerometer	2.80
Capacitor, Tantalum	1.00	Battery Cell	1.40
Connector, 10 pins	.20	Choke	.35
Connector, 50 pins	1.10	RF Choke	.05
Diode, General Purpose	.20	Coil	.05
Diode, Power	1.00	Crystal, Oscillator	.60
Gyroscope	10.00	Diodes, Zener	.15
Klystron	10.00	Duplexer	4.00
Quartz Crystal	.40	Heater	.02
Relay, General Purpose	.25/cs*	Inductor	.40
Relay, Power	.30/cs*	Magnetron, Tunable	400.00
Resistor, Fixed	.10	Sensistor	8.00
Resistor, Variable	.60	Sensor, Temperature	3.30
Transformer, Low Voltage	.50	Solder Joint, etc.	.004
Transistor, Germanium	1.50	Switch	.05/cs*
Transistor, Silicon	1.00	Terminal Board	.06
		Thermostat	.06
		Thyratron	5.00
		Transformer, I.F.	.10
		Tube, Subminiature Tetrode	2.15
		Tube, Subminiature Triode	1.75
		Waveguide, Flexible	2.80

[1]See Reference (3)
[2]See Reference (2)
*cs denotes contact set

6.3 Ordnance Section

For ordnance components, such as for stage separation, nose fairing separation and payload separation, each lot of explosive devices is tested to assure a reliability of .995 with 95 percent confidence.

6.4 Structure

Data employed in predicting structural reliability will be presented in a later section in order to facilitate the discussion of the predictive approach.

6.5 Environmental Stresses

In order to use properly the failure rates presented, two conditions must be accounted for. First, the variations in stresses anticipated during actual flight must be considered; and, second, the differences in stress conditions existing during actual flight and those existing at the time of data collection must be evaluated.

Several environmental conditions of flight (vibration, vacuum, thermal conditions) are common to both propulsion components and electronic units.

For each flight, the Ablestar stage may be described as experiencing five distinct environments from first stage "ride" to final burnout. These five environments are (1) the "ride" on the booster stage below and above maximum dynamic pressure (max q), (2) the "ride" on the booster during max q, (3) the time of first firing of the AJ 10-104 propulsion system, (4) the coast time, and (5) the period of re-start (second firing). These times were shown in Figure 2. The time between first stage burnout and second stage firing is not considered because it is too short to affect the overall calculations.

6.5.1 Stress Factors

The failure rates listed in the tables were not obtained from in-flight vehicles. Consequently, a stress factor designated as K, must be used as a multiplier for these listed failure rates so that the in-flight failure rate may be approximated. Table V E (pg. 93) in Data Source #7 (WSEIAC) shows the various adjustment factors for different environmental stresses, as reflected in vehicle mission.

As discussed below, a somewhat different approach is used in determining the K factor for propulsion components and electronic or forward section components.

The electronic subsystems acceptance checkouts are considered as being at somewhat higher environmental stress than the ground conditions of Data Source #7 (WSEIAC). The environment for electronic subsystems at maximum aerodynamic pressure of the boost phase is considered to be much more severe than the environment for the remainder of the boost and vehicle operation.

Similarly, for each change of environment of the stage, the value of K changes for the propulsion system although somewhat differently from the assumed change of K in the electronics subsystems. Ground tests of the propulsion system components do not compare in many instances with the stress experienced in flight; i.e., nitrogen is used throughout during propulsion acceptance leak checks instead of actual propellants. However, there is no difference in the electrical power applied to the electronic units between systems tests and flight operation.

Propulsion System Factors

The propulsion checkout tests are fairly severe. The boosted flight, however, except at lift-off, during the transonic period and at stage separation, is quite smooth. During these periods -- which in total probably do not exceed 20 seconds -- a K factor of 6.7 is used.

During the period of vehicle operation, a factor of unity is applied. Since during the major portion of the Boost phase the stress is less than during vehicle operation, a factor of 0.8 is employed. The Coast period represents even lower stresses, leading to the assignment of values of 0.1 and 0.2 to K.

The attitude control, however, continues to operate at an operational stress close to the design nominal. Consequently, a value of 1.0 is employed for this device. These factors are summarized in Table VII.

TABLE VII.

STRESS (K) FACTORS APPLIED TO FAILURE RATES FROM VARIOUS SOURCES

Failure Rate Data Source	Boost Phase			Vehicle Operation			Coast Period	
	Non-Pressurized Non-Powered Non-Operating	Pressurized Power-On Non-Operating	Maximum Dynamic Pressure	Pressurized Power-On Non-Operating	Pressurized Power-On Operational	Non-Pressurized Non-Power Non-Operating	Pressurized Power-On Non-Operating	Pressurized Power-On Operational
Able Type, Ablestar Hangar Check-Out R.F. Data	.3	.8	6.7	0.5	1.0	0.1	0.2	1.0
7th Symposium of IRE, Jan. 1961 (Ref. 2)	50	150	500	400	400	10	20	120
Structural Materials 7th Symposium of IRE, Jan. '61 (Ref. 2)	50	50	200	50	50	1	1	1
Aerospace Corp. Rpt. 1923-1-59 Data (Ref. 3)		25	200		40			10

Electronic System Factors

In calculating the failure rates for electronic components, a K factor of 200 (250 for the gyro reference assembly) was selected for the 20 seconds of maximum stress (which occurs at lift-off, maximum aerodynamic pressure and again at stage separation), a K factor of 25 for Booster phase, 40 for vehicle operation and 10 for the coast period for the following reasons: The stress encountered by the electronics during Ablestar operation when the electronics are required is assumed to be somewhat high than the stress encountered during the ground systems test. There is some empirical evidence that the failure rates determined from ground systems testing on the Ablestar electronics are about 25 times the failure rates encountered under laboratory conditions.[1/] On this basis, it is assumed for the boost operation with power on and electronics not required, that the stress factor K is 25, and for the Ablestar operation with power on and electronics required, that the stress factor is 40. During coast period, a stress factor of 10 is used. Vibrational effects are virtually non-existent, but vacuum, temperature, and other space influences may tend to cause deleterious stresses during coast. These factors are also shown in Table VII.

[1/] From Table I in Ablestar Stage Reliability Progress Reports, SGC No. 105R Series.

7.0 Parameter Estimation

In this section, the several reliability characteristics for the system components will be determined.

7.1 Propulsion System

7.1.1 Probability of Perfect Flight

The reliability of the AJ 10-104 propulsion system, including the mechanical components of the attitude control subsystem, is computed from:

$$R = e^{-(t_1 \lambda_1 + t_2 \lambda_2 + t_3 \lambda_3 + t_4 \lambda_4 + t_5 \lambda_5)}$$

where

t_i is the time during which a particular stress is encountered, and

λ_i is the failure rate during that period.

The subscripts indicate the following time periods:

1 - Booster ride

2 - Maximum aerodynamic pressure during booster ride

3 - First spacecraft operation

4 - Coast operation

5 - Re-start operation.

Employing the failure rates and the stress factors discussed in Section 6 the failure rates shown in Table VIII for the several sub-assemblies of the propulsion system may be obtained.

TABLE VIII

FAILURE RATES - ALL COMPONENTS OF THE AJ10-104 PROPULSION SYSTEM

(Failures per Thousand Hours)

	Booster Ride	Maximum "q"	First Firing	Coast Period	Second Firing
Pneumatic System	9.01	74.10	12.14	2.18	14.12
Tank and Transition Assembly	.72	2.87	.72	0.10	.72
AJ10-104 Harness Installation	.39	3.46	0.57	.14	0.57
Hydraulic System Installation	18.14	139.84	26.08	4.26	26.08
Transducer Installation	6.32	52.72	7.90	1.58	7.90
Fill & Drain System Installation	4.90	39.73	6.27	1.16	6.27
Oxidizer Tank Pressurizing Installation	0.07	0.37	0.14	0.01	0.14
Attitude Control & Restart System	4.44	77.74	10.48	7.27	10.48
Thrust Chamber and Support Assembly	8.0	116.19	24.57	3.48	24.57
Components not in Major Subassemblies	3.47	38.57	6.48	1.03	6.48
TOTAL	56.09	545.78	92.14	24.42	94.12

While not shown in this report, the failure rates of individual elements as sub-assembly components are presented in Reference 1 (Table VI). The reference also shows the K factors employed. Generic failure rates for working and pressurized components; e.g., valves and pressure lines, listed in the reference are estimated failure rates during system tests on these items. Failure rates for structural and miscellaneous items; e.g., brackets, gaskets, are from Reference 2.

Employing the summations of failure rates from Table VIII and the times from Figure 2, the probability of perfect operation of the propulsion and related systems is estimated from:

$$R_P = e^{-(56.09)(145)/(3600)(1000)}\ e^{-(545.8)(20)/(3600)(1000)}$$

$$e^{-(92.14)(285)/(3600)(1000)}\ e^{-(24.42)(1620)/(3600)(1000)}$$

$$e^{-(94.12)(10)/(3600)(1000)}$$

$$= e^{-85,811/3,600,000}$$

$$R_P = e^{-.02384}$$

$$R_P = .9764$$

7.1.2 Probability of Acceptable Flight

The following reliability estimate is based on the assumption that items such as propellant and gas fill and drain quick disconnects and the oxidizer and fuel vent valves are items which do not function or operate after initial loading. Any leak in these items will be detected while the vehicle is still on the ground. Also, certain disconnects and valves have redundant features which preclude leakage; therefore, their reliabilities will be approximated by unity in the acceptable flight calculations. These are: (1) fill and drain disconnects and shut-off valves, both fuel and oxidizer; (2) umbilical power disconnects, squib actuated and pull separation; (3) helium and nitrogen fill disconnects, and helium and nitrogen check valves.

During the restart and second Ablestar firing, the pitch and yaw control valves need not operate; one valve of this type, which is used as a settling jet, does not operate beyond this point.

The pressure transducers are not essential for acceptable operation, and the destruct assembly is also not necessary for either "perfect" or "acceptable" flight; for it will be used only if the flight is, in fact, determined to be unsuccessful.

Table IX lists the failure rates of these non-essential items. Subtracting these failure rates from the total failure rates of Table VI, the following estimates of the failure rates results: booster ride 56.09 - 18.12 = 37.97; high vibration 545.8 - 153.1 = 392.7; first operation of Ablestar 92.14 - 27.16 = 64.98; coast period 24.42 - 4.51 = 19.91; and second operation of Ablestar 94.12 - 29.74 = 64.38 failures per 1000 hours. The estimated

TABLE IX

FAILURE RATES - PROPULSION COMPONENTS NOT ESSENTIAL FOR ACCEPTABLE FLIGHT

(Failures per Thousand Hours)

	Booster Ride	Max. Vibr.	First Firing	Coast Period	Second Firing
Pressure Transducers (10)	6.880	57.620	8.600	1.720	8.600
Flowmeter	.022	.075	.060	.003	.060
Sensing Unit (Transonics)	.872	6.972	1.394	.349	1.394
Oxidizer Probe	0.779	17.403	2.597	.260	2.597*
Thermistor Probe	1.800	6.000	4.800	.240	4.800
Valves (Pitch & Yaw)					1.498
Valves (Tank Settling)					0.338
Valves (Low Thrust Roll)					0.749
Oxidizer Vent Valve	1.419	11.887	1.774	.355	1.774
Fuel Vent Valve	1.419	11.887	1.774	.355	1.774
Quick Disconnects (Liquid)	3.255	27.264	4.069	.814	4.069
Quick Disconnects (Gas)	.585	4.898	.731	.146	4.731
Wiggins Valves	1.085	9.088	1.356	.271	1.356
TOTAL	18.116	153.094	27.155	4.513	29.740

*Armed for only last part of second firing.

reliability of the Propulsion System for acceptable flight is thus:

$$R'_P = e^{-(37.97)(145)/(3600)(1000)} \; e^{-(392.7)(20)/(3600)(1000)}$$

$$e^{-(64.98)(285)/(3600)(1000)} \; e^{-(19.91)(1620)/(3600)(1000)}$$

$$e^{-(64.38)(10)/(3600)(1000)}$$

$$= e^{-64,777/3,600,000}$$

$$= e^{-.017994}$$

$$= .9822.$$

7.2 Forward Section

The reliability of the forward section of the Ablestar stage is calculated through the use of the failure rate data summarized in Table VI.

7.2.1 Probability of Perfect Flight

Table X tabulates the failure rates of all electronic components in the Forward Section. For a "perfect" flight, all assemblies listed in Table X must function properly. The following calculations show the reliability estimate for a "perfect" flight of the electronics section.

TABLE X

ELECTRONIC ASSEMBLIES NECESSARY FOR PERFECT FLIGHT

Assembly Name	Generic Fr/10^6 Hrs.
Essential Assemblies (See Table X)	816.81
Telemetry Conditioner	388.82
Telemetry Transmitter	31.86
Telemetry Antenna	2.00
Telemetry Battery	25.30
Vibration Transducers (3)	90.00
Temperature Sensors (4)	13.20
Low Pass Filter	.82
TOTAL (perfect)	1368.81

$$R_F = e^{-\frac{1}{3600\times10^6}\left\{20\left[(200)(1287.98) + 250(80.23)\right] + 1368.81\left[145(20) + (285)(40) + 1620(10) + 10(40)\right]\right\}}$$

$$R_F = e^{-0.0136}$$

$$R_F = .9865.$$

7.2.2 Probability of Acceptable Flight

Table XI lists the electronic subsystems which must work during an "acceptable" flight. Since the static inverter dummy load can fail open and not cause serious degradation of the flight, only half of its failure rate is used for the "acceptable" flight. The BTL guidance package is not included in these calculations because it is Government-furnished and is therefore, treated as being external to the Ablestar stage. The following calculations show the reliability estimate for an "acceptable" flight of the electronic section:

$$R_F' = e^{-\frac{1}{3600\times10^6}\left\{20\left[(200)(737.22) + (250)(80.83)\right] + 816.24\left[(145)(25) + (285)(40) + (1620)(10) + (10(40)\right]\right\}}$$

$$R_F' = e^{-.0081}$$

$$R_F' = .9919.$$

TABLE XI

ELECTRONIC ASSEMBLIES NECESSARY FOR ACCEPTABLE FLIGHT

Assembly Name	Generic Fr/10^6 Hrs.
Gyro Reference Assembly	80.83
Electronics Package	212.91
Programmer	346.96
Accelerometer	75.44
Distribution Box	14.84
Battery Box	23.14
Static Inverter	56.44
Static Inverter Dummy Load	1.13*
Fuel Vent Cable	.12
TOTAL (acceptable)	816.245

*Total generic failure rate of these assemblies is 818.61 failures per million hours but only half the failures of static inverter dummy load (i.e., failed open) will cause an unacceptable flight. Therefore, total generic failure rate is 816.81 - 1.13/2 = 816.245 failures per million hours.

7.2.3 Range Safety System

In determining the reliability of the electronic section, the reliability of the Range Safety System has not been included. This has been done because the functioning of the Range Safety System is wholly dependent on a failure of another part of the stage. The probability of the Range Safety System working when it is not supposed to is very remote and is, therefore, not included in the calculations.

The electronics assemblies of the Range Safety System are listed in Table XII. Since the Range Safety System is only required to function through SECO I, its operating time is 450 seconds. The reliability estimate of the electronics of the Range Safety System is shown in the following calculations.

$$R_{RS(electronics)}$$

$$= e^{-\frac{(20)(200)(665.51)+(145)(25)(665.51)+(285)(40)(665.51)}{3600 \times 10^6}}$$

$$= e^{-.0035}$$

$$= .9965.$$

The safety and arming mechanism is placed in the propulsion section and its reliability is estimated as .9982, apportioned .9984 to the switch and explosive charge and .9998 to the mechanism.

TABLE XII

RANGE SAFETY SYSTEM ASSEMBLIES

Assembly Name	Generic Fr/10^6 Hrs.
Tracking Beacon	505.48
Receiver	81.60
Control Box	10.36
Battery	25.30
Antennas (6)	12.00
378 Beacon	18.67
Beacon Battery	11.28
Destruct Filter	.82
TOTAL	665.51

The main contributors to mechanism unreliability are believed to be the receptacle and spring plunger each having a constant failure rate. The reliability of each of these two items is at least .9999 for the time the destruct assembly is under stress. The switch and explosive charge have an estimated reliability of .9984 based on data from Ordnance Associates, Inc. When the ordnance is considered, the estimated reliability of the Range Safety System becomes:

$$R_{RS} = (.9965)\ (.999)^2\ (.9984)$$
$$= .9947.$$

The probability that a malfunction in Ablestar is of sufficient importance to cause a Range Safety destruct signal and that the Ablestar stage will be successfully destroyed is the product of two probabilities; i.e., (.9947) (probability Ablestar is off-course or lacking in velocity until SECO 1),

or (.9947) (1 - reliability of all propulsion subsystems necessary for acceptable flight and all forward section subsystems necessary for acceptable flight except the integrating accelerometer).

$$(.9947)\left[1-\left(\exp - \left(\frac{[(145)(37.97)+(20)(392.7)+285(64.98]}{3600 \times 10^6}\right)\right)\right.$$
$$\left.\left(\exp - \frac{[(20)(200)+(145)(25)+(285)(40)]\ [(661.6)] + [(20)(250)+145)(25)+(285)(40)]\ [(80.83)]}{3600 \times 10^6}\right)\right]$$

$$= (.9947)\left[1 - (e^{-.0089})(e^{-.0040})\right]$$
$$= (.9947)\left[1 - (.9911)(.9960)\right]$$

$$= (.9947)(1 - .9871)$$

$$= (.9947)(.0129)$$

$$= .0128.$$

Consequently, the probability that destruction will be necessary is 1.29% and the probability of being destroyed is 1.28%. The probability of an Ablestar going off course and not being successfully destroyed is .0129 - .0128 = .0001, or 0.01%.

7.3 Ordnance Items

Excluding range safety ordnance, the ordnance of the Ablestar Stage includes the booster and Ablestar separation, and the nose fairing jettison. Both must function properly for acceptable flight. There are two explosive bolts on the nose fairing, 180° apart, two sets of actuators (two each) on the nose fairing assembly, and three explosive bolts on the interface between the first and second stages. Each of the separation assemblies has a demonstrated reliability of at least .995 with 95% confidence.[2/] The same number of tests without failure which demonstrate a reliability of .995 with 95% confidence also demonstrate a reliability of .9988 with 50% confidence. The inherent reliability is certainly greater than the demonstrated reliability and is estimated at .9988.

[2/] Aerojet Specifications AGC 54006 and AGC 54009 (Per STL Memo #7740.14-21).

Consequently, the reliability of the Ablestar ordnance (excluding range safety) is estimated to be:

$$R_o = (.9988)(.9988)$$
$$= .9976.$$

Two redundant SEV-22M Conax retro system control valves in the propulsion system provide a counter thrust at payload separation so that the Ablestar will not hit the payload and damage it or interfere with its proper orbit. There are also two Conax vent valves in the forward tank which are actuated subsequent to payload separation to minimize possible explosion due to residual propellants. These redundant Conax valve systems are at least as reliable as the other ordnance subsystems, so their reliability also is estimated nominally at .9988. Thus total ordnance reliability, excluding range safety, is (.9976)(.9988) = .9964.

7.4 Structure

In order to demonstrate one approach to the assessment of the reliability of a structure, it is assumed that test data of the types noted below are available.

7.4.1 Test Data

Previous experience has shown that critical structural members exist in the tanks and in the transition stage. Other structures are engineered with sufficiently proved safety factors to warrant assignment of a reliability unity. Testing is concentrated, therefore, on the critical items.

Tank Reliability

In view of a reliability reserve, it was possible to adopt the thoroughly conservative methodology of proving tank yield points against proof pressures in lieu of the looser (but still valid) procedure of proving burst points against maximum expected operating pressures (MEOP). For the helium tank, burst testing has shown that rupture occurs at 6077 psi. The working pressure (MEOP) is taken as 4400 psi for this analysis, and a safety factor of 1.1 is applied to obtain a proof pressure of 4850 psi. In order to evaluate this safety margin on a probability basis, strain gages placed at points of each helium tank during proof testing provided 31 measurements of strain and stress during the burst test. Table XIII shows typical data from the test.

Yield points for these 31 stress-strain variables were determined by graphing strain data from the rupture testing against pressure. Knick points in the graphs determine the strain for this material and tank configuration at which Hooke's law of elasticity ceases to hold, namely, the yield points. These plots and the knick points were assembled in four graphs, an example of which is shown in Figure 3. These graphs indicate that 16 data curves reach yield points near or before the tank burst pressure. These are the most suitable locations for reliability tests.

TABLE XIII

AJ10-104 HELIUM SPHERE (JC5) CALCULATION OF STRESS

Young's Modulus of Elasticity = 16.8×10^6 psi
Poisson's Ratio = 0.33000000

Pressure Psi	3 Longitudinal Strain Micro-In/In	4 Hoop Strain Micro-In/In	Longitudinal Stress Psi	Hoop Stress Psi
*GAGES 3 & 4				
2000	1553	1539	38854	38677
4000	3066	3155	77433	78557
4400	3434	3501	86523	87369
4850	3761	3847	94841	95927
5000	3884	3962	97875	98860
5250	4088	4155	102922	103768
5500	4170	4270	105183	106447
5750	4660	4578	116338	115302
6000	5274	5040	130788	127832
6077	5601	5239	138191	133618
GAGES 7 & 6				
	7	6		
2000	1927	1636	46508	42833
4000	3889	3352	94174	87391
4400	4310	3712	104351	96797
4850	4730	4111	114752	106933
5000	4871	4230	118151	110054
5250	5116	4470	124263	116103
5500	5256	4590	127649	119236
5750	5782	5069	140546	131539
6000	6167	5547	150778	142946
6077	6237	5747	153342	147152

*Gages 1 & 2, Gage 2 Inoperative

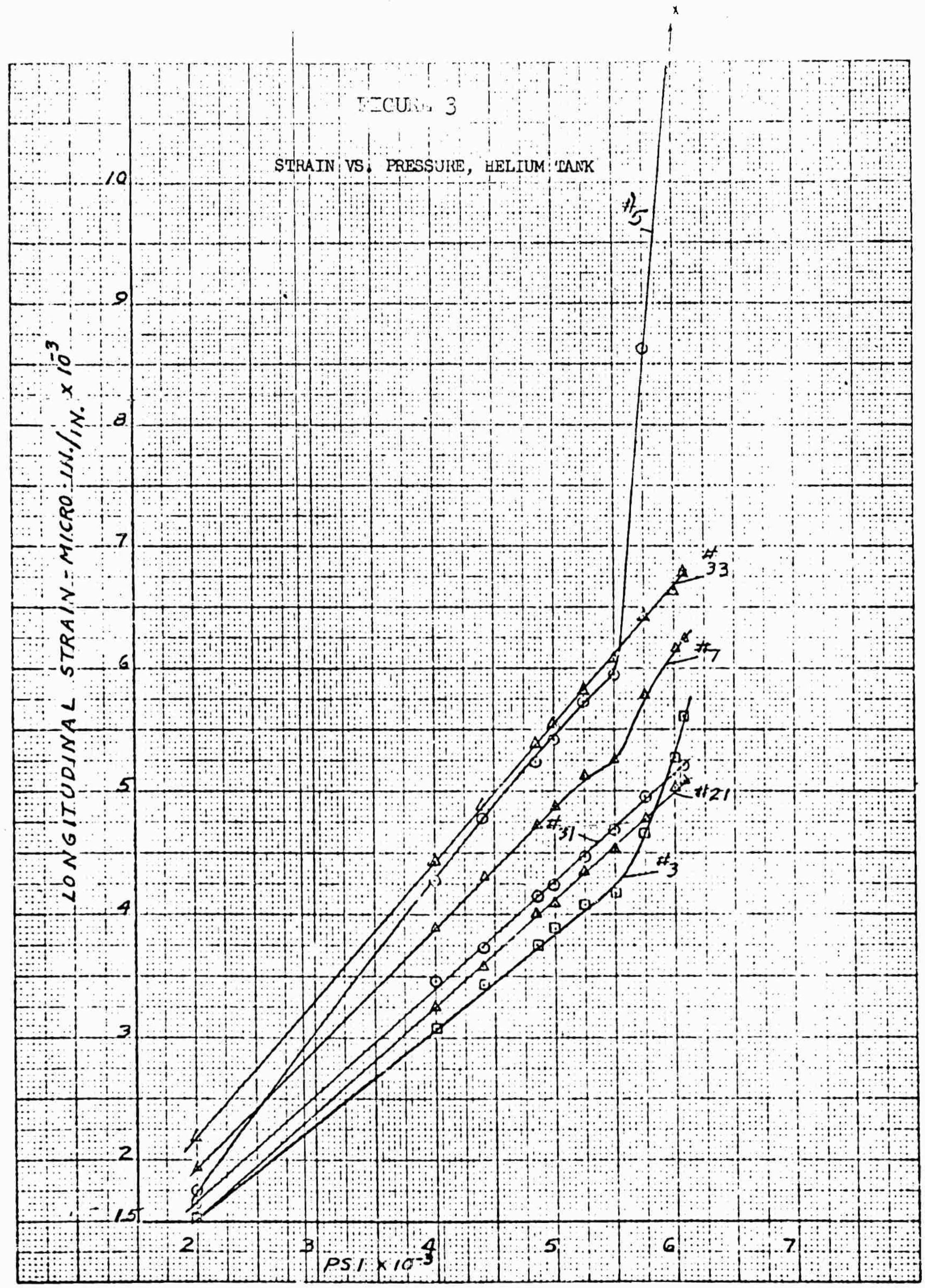
FIGURE 3
STRAIN VS. PRESSURE, HELIUM TANK
LONGITUDINAL STRAIN - MICRO IN./IN. x 10^-3
PSI x 10^-3
#5
#33
#7
#21
#31
#3
1.5
2
3
4
5
6
7
8
9
10

By making 16 comparable series of pressure-strain readings during proof tests on helium tanks for successive vehicles, a series of maximum strain readings could be obtained for each point. Suppositional data listed in Table XIV to permit completion of the calculation example. The maximum strain for each of 8 supposed tests is derived from the Table XIV data and listed in Table VI together with the nick point derived from the Table XIII data. an extreme-value plot of these maxima is presented in Figure 4. From these plots the structural risk determined for each location is assessed and listed in Table XV. The maximum risk, .000015, is at the location which is weakest for the mission analyzed. This risk is the structural unreliability of the tank against the burst mode of failure, and

$$1 - .000015 = .999985$$

is the structural reliability of the tank.

A gage, or "point," represents a location and directional reference of strain, caused by stress properly imposed under simulated environment.

It will be noted from Figure 3 that the traces of strain gages #21, #31 and #33 did not show knick points from the rupture testing. This means that the strains measured by these gages did not extend to include yield points of the tank. Hence, the gage locations were representative of relatively strong points in the tank configuration and will be excluded from the further analysis.

TABLE XIV

EIGHT PROOF TESTS,* HELIUM TANK

Pressure Psi	Gage No.	Long. Strain μ in/in	Gage No.	Hoop Strain μ in/in	Gage No.	Long. Strain μ in/in	Gage No.	Long. Strain μ in/in	Gage No.	Long. Strain μ in/in	Gage No.	Hoop Strain μ in/in
Test #JC 5	3		4		5		7		25		26	
1800		1360		1437		2045		1942		2147		2137
3000		2310		2397		3015		2907		3117		3105
4000		3065		3155		4285		3890		4145		3900
4500		3435		3500		4777		4310		4564		4320
4840		3695		3760		5080		4690		5020		4010
Test #JC 6	3		4		5		7		25		26	
1800		1240		1220		2148		1877		2140		1722
3000		2190		2169		3116		2837		3124		2682
4000		2900		2878		3933		3825		4149		3565
4500		3190		3205		5200		4185		4568		3950
4840		3770		3860		4775		4650		4980		4320
Test #Sup-1	3		4		5		7		25		26	
1800		1300		1360		2385		2048		2745		1785
3000		2250		2300		3355		3018		3715		2747
4000		3040		3110		4175		3838		4535		3540
4500		3415		3492		4593		4252		4955		3940
4840		3670		3840		5065		4700		4655		4410
Test #Sup-2	3		4		5		7		25		26	
1800		1210		1268		2436		2043		2290		1870
3000		2160		2230		3406		2993		3260		2815
4000		2900		3045		4226		3805		4080		3585
4500		3680		3440		4646		4215		4495		3990
4840		3620		3850		5120		4680		4960		4340

*Suppositional Data Only

TABLE XV

CALCULATED RELIABILITY DATA, TANK

No. of Gage Location on Tank	Yield Point (Knick on Strain Trace)	Proof Test Maxima* (Micro-in/in)	Risk of Exceeding Yield Point at Proof Pressure
3	4250	3695, 3768, 3670, 3680, 3836, 3700, 3662, 3740	.000012
4	4190	3760, 3860, 3840, 3850, 3840, 3860, 3790, 3780	.000015**
5	5950	5080, 5200, 5065, 5120, 5060, 5100, 5070, 5110	.000000
7	5250	4690, 4650, 4700, 4680, 4685, 4740, 4640, 4750	.000003
8	5560	-	-
10	4700	-	-
12	4950	-	-
16	4600	-	-
18	4650	-	-
19	5550	-	-
20	5350	-	-
21	NONE	-	-
22	3750	-	-
23	4650	-	-
25	5500	5030, 4920, 4955, 4960, 4910, 4920, 4850, 4920	.000001
26	4780	4320, 4320, 4410, 4340, 4310, 4340, 4280, 4340	.000007
30	4800	-	-
31	NONE	-	-
33	NONE	-	-

*Suppositional data only

**Maximum

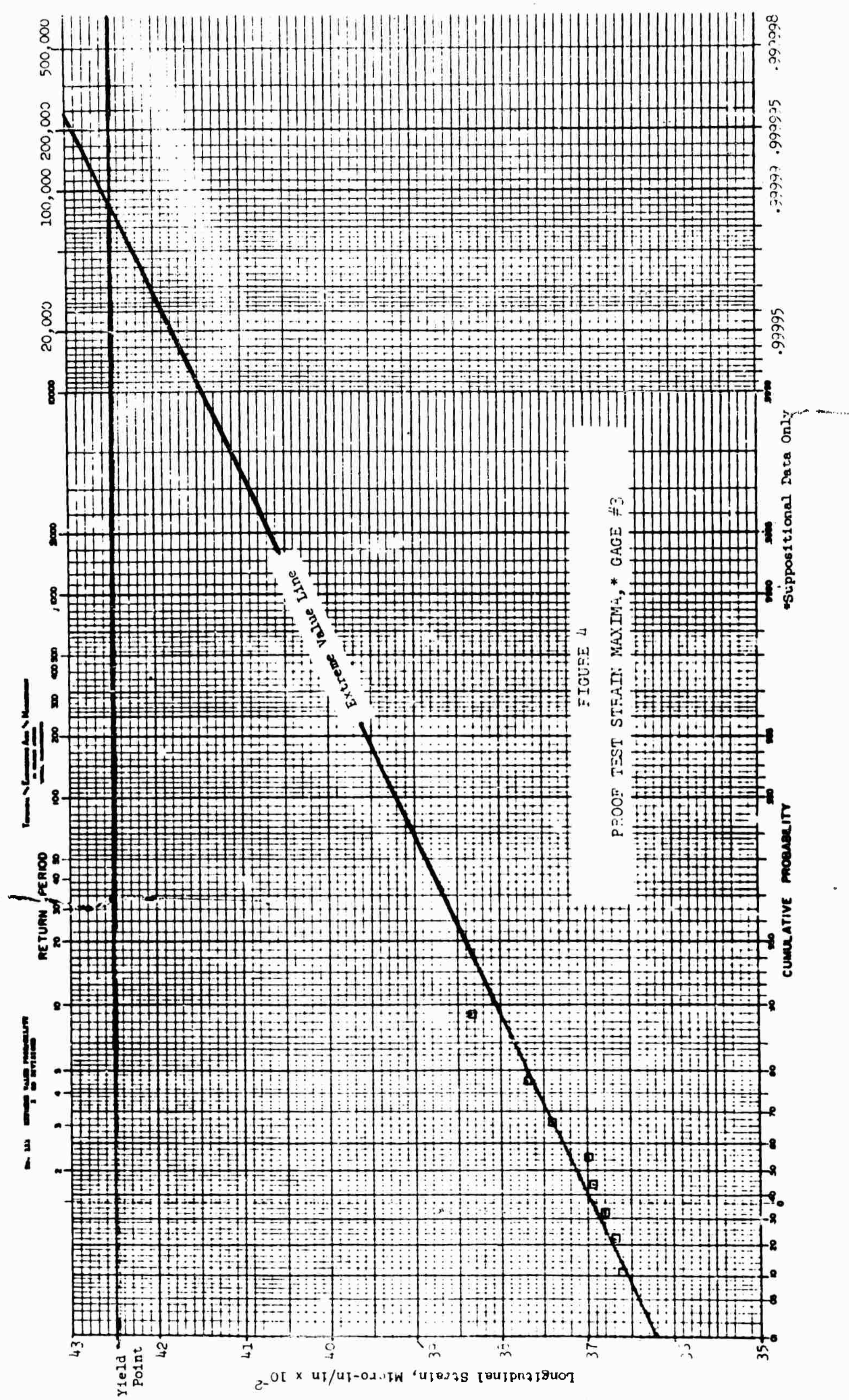

FIGURE 4

PROOF TEST STRAIN MAXIMA,* GAGE #3

In particular, these gage points would not be instrumented in proof testing the tanks of successive production vehicles.

In general, engineering judgment should be used to instrument the suspected weakest locations of the structure being evaluated by destructive testing. The instrumental traces are then examined for yield points. The locations for which yield points appear should be instrumented during future structure reliability proof testing of production items. Reliability of the individual vehicle structures may then be calculated.

Further comments chis approach are presented in Section 9.

Transition Stringer Reliability

Through similar considerations, again using suppositional data in order to provide a ready example (Table XVI), the case is considered in which too few data are available for fitting extreme value functions. For such a case, a graph like Table II in Reference 7 (which is based on normal-curve, or Gaussian, probability theory) may be developed. It is recommended that sample sizes from 3 to 7 be included in this graph at 65% statistical confidence, and that the graph be extented to 6 nines.

The reason for suggesting that a confidence level should automatically be imposed is that, for so few tests, statistical accidents of sampling can occur with unacceptable frequency unless so controlled. The reason for recommending 65% confidence

TABLE XVI TRANSITION STRINGER		
Failure Stress	Design Load x 10^{-3}	*Yield Point x 10^{-3}
Moment(in-lb)	$M_o = 787.5$	1,217.7 1,261.0 1,292.1 $\bar{X} = 1256.93$ $S = 37.4044$ $K = 12.55$ $R = >> .9999$**
Axial Load (LB)	$A_o = 123.9$	250.0 262.5 273.9 $\bar{X} = 262.13$ $S = 11.9542$ $K = 11.56$ $R = >> .9999$**

*Suppositional data only.
**cf Reference 7, Table II.

is that binomial and exponential models result in the same[3/] reliability estimate from the same test data at a confidence level between 62% and 63% (imposing, thus, a lower limit on the most suitable confidence level), while on the other hand all feasible information should be drained from these few and expensive data (imposing an upper limit). For more than 7 points, extreme value curve fitting is deemed potentially more accurate.[4/]

Stringer members of the Ablestar transition-stage structure have a torque design load M_o = 787,500 inch-pounds. From suppositional data, provided in Table 16 to permit presentation of an example of Gaussian-model analysis, it is calculated that

$$K = \frac{\overline{X} - M_o}{S} = 12.55 > 8.$$

Since the mean yield point (calculated for the 3 suppositional tests by methods like the knick-point determination discussed earlier) is more than 8 standard deviations above the design load, any possible unreliability may be ignored in accordance with the suggestions in Reference 12. In other words, structural reliability against this failure mode is estimated conservatively to be insignificantly less than unity and will be taken as unity. Note that the standard deviation was estimated using N-1 as a divisor (small-sample estimator).

[3/] Provided no failures occur during test.

[4/] However, it is conceivable that more than 7 structural readings which are not maxima of sets could be analyzed by the Gaussian stress-strain technique.

Stringer Reliability Under Compression During Boost

Similarly, Table XVI provides the calculations from suppositional axial-load tests of 3 stringers which show the test mean to be more than 8 standard deviations above the design load A_o = 123,900 pounds. Therefore, structural reliability against this failure mode also will be taken as unity. Combining the results obtained above produces (by product rule) a total structural reliability of

$$R_S = (.999985)(1.000000)(1.000000) = .999985$$

$$R_S = .9999.$$

Alternative Techniques

For structures which are simple in the sense that the distribution of mission stresses throughout the structure can be determined analytically, strain data can be related to stress by methods like those in Reference 15. If strengths of all materials are known and can be used to obtain the structural strength of each member against all significant failure modes, it may in some cases be possible to determine reliability by propogating the sample-to-sample variance in yield points into the variance parameter of the density function of the yield points.

For complex structures for which suitable stress-distribution equations and other performance equations can be developed explicitly in compatible transfer-function form, a combination of the methods in Reference 13 (for environmental stress effects) and 14 (for probabilistic determinations) should be developed.

8.0 Model Exercise

It is now possible to evaluate the elements of the Dependability matrix by substituting the parameter estimates made in Section 7.0 in the models presented in Section 5.0.

The several probabilities determined in Section 7.0 are summarized below.

Parameter	Probability
R_P	.9764
R_P'	.9822
R_F	.9865
R_F'	.9919
R_O	.9964
R_S	.9999
$R_{\text{Safety \& Arming}}$	.9982

The models presented earlier will first be evaluated, including the effect of the Safety and Arming device in the d_{11} element

$$
\begin{aligned}
d_{11} &= R_P \; R_F \; R_O \; R_S \; R_{S+A} \\
&= (.9764)(.9865)(.9964)(.9999)(.9982) \\
&= .9579
\end{aligned}
$$

$$d_{12} = R'_P \, R'_F \, R_O \, R_S - d_{11}$$

$$= (.9822)(.9919)(.9964)(.9999) - .9579$$

$$= .9706 - .9579$$

$$= .0127$$

$$d_{13} = 1 - (d_{11} + d_{12})$$

$$= 1 - (.9579 + .0127)$$

$$= .0294$$

$$d_{22} = R'_P \, R'_F \, R_O \, R_S$$

$$= (.9822)(.9919)(.9964)(.9999)$$

$$= .9706$$

$$d_{23} = 1 - d_{22}$$

$$= 1 - .9706$$

$$= .0294$$

The resulting matrix, then, is:

$$[D] = \begin{bmatrix} 0.9579 & 0.0127 & 0.0294 \\ 0 & 0.9706 & 0.0294 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

This result can then be employed in conjunction with vectors for Availability and Capability as demonstrated in previous examples.

9.0 Additional Comments - Structural Reliability

The calculation of structural reliability is recommended for those areas where any substantial compromise with reserve strength has been necessary in order to increase vehicle performance within prescribed weight limitations or similar constraints. It is also recommended where new departures in structural design transcend the availability of solid engineering experience with safety margins. Within the current practice of providing adequate, solidly based engineering safety factors to well analyzed structures of known characteristics, however, structural reliability may customarily be taken as unity.

Because of this, programs may often be funded under restrictions that prevent extensive application of expensive structural tests, without which the variability data necessary for statistical appraisal of the risks of structural failures during mission operations (structural unreliability) cannot be assessed. Consequently, it is appropriate to exemplify two structural calculation models, one for proper statistical analysis based on extreme-value theory, and the other for statistical control of the structural risk when data are insufficient for proper analysis. The general theory of extreme values of structural strain is discussed briefly below, with references. The two examples presented in Section 7.4 illustrate these approaches.

For structures, only two states are considered, namely, not failed and failed. There is at least a third state in which a yield point, or elastic limit, of a structural member has been exceeded without mission degradation. However, state-of-the-art data are not expected to suffice for useful conclusions as to reliability within this marginal region.

9.1 Extreme Value Approach to Evaluation of Structure

Application of the extreme value theory to the validation of structural designs gives the analyst a significant tool in the assessment of test results of various structural members. In turn, it provides a basis for prediction of the reliability of individual components or structural members. It is, therefore, an important ancillary method for the Safety Margin Analysis Theory. Preliminary considerations given to the application of the extreme value theory indicates that it can provide means for rapid graphical analysis of the best results and give valid results on which to derive significant engineering conclusions.

The strength of materials in engineering has been always an important design criterion. The growth of statistical methodology provides important means to further refine and sharpen the analytical and design considerations of equipment structures. For example, for observations based upon strength of materials, if the ratio $s:\bar{x}$ is increased from 5% to 25%, where s is standard deviation from measurement and $\bar{x}$ is mean strength, then the failure rate increases by a factor of 50, approximately once in 500 observations. Hence, the scatter in ultimate strength should be considered. See Figure 5.

Increased scatter can be caused by three sources: (1) The material and surface conditions of the specimen; (2) Errors in the nominal loads of the testing machine; and, (3) Environmental changes. (1) and (2) imply that the machines or specimens are not homogeneous. (1) could also indicate that the stress levels of the test are not sufficiently separated as may occur in the case when observations may be assigned to the wrong stress level. Increased scatter can be caused, for example, by increasing the temperature of the material at different intervals of time during its application.

The scatter is a function, therefore, of the sample size and provides realistic protection in structural validation.

A good book[9] treating the subject contains an article contributed by E. J. Gumbel called "Statistical Estimation of the Endurance Limit, An Application of Extreme-Value Theory." Gumbel defines the probability of survival, l(x), which is the same as reliability, and plots a family of l(x) curves on S-N axes. See Figure 6. Stress is S and the number of cycles is N. These curves are plotted in logarithmic space.

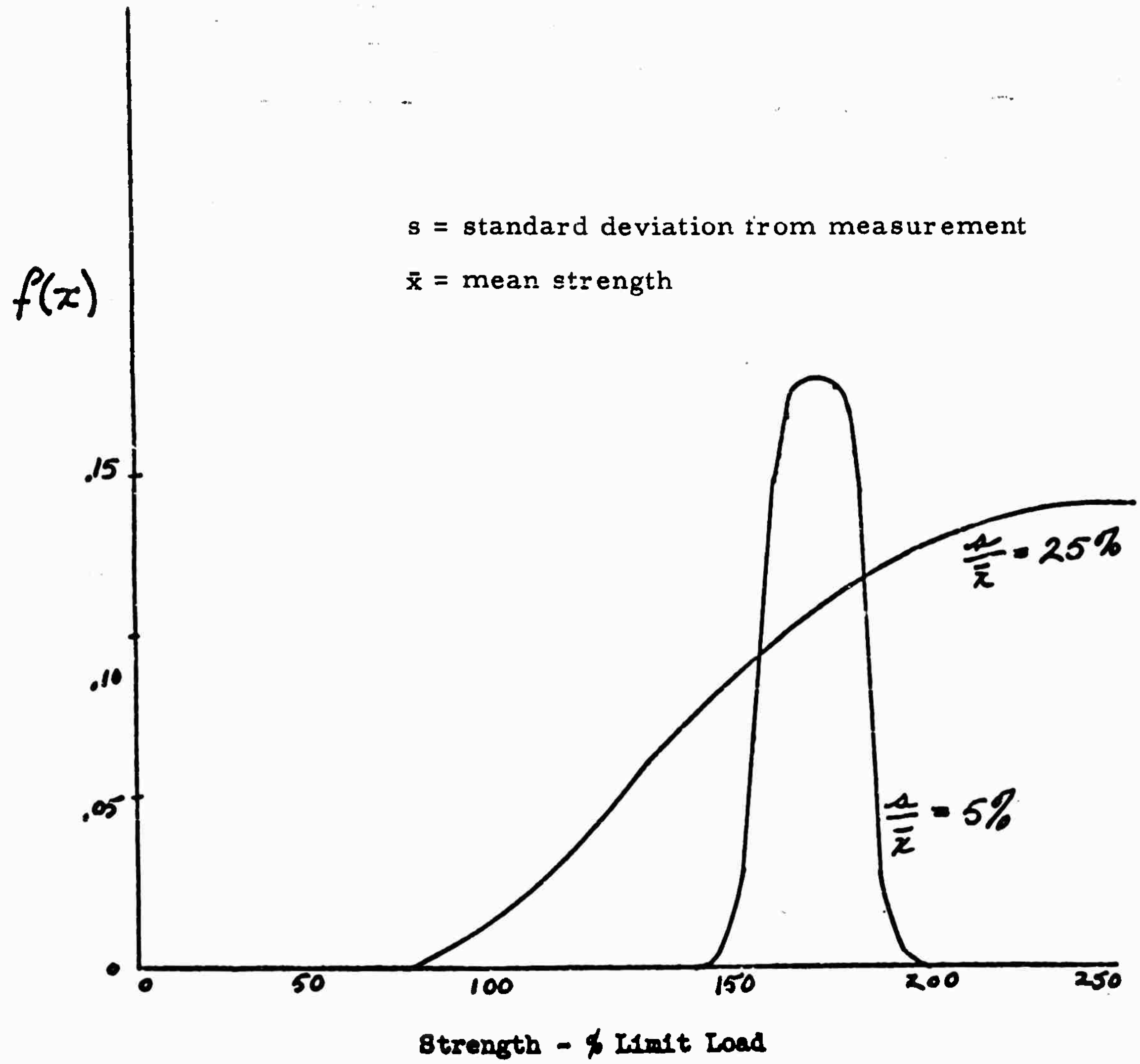

FIGURE 5

ULTIMATE STRENGTH AS A FUNCTION OF $\frac{s}{\bar{x}}$

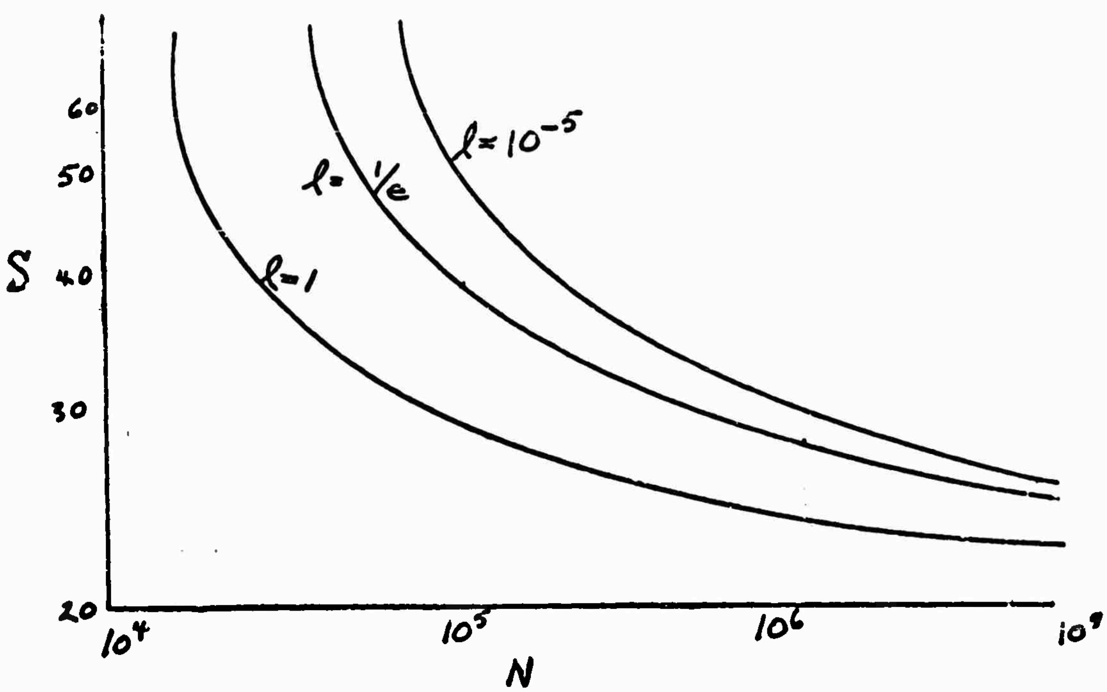

FIGURE 6. A FAMILY OF $\ell(x)$ CURVES

The curves are based on fatigue tests where the variables of interest were minimum life, true endurance limit, endurance limit, and probability of permanent survival.

Freundenthal has also done some extensive work in applying extreme value theory to repeated and ultimate load to improve the reliability. He suggests that a more rational basis for selection of safety factors should be based upon the concept of probability. His article, "Safety, Safety Factors and Reliability of Mechanical Systems," is published in (10).

In this method scatter is a function of the sample size, N. This method clearly points out the "3s" fallacy commonly used by test engineers. The 3s method assumes that, no matter how large the sample size becomes, the risk that some member of the sample will exceed 3 standard deviations stays constant. However, this is not the case. The scatter (i.e., sample range) always tends to increase as more samples are taken. Also, the width of the confidence interval becomes wider for probabilities that approach one. Thus, the extreme value is a good indicator as to the overestimation or underestimation achieved by the assigned safety factor used in the design of a given structure.

Consider N random observations X_1, X_2, ..., X_n that are rearranged in descending order of magnitude and denoted by $X_{(1)}$, $X_{(2)}$, ..., $X_{(n)}$. We assume that these observations are from the following cumulative distribution function (cdf):

$$F(y) = e^{-e^{-y}}, \quad -\infty < y < +\infty$$

where y is the reduced variate defined by:

$$y = \frac{x - \theta_1}{\theta_2}$$

and where θ_1 is the location parameter and θ_2 is the scale parameter.

Now let

$$P = e^{-e^{-y}} \text{ - - - - - - - - - - -(equation A)}$$

where P is defined as $1 - \frac{i}{N+1}$ for smallest value theory and where i is the i^{th} ordered observation. The range of this cumulative distribution function is $\left(\frac{1}{N+1}, \frac{2}{N+1}, \ldots \frac{N}{N+1}\right)$ where $\frac{N}{N+1}$ approaches unity as N becomes large. Taking double logarithms,

$$y = -1_n n \left[-1_n n \; P\right]$$

We now have the line:

$$X = \theta_1 + \theta_2 \, y$$

which is plotted on extreme value probability paper. θ_2 is the slope of the line and θ_1 is the intercept. The slope indicates the scatter of the observations. The shallower the slope, the greater will be the scatter.

By extrapolating the line, the model gives us protection against the weakest link in any furure observed specimens. For example, we can find the smallest value, $X_{(N+K)}$, for K future observations by extrapolating the line to a value that corresponds to the cumulative probability distribution of $P = 1 - \frac{N+K}{N+K+1} = \frac{1}{N+K+1}$. However, the width of the confidence interval becomes larger as K increases. This seems reasonable because the uncertainty increases for statements on, say, the billionth extreme.

9.2 Practical Considerations

As pointed out in the discussion, the problem is to devise a structural validation method which combines minimum safety factor, designer's judgment, lifetime history, and conditions. To emphasize the powerful flexibility of extreme value theory this example looks at one test performed on a spherical tank, where comparisons are made on strain gage readings by looking at three particular pressures of the test; namely, burst, proof, and working pressures.

The proof pressure was chosen before the test by the following method:

Proof Pressure = (safety factor) x (working pressure) where the safety factor was taken as 1.1, hence

working pressure = 4400

proof pressure = 4840.

It turned out that the burst pressure was 6077. Looking at the largest of the extremes of these three lines it can be seen that burst occurs well above the proof pressure line. The designer's safety factor appears to be a very safe value based upon the results of this one test.

If more tests were taken independently and plotted in a similar manner, then from the variance and mean (i.e., from the parameters in the extreme value distribution) the lines corresponding to the level of protection can be drawn if it is assumed that the positioning of the strain gages on the sphere does not influence the reading for any particular sphere. This is one reason why more tests should be taken, because then the various strain gage populations can be separated by ranking procedures in the theory of order statistics. In most cases this would substantiate the test engineer's choice in placing the strain gages.

Figure 7, graphed from the data in Table XVII, shows that the readings of these thirty-one strain gages from this one test should be separated into more than one population. The physical match would be that many of the gages are far below the critical point when the burst occurs. The graph shows that a minimum of 13% of the gages belong to another population. After this separation the variance would decrease and the mode would increase.

In the graph, since the burst line is much higher than the proof line above the working line, the designed safety factor is adequate. The fact that three strain gages were very high makes the others suspect. This burst line would be an upper limit for this test, and can be assigned a reliability of zero (i.e., the

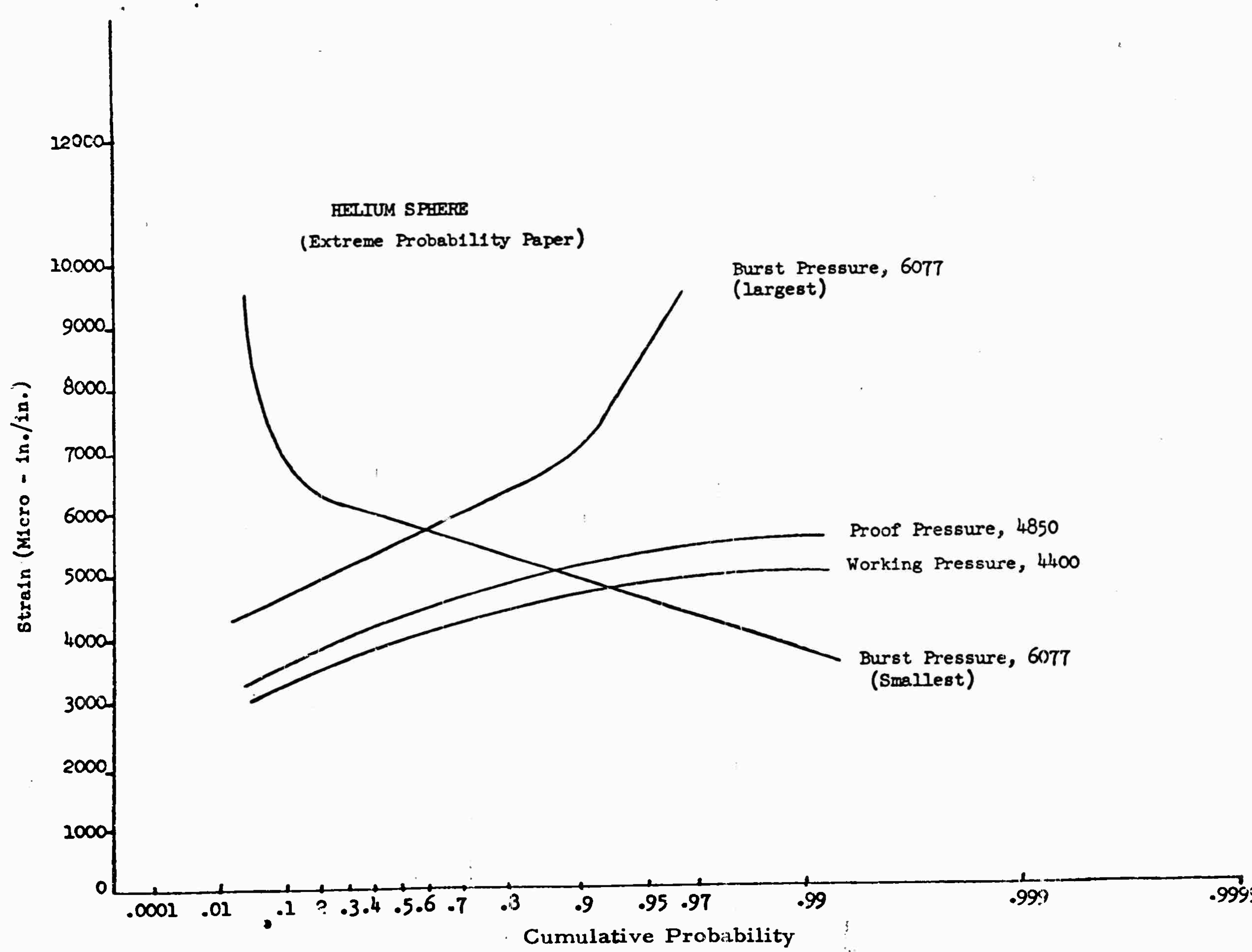

FIGURE 7. PRELIMINARY ANALYSIS OF BURST TEST DATA

TABLE [illegible]

ORDERED STRAIN READINGS

i	Strain at Burst Pressure	Strain at Proof Pressure	Strain at Working Pressure	$\frac{i}{32}$
1	4343	3347	3028	.031
2	4548	3491	3124	.063
3	4824	3542	3220	.094
4	4903	3722	3399	.125
5	5017	3761	3434	.156
6	5065	3814	3450	.188
7	5211	3847	3501	.219
8	5239	4010	3588	.250
9	5601	4053	3656	.281
10	5682	4094	3685	.313
11	5747	4111	3712	.344
12	5769	4160	3722	.375
13	5845	4177	3759	.406
14	5897	4208	3834	.438
15	5931	4255	3846	.469
16	5952	4338	3858	.500
17	5992	4348	3925	.531
18	6004	4362	3926	.563
19	6016	4370	3940	.594
20	6023	4390	3940	.625
21	6032	4434	4002	.656
22	6227	4555	4108	.688
23	6237	4651	4224	.719
24	6266	4730	4310	.750
25	6427	4907	4432	.781
26	6494	5008	4499	.813
27	6798	5020	4553	.844
28	7861	5038	4564	.875
29	9460	5068	4580	.906
30		5232	4777	.938
31		5375	4869	.969

line of sure failure). More tests would give the variance of this sure failure line and the 3-S limits of this sure failure line must not intersect the 3-S limits of the working line.

Analysis by Calculation and/or Graphing

As explained in the discussion of the method of analysis, rapid results of analysis can be achieved. Moreover, this method affords graphical means of analysis which provides the designer the additional assurance of confirming his own engineering experience.

For $N < 18$, the Type I extreme-value function should be fitted by a computer program using modified[16] least-squares equations, and checked by graphing. Graphical solutions may be feasible for larger samples. For extreme-value calculations and graphing, the maxima (or minima) used as observations are ordered in size, and the j^{th} cumulative frequency is determined by the formula,

$$F_j = \frac{j}{N+1}, \quad j = 1, 2, 3, ---N.$$

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APPENDIX I

A MODEL FRAMEWORK FOR SYSTEM EFFECTIVENESS

February 1964

(Revised July 1964)

Prepared for
Weapon System Effectiveness
Industry Advisory Committee
Task Group IV

By
Harold S. Balaban

NOTE: This paper was revised in July, 1964, with the permission of the author. The revision represents changes in terminology and notations only. The revised paper is now in agreement with the notation and terms employed by Task Group II.

Specifically, the transition matrix is denoted by D, rather than R; and the capability vector (previously called Design Adequacy) is denoted by C rather than D.

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A MODEL FRAMEWORK FOR SYSTEM EFFECTIVENESS[1]

Prepared for the Weapon System Effectiveness Industry Advisory Committee, Task Group IV by H. Balaban, ARINC Research Corporation, February 1964.

1. INTRODUCTION

The model framework for system effectiveness that is summarized in this paper was developed primarily for aircraft systems; application to other system types will, for many cases, be quite direct. The approach used for quantifying a probabilistic or expected-value figure of merit for system effectiveness is based on matrix representation of system-state probabilities which may be analyzed through the theory of Markov chains.

The framework will be developed through first considering a probabilistic figure-of-merit for a system for which there exists just a single performance-point in time. This restriction is then relaxed to include a finite number of such points. This vector of discrete performance points is then extended to one which contains discrete performance intervals. The final extension is consideration of an expected value figure-of-merit. Sub-models for quantifying some of the required inputs to the overall model are also discussed.

2. BASIC ASSUMPTIONS AND CONDITIONS

(1) Aircraft systems with a known mission length are considered. After mission completion, facilities are available for performance of maintenance actions.

(2) Units in the system can be classified into two states -- success or failure -- in consonance with the usual definition of reliability. That is, a successful unit is defined to be one that meets its design specification. Capability accounts for the adequacy of the specification for the mission under consideration.

(3) The state transition process is Markovian; that is, the state of the system at some future time is dependent only on the present state and future performance, and not on how the system reached its present state.

(4) The system level at which the system states are represented comprises only units that are mutually independent with respect to their performance (output) and their effect on mission accomplishment (capabilities).

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[1] Abstracted from the report, "System Effectiveness: Concepts and Analytical Techniques", H. Balaban, D. Costello, ARINC Research Corporation Publication 267-01-7-419, under Air Force Contract AF 33(657)-10594, January 1964.

3. SINGLE PERFORMANCE-TIME POINT

Consider a system which is to perform a mission of t hours duration, over which each unit may be prone to failure. At time t all required functions must be successfully performed. Effectiveness is defined to be the probability that the system will be "ready" at the beginning of the mission and will be able to successfully perform all functions.

The following vectors and matrices are defined, assuming four possible system states.

$$V = (v_1, v_2, v_3, v_4),$$

where v_i = probability that the system is in State i at time 0, the beginning of the mission;

$$W = \begin{bmatrix} w_1 & & & 0 \\ & w_2 & & \\ & & w_3 & \\ 0 & & & w_4 \end{bmatrix},$$

where w_i = probability that the system will be used for the mission, given State i at time 0;

$$D(0, t) = \begin{bmatrix} D_{11}(0, t) & D_{12}(0, t) & D_{13}(0, t) & D_{14}(0, t) \\ D_{21}(0, t) & D_{22}(0, t) & D_{23}(0, t) & D_{24}(0, t) \\ D_{31}(0, t) & D_{32}(0, t) & D_{33}(0, t) & D_{34}(0, t) \\ D_{41}(0, t) & D_{42}(0, t) & D_{43}(0, t) & D_{44}(0, t) \end{bmatrix},$$

where $D_{ij}(0, t)$ = transition probability that the system is in State j at time t, given State i at time 0; and

$$C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix},$$

where c_i = probability that the i^{th} system state will lead to successful mission accomplishment.

These vectors and matrices are characterized as follows:

V vector (state readiness) - a function of reliability on previous missions and during ground maintenance, maintenance diagnostic and repair capability, logistics and other ground support factors

W matrix (mission readiness) - a function of diagnosis of system state (during alert phase), operational policy, system flexibility, system backup

D matrix (state transition) - a function of reliability and in-flight repair capability

C vector (capability) - a function of design specifications, mission requirements, performance capabilities, external environment.

Under the basic restrictions and assumptions given in Section 2, effectiveness is represented by the equation

$$E = VWDC, \qquad \text{(Equation 1)}$$

which, by performing the indicated matrix multiplications, yields the equation

$$E = \sum_{j=1}^{4} \sum_{i=1}^{4} v_i w_i D_{ij}(0, t) c_j. \qquad \text{(Equation 2)}$$

The general term of Equation 2 is

P(State i at t = 0)·P(system is used, given State i at t = 0)·
P(state transition from i to j during the mission)·
P(all required functions are performed, given State j).

Even with the extreme simplicity imposed on this example, there still remains the problem of quantifying the elements in the V, W, D, and C matrices. Several pertinent considerations are presented in Section 7.

4. MULTIPLE PERFORMANCE-TIME POINTS

In this section we relax the assumption that only a single performance-time point exists. In order to maintain the discrete-parameter case for purposes of simplicity, we now assume that m performance-time points are established, t_1, $t_2, \ldots . t_m$, and that t_0 is time 0 and t_m is the mission length. For each t_i there exists a set of required functions $\{F_i\}$, which will generally vary with t_i.

The following matrices are now defined:

$$D(t_{i-1},t_i) = \begin{bmatrix} D_{11}(t_{i-1},t_i) & D_{12}(t_{i-1},t_i) & \cdots & D_{1s}(t_{i-1},t_i) \\ D_{21}(t_{i-1},t_i) & D_{22}(t_{i-1},t_i) & \cdots & D_{2s}(t_{i-1},t_i) \\ \vdots & \vdots & & \vdots \\ D_{s1}(t_{i-1},t_i) & D_{s2}(t_{i-1},t_i) & \cdots & D_{ss}(t_{i-1},t_i) \end{bmatrix},$$

where $1 \leqq i \leqq m$, $D_{jk}(t_{i-1},t_i)$ is the probability of a transition from State j to State k during the interval (t_{i-1},t_i), and s is the total number of system states. [Note that if $t_1 = 0$, $D(t_0,t_1)$ is the identity matrix.]

$$C(t_i) = \begin{bmatrix} c_1(t_i) & & & O \\ & c_2(t_i) & & \\ & & \ddots & \\ O & & & c_s(t_i) \end{bmatrix},$$

where $c_k(t_i)$ is the capability of the system at time t_i if it is in State k. The $c_k(t_i)$ elements are thus related to the set of required functions $\{F_i\}$.

$$C(t_m) = \begin{bmatrix} c_1(t_m) \\ c_2(t_m) \\ \cdot \\ \cdot \\ \cdot \\ c_s(t_m) \end{bmatrix},$$

where $c_k(t_m)$ is defined in the same manner as $c_k(t_i)$.

Then we have the following equation for effectiveness, E:

$$E = VW \prod_{i=1}^{m} \left[D(t_{i-1}, t_i) C(t_i) \right], \qquad \text{(Equation 3)}$$

where V and W are as defined previously.

All comments made in Section 7 pertaining to quantification of the elements in the V, W, D, and C matrices apply to Equation 3. The algebraic manipulations for a two-state, two-time-interval situation are shown below.

For display purposes, let

$$c_k^{(i)} = c_k(t_i) \text{ and } D_{jk}^{(i)} = D_{jk}(t_{i-1}, t_i).$$

Then the equation

$$E = VWD(t_0, t_1)C(t_1)D(t_1, t_2)C(t_2)$$

reduces, by pairwise matrix multiplication, to

$$E = \begin{bmatrix} v_1 w_1, v_2 w_2 \end{bmatrix} \cdot \begin{bmatrix} D_{11}^{(1)} c_1^{(1)} & D_{12}^{(1)} c_2^{(1)} \\ D_{21}^{(1)} c_1^{(1)} & D_{22}^{(1)} c_2^{(1)} \end{bmatrix} \cdot \begin{bmatrix} D_{11}^{(2)} c_1^{(2)} + D_{12}^{(2)} c_2^{(2)} \\ D_{21}^{(2)} c_1^{(2)} + D_{22}^{(2)} c_2^{(2)} \end{bmatrix},$$

which then reduces to

$$E = \left[v_1 w_1 D_{11}^{(1)} c_1^{(1)} + v_2 w_2 D_{21}^{(1)} c_1^{(1)} \right] \cdot \left[D_{11}^{(2)} c_1^{(2)} + D_{12}^{(2)} c_2^{(2)} \right] + \left[v_1 w_1 D_{12}^{(1)} c_2^{(1)} + v_2 w_2 D_{22}^{(1)} c_2^{(1)} \right] \cdot \left[D_{21}^{(2)} c_1^{(2)} + D_{22}^{(2)} c_2^{(2)} \right] .$$

It is noted that for the case in which there is no in-flight repair, the matrix product $P_i = D(t_{i-1}, t_i) c(t_i)$ is triangular for $i = 1,2,\ldots\ldots,m-1$, if system-state numbers are assigned by the rule described in Section 7.3. Then the product

$$\prod_{i=1}^{m-1} P_i$$

is also triangular; if programming advantage is made of this fact, significant savings in computer time can result.

5. MULTIPLE PERFORMANCE-TIME INTERVALS

A further extension would be to relax the restriction that functional performance is required only at discrete points in time. We shall now consider a vector of performance-time intervals T_i which represents the intervals from t_i to $t_i + \Delta_i$, where $i = 1, 2, \ldots, m$. In order to maintain a discrete-parameter Markov chain, we shall assume that for system success no state transitions are allowed during such intervals.

Define the matrix $G(T_i)$, which represents state continuance as follows:

$$G(T_i) = \begin{bmatrix} g_1(T_i) & & & \bigcirc \\ & g_2(T_i) & & \\ & & \ddots & \\ \bigcirc & & & g_s(T_i) \end{bmatrix},$$

where $c_k(T_i)$ represents the probability that, given State k at time t_i, no state transition occurs before time $t_i + \Delta_i$. If the intervals $T_1, T_2, \ldots, T_m$ can be constructed without resulting in overlap in the t_i's, which might occur because of varying functional performance-time requirements, we then have, from Equation 3,

$$E = VW \prod_{i=1}^{m} \left[D(t_{i-1}+\Delta_{i-1}, t_i) G(T_i) C(T_i) \right]. \quad \text{(Equation 4)}$$

For cases where overlap does occur, e.g., Function 1 is required for time-period 0-2, and Function 2 is required for time-period 1 to 3, it is possible to obtain the value of $E(F_j)$, the effectiveness of the j^{th} function. Overall system effectiveness is then an appropriate combinatorial function of the $E(F_j)$.

6. EXTENSION TO AN EXPECTED VALUE FIGURE-OF-MERIT

The model for quantifying a probabilistic figure of merit for effectiveness can be extended to one that quantifies an expected-value figure of merit. This extension is accomplished through appropriate modification of the capability vector.

Instead of defining c_k in the Vector C to be the probability of mission accomplishment, given the k^{th} state, let us define c'_k in the Vector C' to be some value coefficient corresponding to performance in the k^{th} state. Thus, for State k, c'_k might be the percentage of information return; the expected target destruction; possibly a number on a value scale of 0 to 10; or some other value.

This definition of the C' vector applies directly to the simple model given by Equation 1. For the extension of the model to multiple performance times, this vector must be mission-time-dependent, i.e., of the form $C'(t_i)$. This requires that the time-dependent functions $c'_k(t_i)$ have meaning. For the examples given above, the percentage of information return might be a figure of merit for a reconnaissance mission in which information is related to mission time by depth of penetration into enemy territory. The expected target destruction might be time-dependent if a plane is to bomb more than one target. These examples, of course, are by no means complete, since some artificiality must be introduced for the discrete-parameter case.

We then have the following equation:[2]

$$E(t_j) = VW \prod_{i=1}^{j} D(t_{i-1}, t_i) C'(t_j)$$

$$= VWD(t_0, t_j) C'(t_j), \qquad \text{(Equation 5)}$$

where $E(t_j)$ represents the expected-value figure of merit for effectiveness at time t_j. Assuming well-behaved functions, the time average of $E(t_j)$ over the mission length t_m may then be used as an overall-effectiveness figure of merit.

For the discrete case,

$$E = \frac{1}{m} \sum_{j=1}^{m} E(t_j). \qquad \text{(Equation 6)}$$

For the continuous case,

$$E = \frac{1}{t_m} \int_0^{t_m} E(\tau) d\tau. \qquad \text{(Equation 7)}$$

Note that if the value coefficient $c_k(t)$ equals 1, if State k belongs to the set of satisfactory states and $c_k(t)$ is 0 otherwise, Equations 6 and 7 represent the expected fraction of mission time during which the system is in a satisfactory state.

[2] $C'(t_j)$ in this equation is a column vector with s rows.

7. QUANTIFICATION OF ELEMENTS

This section discusses means for quantifying elements in the model presented for a single performance-time point, Equation 1. Extension to the other models is fairly direct.

7.1 Quantification of the State Readiness Vector, V

Quantifying the v_i could very well involve a model much more complex than the effectiveness model to which it is an input. If we consider the V vector as one composed of steady-state probabilities,[3] the well-known formula for availability or readiness can be used. It is expressed by the equation

$$A = \frac{MTBF}{MTBF + MDT}, \qquad \text{(Equation 8)}$$

where MTBF is mean time between failures and MDT is mean down time. If we assume complete independence both during maintenance and during flight, we have

$$v_1 = A_1 A_2$$
$$v_2 = A_1(1 - A_2)$$
$$v_3 = (1 - A_1)A_2$$
$$v_4 = (1 - A_1)(1 - A_2),$$

where A_i is the availability of the i^{th} unit (i = a or b).

There exist, of course, much more complex models for quantifying the system state-readiness parameters. These models may involve such disciplines as queuing theory, inventory theory, renewal theory, and Markov processes. If the steady-state situation is assumed to hold, one can use reliability theory to estimate MTBF's, and the above disciplines to estimate MDT's at the system level for which the independence assumption holds. It is also possible that knowledge of the interdependencies at a particular system level will indicate the appropriate combination of unit-availability parameters, thus obviating the need for the assumption of independence.

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[3] The term steady state refers to the limiting probability distribution of a Markov process in which the distribution is independent of initial conditions.

It is emphasized, however, that the readiness vector is generally dependent on the D and W matrices as shown by the general Markovian equation

$$P(t_n) = P(t_i)P(t_i,t_n) \qquad \text{(Equation 9)}$$

where $P(t_i)$ is an unconditional probability vector and $P(t_i,t_n)$ is a transition probability matrix. $P(t_n)$ corresponds to the vector at calendar time t_n and $P(t_i,t_n)$ is some function of the W and D matrices.

To view this dependence more directly, assume that we are considering states only at the system level; thus two states are possible at the beginning of a mission: success, S, or failure, F. The following matrices are defined:

$$V = \begin{bmatrix} v_s, & v_f \end{bmatrix} \qquad W = \begin{bmatrix} w_s & 0 \\ 0 & w_f \end{bmatrix}$$

$$D = \begin{bmatrix} D_{ss} & D_{sf} \\ D_{fs} & D_{ff} \end{bmatrix} \qquad C = \begin{bmatrix} c_s \\ c_f \end{bmatrix}.$$

The subscript s corresponds to success, and the subscript f corresponds to failure. Stationary transition probabilities corresponding to a constant mission length are assumed for elements in the D matrix.

Assume that missions are scheduled to take place at calendar times $t_1, t_2, \ldots, t_m, \ldots$, which are independent of previous system performance. It is now desired to obtain the vector V for a particular mission. [Since $v_f = 1 - v_s$, we actually need only find the probability that at the beginning of the m^{th} mission (time t_m) the system is in a successful state.]

The dependencies now become obvious. Whether the system is successful at t_m depends on whether it failed during the previous mission. If it failed, the repair capability is introduced. Whether it failed during the previous mission is a function of its state at time t_{m-1}; etc.

To obtain the state-readiness vector at the beginning of the m^{th} mission, Equation 9 can be used if a transition-probability matrix, $P(t_{m-1},t_m)$, can be obtained, where the transition occurs from the beginning of the $(m - 1)^{st}$ mission to the beginning of the m^{th} mission. This matrix is obtainable from the W and D matrices, with the addition of the following definition:

Let γ = probability that maintenance will restore a failed system to a successful state.

Note that in the transition matrix that follows, since W and D are assumed to be stationary transition mechanisms and γ is not a function of time, P is also a stationary transition matrix:

$$\begin{bmatrix} P_{ss} = w_s(D_{ss} + D_{sf}\gamma) & P_{sf} = w_s D_{sf}(1-\gamma) \\ \quad +(1-w_s) & \\ P_{fs} = w_f(D_{fs} + D_{ff}\gamma) & P_{ff} = w_f D_{ff}(1-\gamma) \\ \quad +(1-w_f)\gamma & \quad +(1-w_f)(1-\gamma) \end{bmatrix}.$$

If the initial condition at t_1 is $V(t_1) = [p, q]$, where $q = 1 - p$, from Equation 9 we have

$$\begin{aligned} V(t_2) &= V(t_1)P(t_1, t_2) \\ &= [p, q]\begin{bmatrix} P_{ss} & P_{sf} \\ P_{fs} & P_{ff} \end{bmatrix} \\ &= [pP_{ss} + qP_{fs}, pP_{sf} + qP_{ff}]; \end{aligned}$$

and,

$$\begin{aligned} V(t_3) &= V(t_2)P(t_2, t_3) \\ &= [pP_{ss} + qP_{fs}, pP_{sf} + qP_{ff}]\begin{bmatrix} P_{ss} & P_{sf} \\ P_{fs} & P_{ff} \end{bmatrix}, \end{aligned}$$

and the recursion is established.

Since we are considering a finite transition matrix with stationary transition probabilities, we can employ the equation

$$\Pi = \Pi P(t, t+\Delta), \qquad \sum_i \pi_i = 1.0 \qquad \text{(Equation 10)}$$

to obtain the steady-state vector or stationary distribution of the system states at mission start time. Let π_s and π_f represent the steady-state probabilities of the success and failure states, respectively. Then, from Equation 10,

(a) $\pi_s = \pi_s P_{ss} + \pi_f P_{fs}$,

and

(b) $\pi_f = \pi_s P_{sf} + \pi_f P_{ff}$.

Since $\pi_f = 1 - \pi_s$, we have, from (a)

$$\pi_s = \pi_s P_{ss} + P_{fs} - \pi_s P_{fs},$$

or

$$\pi_s = \frac{P_{fs}}{P_{sf} + P_{fs}} \qquad \text{(Equation 11)}$$

and

$$\pi_f = \frac{P_{sf}}{P_{sf} + P_{fs}} . \qquad \text{(Equation 12)}$$

π_s and π_f can then be expressed in terms of γ and the elements of the W and D matrices.

Note that Equations 11 and 12 hold for any system for which the stationary transition mechanism exists and for which the S-F classification is made. The quantification of the transition probabilities will, of course, vary for different systems and missions.

It is possible to employ similar techniques at the unit level. However, as the number of units increases, the number of system states increases geometrically and mathematical and computational complexity becomes a serious problem. One approach often employed is to use simulation models in conjunction with inputs obtained through lower level analytical models to obtain the state readiness vector.

7.2 Quantification of the Mission-Readiness Matrix, W

The elements, w_i, in the mission-readiness matrix are the conditional probabilities of using the system for a particular mission if the system is in the i^{th} state. Thus, unlike the

v_i, which are independent of the next mission to be performed (the v_i might depend on the previous missions), the w_i are generally dependent on the next mission.

Secondly, the w_i elements depend on the maintenance and checkout procedures and capabilities. Normally, a plane that that has returned from a mission will be checked out, and required maintenance will be performed. It is possible, for example, that a plane which has successfully completed a mission will experience an equipment failure during landing; this failure would classify the system in an unacceptable state (the corresponding $w_i = 0$). Because of the previously successful mission, however, perhaps only cursory maintenance functions are performed and the equipment failure remains undetected. Then the plane is incorrectly though to be in an acceptable state.

These two major factors, mission dependence and the maintenance and checkout capabilities, lead to the following mathematical decomposition of the elements of the W matrix:

$$w_i = P[\text{use system} \mid \text{State } i]$$

$$= \sum_j P[\text{use system} \mid \text{think State } j]\, P[\text{think State } j \mid \text{State } i] \,. \qquad \text{(Equation 13)}$$

The first conditional probability under the summation is primarily dependent on the mission to be performed, while the second is primarily dependent on maintenance and checkout capabilities.

7.3 Quantification of the State-Transition Matrix, D

It is first noted that for the basic model under consideration, the D matrix represents state transition <u>during the mission</u>. As shown in Section 7.1, the D matrix is just one of the inputs for obtaining the state-transition matrix <u>between missions</u>.

The simplest (and not wholly unrealistic) case to consider under the basic model is to assume that no in-flight repairs or, more generally, no restorations of failed units to successful states are possible. Then the in-flight state transitions are wholly dependent on the reliability parameter. A transition from a state with k failed units can be made only to states with (k + 1) or more failed units.

For computational purposes, the following scheme for assigning state numbers is suggested:

Represent a state by a code consisting of 0's and 1's, where 0 represents unit failure and 1 represents unit success. Thus, 101 represents the system state: Unit 1 successful, Unit 2 failed, and Unit 3 successful. These system-state codes are then equivalent to binary numbers, and the system-state numbers to be assigned are then the equivalent decimal numbers plus one, so that the last state number equals the number of states. Thus, if there are three units, there are $2^3 = 8$ system states, and the state codes and numbers are as follows:

State Code	State Number	State Code	State Number
000	1	100	5
001	2	101	6
010	3	110	7
011	4	111	8

The reason for this particular scheme is that for the assumption of no in-flight repairs, the R matrix is triangular -- a computationally useful property, since all elements above the diagonal are zero. The triangular matrix occurs because it is not possible to go from State i to State j for $i < j$, since such transition would presume that a failed unit was restored to operating condition either through repair or some intermittency condition. Note that zero-valued elements will also appear below the diagonal. For example, $._{32}$ in the above example is equal to zero since the transition from State 3 to State 2 involves a transition of Unit 3 from a failed to a successful state.

The elements in the D matrix, D_{ij} $(i \geq j)$, are then solely a function of unit reliabilities. Let D_k equal the reliability of the k^{th} unit for the mission length being considered, and $\overline{D}_k = 1 - D_k$, the k^{th} unit unreliability. Then, for the above example, we have

$$
D = \begin{array}{c|cccccccc}
\text{State} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & \bar{D}_3 & D_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & \bar{D}_2 & 0 & D_2 & 0 & 0 & 0 & 0 & 0 \\
4 & \bar{D}_2\bar{D}_3 & \bar{D}_2 D_3 & D_2\bar{D}_3 & D_2 D_3 & 0 & 0 & 0 & 0 \\
5 & \bar{D}_1 & 0 & 0 & 0 & D_1 & 0 & 0 & 0 \\
6 & \bar{D}_1\bar{D}_3 & \bar{D}_1 D_3 & 0 & 0 & D_1\bar{D}_3 & D_1 D_3 & 0 & 0 \\
7 & \bar{D}_1\bar{D}_2 & 0 & \bar{D}_1 D_2 & 0 & D_1\bar{D}_2 & 0 & D_1 D_2 & 0 \\
8 & \bar{D}_1\bar{D}_2\bar{D}_3 & \bar{D}_1\bar{D}_2 D_3 & \bar{D}_1 D_2\bar{D}_3 & D_1\bar{D}_2\bar{D}_3 & D_1\bar{D}_2 D_3 & D_1\bar{D}_2 D_3 & D\ D_2\bar{D}_3 & D_1 D_2 D_3
\end{array}
$$

Extension of this model to include in-flight repair is normally quite difficult. In addition to the complex factors affecting maintenance capability, the model must consider when a failure occurs, since restoration of a failed unit will depend on how much time is available before the unit's function is required. This type of consideration will normally involve a continuous-parameter Markov chain (time is considered as a continuous parameter), and renewal-theory approaches become applicable.[4/]

The following are several of the more important equations applicable to the in-flight repair situations for a two-state model:[5/]

Let

$f(t)$ = time-to-failure density function of a unit
$r(t)$ = repair-time density function
$y(t)$ = density function for the event "end of operation of the unit"
$z(t)$ - density function for the event "end of repair of the unit".

4/ A good introduction to renewal theory is: D. R. Cox, Renewal Theory, John Wiley and Sons, Inc., 1962.

5/ Adapted from: Statistical Theory of Reliability, M. Zelen, Editor, the University of Wisconsin Press, 1963; Chapter 1, "A Survey of Some Mathematical Models in the Theory of Reliability," G. H. Weiss.

Then the following renewal equations apply, assuming operation at time 0:

$$y(t) = f(t) + \int_0^t z(\tau)f(t-\tau)d\tau \quad \text{(Equation 14)}$$

$$z(t) = \int_0^t y(\tau)r(t-\tau)d\tau. \quad \text{(Equation 15)}$$

The expected number of failures during a mission of T hours is then

$$E_T(F) = \int_0^T y(\tau)d\tau, \quad \text{(Equation 16)}$$

and, similarly, the expected number of completed repair activities is

$$E_T(R) = \int_0^T z(\tau)d\tau. \quad \text{(Equation 17)}$$

We can also find the probability that the system is in operation at time t from the equation

$$\eta(t) = D(t) + \int_0^t z(\tau)D(t-\tau)d\tau, \quad \text{(Equation 18)}$$

where $D(t)$ is the unit reliability function.

It is interesting to note that as $t \rightarrow \infty$, $\eta(t)$ approaches the availability formula, MTBF/(MTBF + MDT). Asymptotic results have also been obtained for the distribution of down time, and exact results have been obtained for the case of exponential failure and repair density functions.[6] These latter considerations are important for the expected-value figure of merit if the function of a unit is required continuously, but some contribution to effectiveness will be made if the unit operates only intermittently.

The applicability of the above equations will depend, of course, on the particular operation and background involved. Many references are available for quantifying failure and repair

[6] M. Zelen, op. cit.

density functions. Thus, the densities y(t) and z(t) can be obtained, either analytically through techniques such as Laplace transforms, or through numerical procedures. There still remains the problem of integrating the unit parameters into a system model; for example, an obvious problem is the dependence of repair capability on the number of failures. In-flight repair can only presume a limited maintenance capability, and thus the repair function of a unit will be dependent on the state of other units. These problems will require a great deal of research effort.

7.4 Quantification of the Capability Vector, C

Capability is the least researched concept of the major factors affecting system effectiveness; thus its quantification is still in the early stages of development. A basic approach for obtaining the capability vector is presented in this section.

Capability is defined as the probability of successful mission accomplishment, given satisfactory system operation. This definition is appropriate if the analysis is performed at the system level, and it implies that unsatisfactory operation (that is, performance outside of design specifications) cannot lead to fulfillment of mission requirements.

In the performance of an effectiveness analysis at system sublevels, however, the condition of satisfactory performance loses its meaning since system performance is now represented by system states, which are representations of particular combinations of unit successes and failures. The translation of the capability concept to this level of analysis is accomplished in a straightforward manner by the introduction of system-state capabilities, which represent the probability of successful mission accomplishment when a particular system state exists. These probabilities are the elements c_k in the capability vector.

Since the overall mission requirement is usually a vector of functional requirements, we shall first consider how one may synthesize the C elements from a decomposition of the requirement vector.[1] Assume that a system is to perform a mission that requires accomplishment of m functions, e.g., power generation, communication, and navigation. It is noted that this

---

[1] The rationale for such synthesis is presented in the following publication: H. Leuba, R. Boteilho, Evaluation of System Design Adequacy, ARINC Research Publication No. 267-07-5-416, December 1963.

decomposition is dictated to a large extent by the system level of the analysis. This level, according to assumption (4), Section 2, will depend on the capability to synthesize the system capability from sublevel analyses.

Consider a functional capability matrix as shown below:

$$
\begin{array}{cc}
 & \text{Function} \\
 & \begin{array}{ccccccc} 1 & 2 & \cdots & j & \cdots & m \end{array} \\
\text{Unit} \begin{array}{c} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{array} &
\begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1m} \\
c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2m} \\
\vdots & \vdots & & \vdots & & \vdots \\
c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{im} \\
\vdots & \vdots & & \vdots & & \vdots \\
c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nm}
\end{bmatrix}
\end{array},
$$

where c_{ij} is the probability that Unit i can perform the j^{th} function if it is operating within design specifications. Note that from the rule given for choosing the system level we assume that the c_{ij} elements are mutually independent. In the incorporation of the design-adequacy concept, it is also implied that performance outside of design specifications cannot lead to function accomplishment.

Two modes of functional operation are now considered for obtaining the elements in the C vector:

<u>Committed Mode</u> - Only one successful unit may attempt to perform the function, and that unit is the one which has the maximum functional design adequacy.

<u>Uncommitted Mode</u> - All successful units may attempt to perform the function.

Let $A_k(j)$ represent the capability of the system in State k for the j^{th} function; that is $A_k(j)$ is the probability that the j^{th} function will be successfully performed, given that the system is in State k. Assume for State k that Units $i_1, i_2, \ldots, i_p$ are the successful units. Then:

(a) For a function in the committed mode,

$$A_k(j) = \max_{i_\alpha} c_{i_\alpha j} \quad \alpha = 1, 2, \ldots, p. \qquad \text{(Equation 19)}$$

(b) For a function in the uncommitted mode,

$$A_k(j) = 1 - \prod_{\alpha=1}^{p} \left(1 - c_{i_\alpha j}\right). \qquad \text{(Equation 20)}$$

The capability element c_k in the vector C is then

$$c_k = \prod_{j=1}^{m} A_k(j). \qquad \text{(Equation 21)}$$

The differentiation between committed modes and uncommitted modes is quite elementary, but it does provide a basis for more realistic considerations. For example, an operational sequence pertaining to a particular function may involve a partially committed mode in the sense that a certain unit may attempt to perform the function, provided it is not attempting to perform another function. Analytical expression of $A_k(j)$ for such sequences is cumbersome, but computer procedures for quantifying this probability can be easily developed.

8. CONCLUSIONS

The model framework presented in this document does not represent any new concepts and, in fact, is relatively unsophisticated because of the restrictive assumptions. It does, however, present a point of departure for effectiveness-model building by incorporating in a logical manner, sub-models relating to dependability, availability and capability.

Although this framework can be used to develop much more mathematically complex models, there is the danger that such models require data inputs that are presently unobtainable or that working the model is computationally unfeasible. While it is believed that the specific models presented are workable and, in many cases, quite valid, careful examination of the assumptions is a mandatory requisite before application to a specific system. The effects of any possible violation of assumptions must be ascertained so that numerical outputs may be appropriately modified.

Extensions of the models to more complex cases, e.g., continuous parameter Markov chains, non-stationary transition mechanisms, renewal theory approaches for treating in-flight repair capabilities, etc., have been indicated. A great deal of theoretical research and data collection will be necessary before such problems can be treated practically and realistically.

APPENDIX II

CONCEPTS AND MODELS OF SYSTEM EFFECTIVENESS

30 JANUARY 1964

Prepared for
Weapon System Effectiveness
Industry Advisory Committee
Task Group II

by
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CONCEPTS AND MODELS OF SYSTEM EFFECTIVENESS

As electronic weapon systems continue to evolve into more and more complex structures, the search for efficient prediction and measurement techniques takes on added momentum. Needed are foolproof methods that are easy to implement and that allow management decisions to be reached with minimum risk. Many of the techniques proposed utilize mathematical models to simulate system performance thus allowing a quantitative as well as qualitative measurement and prediction to be made. Most of these mathematical models suffer in that they can only approximate the system in the real world; however, if these models have sufficient detail, a one to one correspondence can be approached. Specific models that have recently received attention are those characterized by Markov processes. These models address themselves to the following fundamental questions.

(1) Is the system working?

(2) If it is working, what is the probability that it will continue working throughout its mission?

(3) Given that the system worked throughout its mission, what is the probability of the mission achieving success?

For systems where the above questions are appropriate, it is possible to define each question as a probability, and the product of all three defines a measure of system effectiveness. Formal definitions of the above probabilities are the following:

(1) "Availability," or "pointwise availability," or "operational readiness" is the probability of the system being operational at any time (t).

(2) "Reliability," or "mission reliability" is the probability of the system surviving an increment of time.

(3) "Design capability," or "design adequacy" is the probability of accomplishing the mission objective given that the system works throughout its mission.

These measures may be determined either analytically (state space analysis) or synthetically (Monte Carlo) or by a combination of both.

The first technique (state space or phase space) will be described by presenting a case history, then developing a general model, followed by additional case histories.

This particular approach depends on developing a framework of reference "state space" which characterizes the system as a function of the condition of its component subsystems.

By classifying the system into a number of "states" reflecting its operating condition, a "state space" is defined and the state of a system can be used to determine the status of the system at any given time. This classification may or may not describe all of subsystem states individually as "working" or "not working." In general, if a complete classification were followed, a system having n units, each of which can be in only one of two possible states, the total number of possible states is 2^n. However, some of these states may be similar and therefore not distinguishable. In these cases the analysis is simplified by restricting the number of "states" describing the system. Additional states may be added if the number of repair crews are limited. The systems' analyst must trade off the requirement for increased system knowledge resulting from a comprehensive listing of system states vs. a more limited classification (few states) which results in a mathematically tractable set of equations. These points are illustrated by the following case histories.

Case I

This is a system composed of three subsystems, Unit A, B, and C. The configuration of these units whether they are in series, parallel, or in combination of both affects the system classification. The acceptable states depend on system configuration and function. A number of system configurations are shown in Figure 1.

Each unit is defined to have only two states, working or non-working. The working state is designated by the letter name of the unit; the non-working state is designated by the letter with a bar above. Defining system states this way, there are 2^n combinations as shown in Figure 2, and Table ..

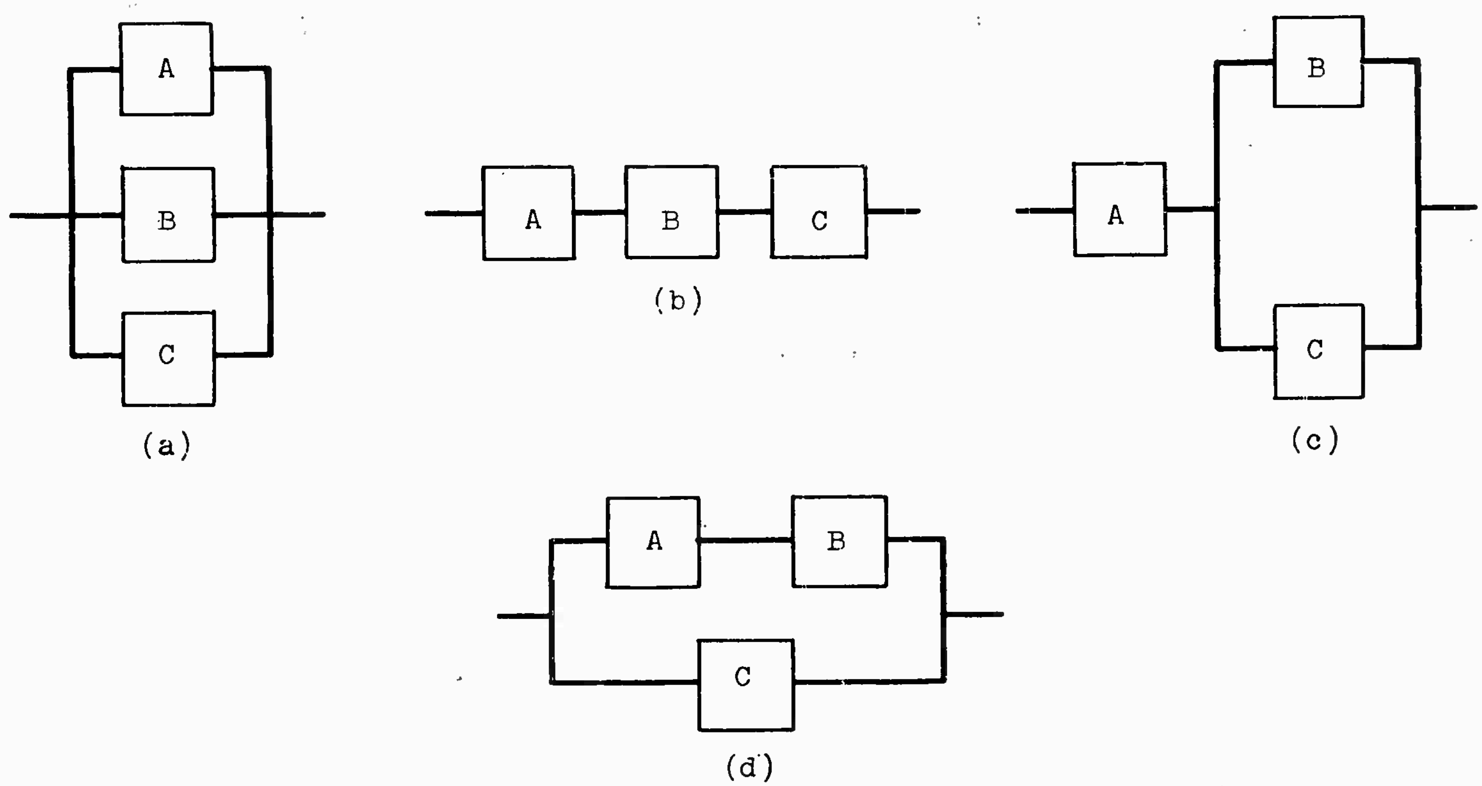


FIGURE 1

3 UNIT SYSTEM CONFIGURATION

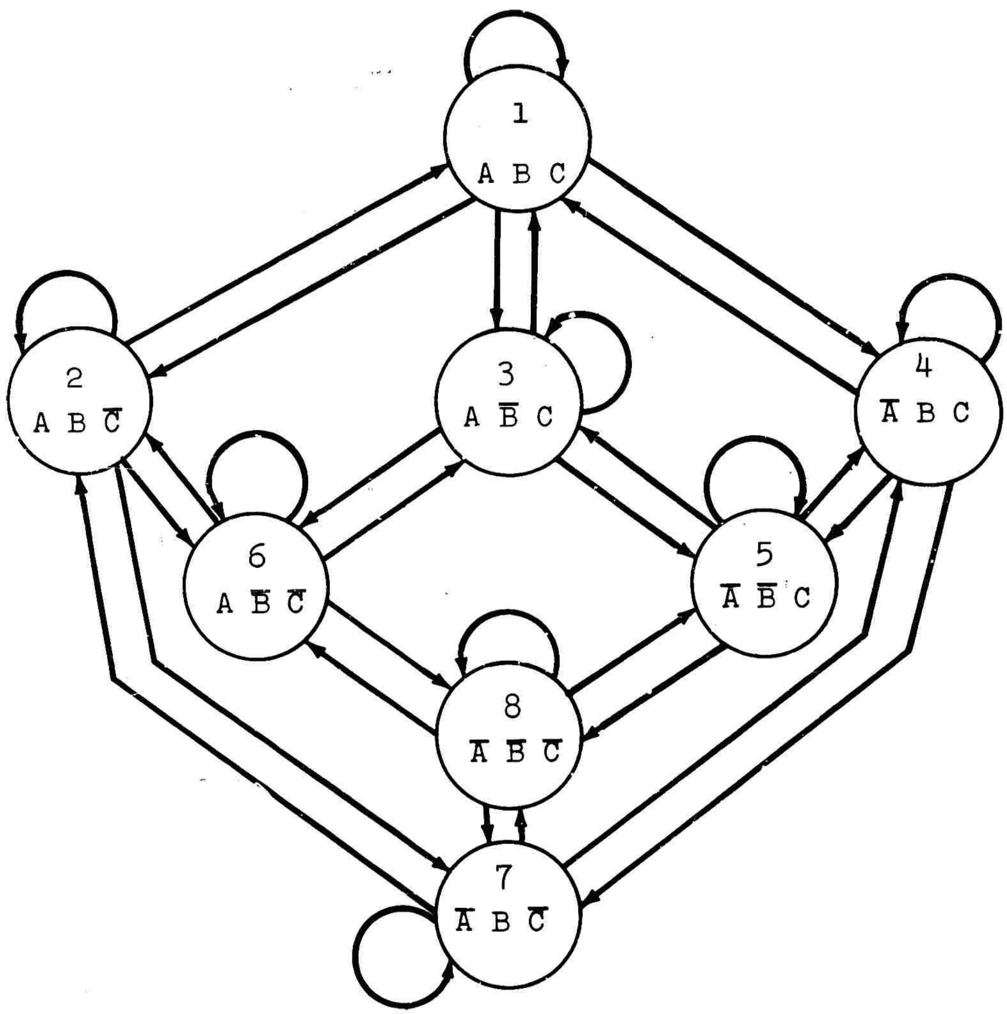


FIGURE 2

STATE SPACE OF THREE UNIT SYSTEM

Table I Tabulation of System States	
State	Status
1	A B C
2	A B $\overline{C}$
3	A $\overline{B}$ C
4	$\overline{A}$ B C
5	$\overline{A}$ $\overline{B}$ C
6	A $\overline{B}$ $\overline{C}$
7	$\overline{A}$ B $\overline{C}$
8	$\overline{A}$ $\overline{B}$ $\overline{C}$

Such an ordered set or array of states of a system constitutes the "state space" of the system. Note in some systems, e.g. Figure 1a, any one unit down or any two units down may not be distinguished. Therefore, states 2, 3, and 4 as well as 5, 6, and 7, listed in Table I, may be considered if fine detail is required by the analysis. In fact, a state must be added if there are a limited number of repair crews, e.g., see Figure 3.

A probability is assigned to each state i (i = 1, 2, ...,8); namely, the probability that the system will be in that state for a time duration Δt sometimes shortened to Δ. The system is defined to start operation at time $t_0 = 0$, t_m is a later instant of time as is $t_m + \Delta t$. The expression $t_m \leq t \leq t_m + \Delta t$ thus

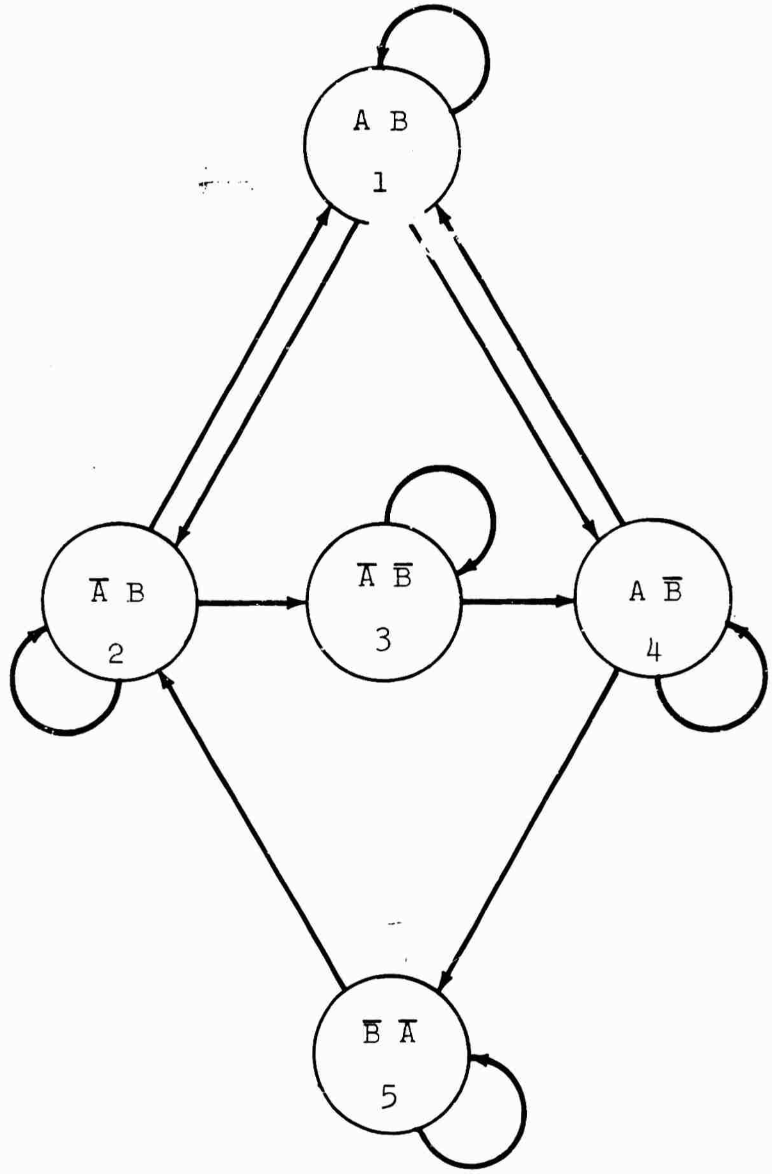


FIGURE 3

TWO UNIT SYSTEM HAVING 5 STATES

represents an interval of time, Δt in length, and occurring from t to $t + \Delta t$. The expression $P(i, t: t_m \leq t \leq t_m + \Delta t)$ represents the probability that the system is in state i at any instant t between t_m and $t_m + \Delta t$. For the sake of convenience, this expression will be shortened to:

$$Pi(t + \Delta t)$$

where t is an arbitrary but fixed instant in time like t_m.

Since there are a number of ways to enter this state $Pi(t + \Delta t)$ (see Figure 3), the sum of these probabilities will give the probability of being in state i; that is,

$$Pi(t + \Delta t) = \sum_{h=1}^{n} P_{hi}\ (t)$$

where

P_{hi} is the probability of going from state h to i

$n = 8$ for the three unit system

and double transitions are excluded.

For this three unit system which admits repair, these individual single transition probabilities are calculated as follows. A probability for any single transition is the product of three other probabilities: the probability that one of the units remains in the same state; the probability that a 2nd unit remains in the same state; and the probability of a 3rd unit changes its state. There are only two possible ways to change state: from working to non-working (a failure) and from non-working to working (a repair). There are only two possible ways to remain in the same state: from working to working, and from non-working to non-working.

To determine these probabilities, the failure time and repair time distributions must be known. For the case being considered, they are exponential and have the following constant repair and failure rates.

Table II

Units	Failure Rate	Repair Rate
A	λA_{ij}	μA_{ij}
B	λB_{ij}	μB_{ij}
C	λC_{ij}	μC_{ij}

The subscripts allow different rates to be assigned to each unit as a function of the state of the system and the number of repair crews available. In the case being studied, the number of repair crews is unlimited; i.e., a repair is initiated immediately upon failure, and the failure rate is independent of the system state.

The transition probabilities are generated as follows:

(1) The probability that the system will go from state 1 to state 1 between t and t + Δt is the product of the following four probabilities.

(a) The probability that the system is in state 1 at t -

$$P_i(t) \quad i = 1$$

(b) The probability that A does not fail during the interval Δt -

$$R_A(\Delta t) = e^{-\lambda_{A11}\Delta t} \approx 1 - \lambda_{A11}\Delta t$$

(c) The probability that B does not fail during the interval Δt -

$$R_B(\Delta t) \approx 1 - \lambda_{B11}\Delta t$$

(d) The probability that C does not fail during the interval Δt -

$$R_C(\Delta t) \approx 1 - \lambda_{C11}\Delta t$$

Thus the probability of the system remaining in state 1 is

$$P_1(t)(1 - \lambda_A\Delta t)(1 - \lambda_B\Delta t)(1 - \lambda_C\Delta t)$$

(2) The probability that the system will go from state 2 to state 1 between t and t + Δt is the product of the following four probabilities.

(a) The probability that the system is in state 2 at t -

$$P_2(t)$$

(b) The probability that A does not fail during the interval Δt -

$$(1 - \lambda_{A21}\Delta t)$$

(c) The probability that B does not fail during Δt -

$$(1 - \lambda_{B12} \Delta t)$$

(d) The probability that C is repaired during the interval Δt -

$$1 - M_C(\Delta t) = 1 - e^{-\mu_{C12} \Delta t} \approx 1-(1-\mu_{C12} \Delta t)$$

$$\approx \mu_{C12} \Delta t$$

This procedure is the same for all other transition probabilities which when calculated result in the following eight equations.

$$\begin{aligned} P_1(t + \Delta t) = {} & P_1(t)(1-\lambda_A \Delta t)(1-\lambda_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_2(t)(1-\lambda_A \Delta t)(1-\lambda_B \Delta t)(\mu_C \Delta t) \\ & + P_3(t)(1-\lambda_A \Delta t)(1-\lambda_C \Delta t)(\mu_B \Delta t) \\ & + P_4(t)(1-\lambda_B \Delta t)(1-\lambda_C \Delta t)(\mu_A \Delta t) \end{aligned} \quad (1)$$

$$\begin{aligned} P_2(t + \Delta t) = {} & P_1(t)(1-\lambda_A \Delta t)(1-\lambda_B \Delta t)(\lambda_C \Delta t) \\ & + P_2(t)(1-\lambda_A \Delta t)(1-\lambda_B \Delta t)(1-\mu_C \Delta t) \\ & + P_6(t)(1-\lambda_A \Delta t)(\mu_B \Delta t)(1-\mu_C \Delta t) \\ & + P_7(t)(\mu_A \Delta t)(1-\lambda_B \Delta t)(1-\mu_C \Delta t) \end{aligned} \quad (2)$$

$$\begin{aligned} P_3(t + \Delta t) = {} & P_1(t)(1-\lambda_A \Delta t)(\lambda_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_3(t)(1-\lambda_A \Delta t)(1-\lambda_C \Delta t)(1-\mu_B \Delta t) \\ & + P_5(t)(\mu_A \Delta t)(1-\mu_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_6(t)(1-\lambda_A \Delta t)(1-\mu_B \Delta t)(\mu_C \Delta t) \end{aligned} \quad (3)$$

$$\begin{aligned} P_4(t + \Delta t) = {} & P_1(t)(\lambda_A \Delta t)(1-\lambda_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_4(t)(1-\mu_A \Delta t)(1-\lambda_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_5(t)(1-\mu_A \Delta t)(\mu_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_7(t)(1-\mu_A \Delta t)(1-\mu_B \Delta t)(\mu_C \Delta t) \end{aligned} \tag{4}$$

$$\begin{aligned} P_5(t + \Delta t) = {} & P_3(t)(\lambda_A \Delta t)(1-\mu_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_4(t)(1-\mu_A \Delta t)(\lambda_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_5(t)(1-\mu_A \Delta t)(1-\mu_B \Delta t)(1-\lambda_C \Delta t) \\ & + P_8(t)(1-\mu_A \Delta t)(1-\mu_B \Delta t)(\mu_C \Delta t) \end{aligned} \tag{5}$$

$$\begin{aligned} P_6(t + \Delta t) = {} & P_2(t)(1-\lambda_A \Delta t)(\lambda_B \Delta t)(1-\mu_C \Delta t) \\ & + P_3(t)(1-\lambda_A \Delta t)(1-\mu_B \Delta t)(\lambda_C \Delta t) \\ & \quad P_6(t)(1-\lambda_A \Delta t)(1-\mu_B \Delta t)(1-\mu_C \Delta t) \\ & + P_8(t)(\mu_A \Delta t)(1-\mu_B \Delta t)(1-\mu_C \Delta t) \end{aligned} \tag{6}$$

$$\begin{aligned} P_7(t + \Delta t) = {} & P_2(t)(\lambda_A \Delta t)(1-\lambda_B \Delta t)(1-\mu_C \Delta t) \\ & + P_4(t)(1-\mu_A \Delta t)(1-\lambda_B \Delta t)(\lambda_C \Delta t) \\ & \quad P_7(t)(1-\mu_A \Delta t)(1-\lambda_B \Delta t)(1-\mu_C \Delta t) \\ & + P_8(t)(1-\mu_A \Delta t)(\mu_B \Delta t)(1-\mu_C \Delta t) \end{aligned} \tag{7}$$

$$\begin{aligned} P_8(t + \Delta t) = {} & P_5(t)(1-\mu_A \Delta t)(1-\mu_B \Delta t)(\lambda_C \Delta t) \\ & + P_6(t)(\lambda_A \Delta t)(1-\mu_B \Delta t)(1-\mu_C \Delta t) \\ & \quad P_7(t)(1-\mu_A \Delta t)(\lambda_B \Delta t)(1-\mu_C \Delta t) \\ & + P_8(t)(1-\mu_A \Delta t)(1-\mu_B \Delta t)(1-\mu_C \Delta t). \end{aligned} \tag{8}$$

The transitional probabilities along the diagonal are all of the form

$$P_i(t)(1 - a)(1 - b)(1 - c)$$

These can be expanded to separate out the $P_i(t)$ which can be brought over to the left side so that the general result is

$$P_i(t+\Delta t) - P_i(t) = P_i(t)\left[-(a+b+c) + ab + cb + ac - abc\right]$$

and if one neglects the higher order terms and divides both sides by Δt, the result is a series of equations where the left side is of the general form:

$$\frac{P_i(t + \Delta t) - P_i(t)}{\Delta t}$$

which is defined as the derivative of $P_i(t)$ in the limit as $t \longrightarrow 0$

$$\dot{P}_i(t) = \lim_{t\to 0} \frac{P_i(t + \Delta t) - P_i(t)}{\Delta t}$$

The limit taken on both sides results in the following equations

$$\dot{P}_1(t) = -(\lambda_A+\lambda_B+\lambda_C)\, P_1(t) + \mu_C P_2(t) + \mu_B P_3(t) + \mu_A P_4(t) \tag{9}$$

$$\dot{P}_2(t) = \lambda_C P_1(t) - (\lambda_A+\lambda_B+\mu_C)P_2(t) + \mu_B P_6(t) + \mu_A P_7(t) \tag{10}$$

$$\dot{P}_3(t) = \lambda_B P_1(t) - (\lambda_A+\lambda_C+\mu_B)P_3(t) + \mu_A P_5(t) + \mu_C P_6(t) \tag{11}$$

$$\dot{P}_4(t) = \lambda_A P_1(t) - (\mu_A+\lambda_B+\lambda_C)\, P_4(t) + \mu_B P_5(t) + \mu_C P_7(t) \tag{12}$$

$$\dot{P}_5(t) = \lambda_A P_3(t) + \lambda_B P_4(t) - (\mu_A+\mu_B+\lambda_C)P_5(t) + \mu_C P_8(t) \tag{13}$$

$$\dot{P}_6(t) = \lambda_B P_2(t) + \lambda_C P_3(t) - (\lambda_A+\mu_B+\mu_C)P_6(t) + \mu_A P_8(t) \tag{14}$$

$$\dot{P}_7(t) = \lambda_A P_2(t) + \lambda_C P_4(t) - (\mu_A+\lambda_B+\mu_C)P_7(t) + \mu_B P_8(t) \tag{15}$$

$$\dot{P}_8(t) = \lambda_C P_5(t) + \lambda_A P_6(t) + \lambda_B P_7(t) - (\mu_A+\mu_B+\mu_C)P_8(t) \tag{16}$$

As linear equations they can be written in matrix form:

$$\begin{bmatrix} -(\lambda_A+\lambda_B+\lambda_C) & \mu_C & \mu_B & \mu_A & 0 & 0 & 0 & 0 \\ \lambda_C & -(\lambda_A+\lambda_B+\mu_C) & 0 & 0 & 0 & \mu_B & \mu_A & 0 \\ \lambda_B & 0 & -(\lambda_A+\mu_B+\lambda_C) & 0 & \mu_A & \mu_C & 0 & 0 \\ \lambda_A & 0 & 0 & -(\mu_A+\lambda_B+\lambda_C) & \mu_B & 0 & \mu_C & 0 \\ 0 & 0 & \lambda_A & \lambda_B & -(\mu_A+\mu_B+\lambda_C) & 0 & 0 & \mu_C \\ 0 & \lambda_B & \lambda_C & 0 & 0 & -(\lambda_A+\mu_B+\mu_C) & 0 & \mu_A \\ 0 & \lambda_A & 0 & \lambda_C & 0 & 0 & -(\mu_A+\lambda_B+\mu_C) & \mu_B \\ 0 & 0 & 0 & 0 & \lambda_C & \lambda_A & \lambda_B & -(\mu_A+\mu_B+\mu_C) \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \end{bmatrix} = \begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \\ \dot{P}_4(t) \\ \dot{P}_5(t) \\ \dot{P}_6(t) \\ \dot{P}_7(t) \\ \dot{P}_8(t) \end{bmatrix} \quad (17)$$

or more generally as:

$$\begin{bmatrix} \\ \lambda,\mu \\ \\ \end{bmatrix}^T \begin{bmatrix} P_1(t) \\ P_2(t) \\ \vdots \\ P_n(t) \end{bmatrix} = \begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \\ \vdots \\ \dot{P}_n(t) \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} P_1(t) & P_2(t) & \dots & P_n(t) \end{bmatrix} \begin{bmatrix} \lambda,\mu \end{bmatrix} = \begin{bmatrix} \dot{P}_1(t) & \dot{P}_2(t) & \dots & \dot{P}_n(t) \end{bmatrix} \tag{19}$$

Where $\left[\lambda,\mu\right]$ is defined as a Q matrix and $\left[\lambda,\mu\right]^T$ is its transpose.

The necessary constraints or boundary conditions required to solve these equations are:

$$\sum_{i=1}^{n} P_i(t) = 1 \tag{20}$$

which says that the system must be in at least one of the eight possible system states (this holds for all t)

$$P_1(0) = 1$$
$$P_i(0) = 0 \quad (i = 2,3, \ldots n) \tag{21}$$

This gives the initial conditions (t=0) of the system.

If only a steady state solution is desired, then one can immediately set the derivatives $\dot{P}_i(t) = 0$, thus leaving a system of algebraic equations which can be solved.

To accomplish this, we omit any one of homogeneous equations and substitute the above constraint for that equation and then solve this system of equations for the $P_i(t)$.

The solution for cases of lower order systems are classical and are given in the literature. A larger system of differential equations can be solved by the Runge-Kutta method on a computer, although the computer memory will place a limit on the number of units an analyzable system may have.

The preceding set of equations provides the necessary structure for defining availability in a formal manner. Availability is defined as the column vector (equation 18) or the row vector (equation 19). This vector contains all of the system component availabilities and by summing the acceptable (favorable) availabilities one obtains a measure of system performance.

It should be noted that in computing the availability of the system no consideration was given beforehand to the system configuration. Only upon describing all possible states of the system via difference equations and subsequently obtaining the differential equations, was consideration given to system configuration. Then computation of system availability is found by summing up the probabilities of acceptable system states.

To compute reliability, $R(\tau)$, one proceeds in a similar manner as in the availability analysis. However, the system configuration must be established beforehand since the absorbing states (those causing system failure) must be defined. As previously mentioned, once an absorbing state is reached, one cannot make a repair and come out of that state; i.e., one remains in that state with probability 1.

The following table lists the failed absorbing states corresponding to the configurations (a) - (d) in Figure 1.

Table III

Absorbing States of Systems

Configuration	Absorbing State
(a)	8
(b)	2,3,4,5,6,7,8
(c)	4,5,6,7,8
(d)	5,7,8

To illustrate this concept, we select the configuration (a) from Figure 1. As can be seen from this configuration, the absorbing state occurs only when units A, B and C are failed simultaneously, which in this case would be state 8. Only the final matrix notation is given for this analysis and is given in equation (22).

$$
\begin{bmatrix}
-(\lambda_A+\lambda_B+\lambda_C) & \mu_C & \mu_B & \mu_A & 0 & 0 & 0 & 0 \\
\lambda_C & -(\lambda_A+\lambda_B+\mu_C) & 0 & 0 & 0 & \mu_B & \mu_A & 0 \\
\lambda_B & 0 & -(\lambda_A+\mu_B+\lambda_C) & 0 & \mu_A & \mu_C & 0 & 0 \\
\lambda_A & 0 & 0 & -(\mu_A+\lambda_B+\lambda_C) & \mu_B & 0 & \mu_C & 0 \\
0 & 0 & \lambda_A & \lambda_B & -(\mu_A+\mu_B+\lambda_C) & 0 & 0 & 0 \\
0 & \lambda_B & \lambda_C & 0 & 0 & -(\lambda_A+\mu_B+\mu_C) & 0 & 0 \\
0 & \lambda_A & 0 & \lambda_C & 0 & 0 & -(\mu_A+\lambda_B+\mu_C) & 0 \\
0 & 0 & 0 & 0 & \lambda_C & \lambda_A & \lambda_B & 0
\end{bmatrix}
\begin{bmatrix}
P_1(\tau) \\ P_2(\tau) \\ P_3(\tau) \\ P_4(\tau) \\ P_5(\tau) \\ P_6(\tau) \\ P_7(\tau) \\ P_8(\tau)
\end{bmatrix}
=
\begin{bmatrix}
\dot{P}_1(\tau) \\ \dot{P}_2(\tau) \\ \dot{P}_3(\tau) \\ \dot{P}_4(\tau) \\ \dot{P}_5(\tau) \\ \dot{P}_6(\tau) \\ \dot{P}_7(\tau) \\ \dot{P}_8(\tau)
\end{bmatrix}
\qquad (22)
$$

As can be seen in the above transition matrix, the column representing the absorbing state contains all zeros. This will be true for all those states that are absorbing. The above equations can be solved by the method of Laplace transforms, or on a computer knowing the initial condition of the system. We also know that

$$\sum_{i=1}^{8} P_i(t) = 1$$

This example has illustrated the technique of obtaining a system of differential equations, both for the availability and reliability models.

Since this analysis has been performed under the assumption that the units have only two possible states, working and non-working, it may be objected that this assumption places a great restriction on the method, since most units used in practice have continuous performance parameters, as opposed to the simple on-off type. But this objection does not hold because such continuous parameters can be quantized; i.e., their ranges can be precisely divided and limited, so that a strict division between working and non-working states is obtained.

Although an underlying exponential distribution for the failures and repairs have been assumed for the units in the system analyzed, so that λ and μ are constants, the analysis can be performed as noted using other distributions and using other weaker assumptions to give more general cases. (See J. Kielson and A. Kooharian "On Time Dependent Queuing Processes," Annals of Math. Stat., March 1960.)

At this point a brief explanation is presented on the effect of having a different number of repair crews. In normal practice, an individual repair crew for each unit of the system is rarely the case. Usually one is limited to one or two repair crews and accordingly the transition probabilities associated with a repair must be modified. To illustrate this we again consider a three unit system made up of identical units (each with failure rate λ), operating in active redundancy as shown in Figure 1 configuration (a). We first consider the case of only one repair crew (with repair rate μ). In this case the system has only four states if we assume that units are indistinguishable. These are:

State 1 - all units working

State 2 - one unit not working

State 3 - two units not working

State 4 - all units not working.

The difference equations for the availability model in this case are:

$$P_1(t + \Delta) = P_1(t)\ (1 - 3\lambda\Delta) + P_2(t)\mu\Delta \tag{23}$$

$$P_2(t + \Delta) = P_1(t)\ 3\lambda\Delta + P_2(t)\ \left[1 - (2\lambda + \mu)\Delta\right] + P_3(t)\mu\Delta \tag{24}$$

$$P_3(t + \Delta) = P_2(t)\ 2\lambda\Delta + P_3(t)\ \left[1 - (\lambda + \mu)\Delta\right] + P_4(t)\mu\Delta \tag{25}$$

$$P_4(t + \Delta) = P_3(t)\lambda\Delta + P_4(t)\ (1 - \mu\Delta) \tag{26}$$

If two repair crews were available (each having repair rate μ), the difference equations would become:

$$P_1(t + \Delta) = P_1(t)\ (1 - 3\lambda\Delta) + P_2(t)\mu\Delta \tag{27}$$

$$P_2(t + \Delta) = P_1(t)\ 3\lambda\Delta + P_2(t)\ \left[1 - (2\lambda + \mu)\Delta\right] + P_3(t)\ 2\mu\Delta \tag{28}$$

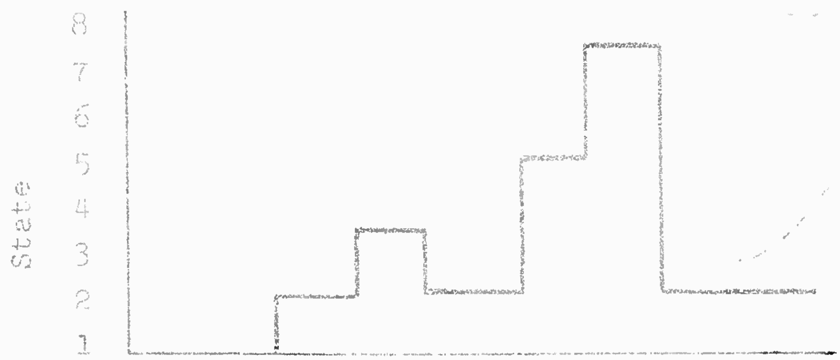
$$P_3(t + \Delta) = P_2(t)\ 2\lambda\Delta + P_3(t)\ \left[1 - (\lambda + 2\mu)\Delta\right] + P_4(t)\ 2\mu\Delta \tag{29}$$

$$P_4(t + \Delta) = P_3(t)\lambda\Delta + P_4(t)\ (1 - 2\mu\Delta) \tag{30}$$

As can be seen, the advantage between one repair crew or two repair crews is when the system is in states 3 or 4. When two repair crews are available and two or three units are inoperative, the probability of completing a repair in Δ is

$2\mu\Delta$ where with one repair crew this probability is $\mu\Delta$. This is intuitively clear since if two or three units have failed and two repair crews are working, the probability of completing a repair is twice as great as the probability when only one repair crew is available. However, the more repair crews one has on hand for repair capability, the greater the idle time of each. Therefore, there is a certain trade-off which must be made both from the standpoint of maximizing uptime and minimizing the idle time of repair crews.

Design capability is defined as the probability that a system will successfully accomplish its mission given the system states; during the mission. The capability of the system can be directly related to the system state. For example, consider a three-unit system (i.e., three radars) Figure 4 - where the probability of detecting a target is .9 if all three radars are working, .8 if two are working, .5 if one is working then the system states can be weighted as follows: -



------ .ission Time ------

FIGURE 4

STATES OCCUPIED DURING MISSION

State	Status	Design Capability Radar
1	A B C	.9
2	A B $\bar{C}$	.8
3	A $\bar{B}$ C	.8
4	$\bar{A}$ B C	.8
5	$\bar{A}$ $\bar{B}$ C	.5
6	A $\bar{B}$ $\bar{C}$	.5
7	$\bar{A}$ B $\bar{C}$	.5
8	$\bar{A}$ $\bar{B}$ $\bar{C}$	0

Then the <u>average</u> capability per mission would provide a useful system measure.

In some systems such an average measure may not be useful, e.g., a missile where a specific lower bound on performance is essential to success.

The previous analysis has been made on a three-unit system. In theory the same mathematical analysis could be made on a system with n units, although if n is too large the method becomes computationally unwieldy. The following analysis is the application of essentially the previous approach to a system composed of any n sub-units, so that the preceding analysis of the three unit system may be regarded as an illustrative example of this more generalized approach.

General Analysis on n-Unit System

The following discussion will formalize the procedures discussed above. A mathematical model of system availability is developed.

We define

$$\dot{P}(t) = \dot{A}(t) = \lim_{t \to 0} \frac{A(t+\Delta t) - A(t)}{\Delta t} \qquad (31)$$

We further define $P(\Delta t)$, a matrix, as the transition probability; i.e., the probability of going from $A(t)$ to $A(t + \Delta t)$ in time Δt. Therefore:

$$A(t + \Delta t) = P(\Delta t)\, A(t) \tag{32}$$

Substituting equation (32) into (31) results in

$$\dot{A}(t) = \lim_{\Delta t \to 0} \frac{P(\Delta t)\, A(t) - A(t)}{\Delta t} \tag{33}$$

and

$$\dot{A}(t) = \lim_{t \to 0} \left(\frac{P(\Delta t) - I}{\Delta t} \right) A(t) \tag{34}$$

where I is the identity (unit) matrix.

The matrix I is required because it enables $A(t)$ to be factored out of the expression.

Define the matrix Q by

$$Q = \lim_{\Delta t \to 0} \frac{P(\Delta t) - I}{\Delta t} \tag{35}$$

then

$$\dot{A}(t) = QA(t) \tag{36}$$

This is a set of first order linear differential equations. The solution of this system of equations is given by:

$$A(t) = A(0)e^{Qt} \tag{37}$$

where A(0) is the initial probability vector and e^{Qt} is defined by its Taylor series. Therefore,

$$A(t) = A(0) \left[1 + Qt + \frac{(Qt)^2}{2!} + \ldots \right]^{1/} \tag{38}$$

This result can be verified by substitution in equation (36).

In a stationary system the rate of change of availability, defined by $\dot{A}(t)$, will approach zero as time increases. Specifically at $t = \infty$, $\dot{A}(t) = 0$ and therefore from equation (36) we have

$$0 = A(\infty)Q \tag{39}$$

Thus, this steady state solution can be satisfied only if the matrix Q is singular; i.e., its determinant is zero.

Additional examples of one and two unit systems further illustrate the above techniques.

1/ $e^{Qt} = \left[\frac{Qt}{e^n} \right]^n$ may be useful for computer programs if t is large.

Case II: Sample Analysis of One-Unit System

Figure 5 is a one unit system.

Unit 1
R = probability of unit working
M = probability of repairing unit if not working

One Unit System

Figure 5

This system has 2^n possible states where n is the number of units in the system tabulated in Table IV.

Table IV

Tabulation of System States of One Unit System

State	Unit
1	working
2	failed

The transition probabilities, $p_{ij}(\Delta t)$, are the probabilities of being in a specific state and either remaining in that state or going to another. These are given in Table 5.

Table V Transition Probabilities		
From \ To	1	2
1	p_{11}	p_{12}
2	p_{21}	p_{22}

where

p_{11} = the probability of being in state one and remaining there.

p_{12} = the probability of being in state one and going to state two, etc.

Substituting the probabilities shown in Figure 5 into Table V results in Table VI.

Table VI Transition Probabilities		
From \ To	1	2
1	R	(1-R)
2	M	(1-M)

where

(1-R) = probability of unit failing

(1-M) = probability of unit not being repaired.

If the units are independent and the chance of failure or repair does not depend on past history, the exponential functions can be used to describe the probabilities of Table 6.

$$R(\Delta t) = e^{-\lambda \Delta t} = 1 - \lambda \Delta t \qquad (40)$$

$$M(\Delta t) = 1 - e^{-\mu \Delta t} = \mu \Delta t \qquad (41)$$

where

λ = failure rate

μ = repair rate

$e^{-\lambda \Delta t}$ = probability of zero failures in Δt

$1 - e^{-\mu \Delta t}$ = probability of at least one repair in Δt.

We now solve for the terms of the Q matrix from the following relationships:

$$Q = \lim_{\Delta t \to 0} \frac{[P(\Delta t) - I]}{\Delta t} \qquad (42)$$

therefore

$$q_{11} = \lim_{\Delta t \to 0} \frac{p_{11} - 1}{\Delta t} = \lim_{\Delta t \to 0} \frac{1 - \lambda \Delta t - 1}{\Delta t} = -\lambda \qquad (43)$$

$$q_{21} = \lim_{\Delta t \to 0} \frac{p_{21}}{\Delta t} = \lim_{t \to 0} \frac{\mu \Delta t}{\Delta t} = \mu \qquad (44)$$

$$q_{12} = \lim_{\Delta t \to 0} \frac{p_{12}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\lambda \Delta t}{\Delta t} = \lambda \tag{45}$$

$$q_{22} = \lim_{\Delta t \to 0} \frac{p_{22} - 1}{\Delta t} = \lim_{\Delta t \to 0} \frac{1 - \mu \Delta t - 1}{\Delta t} = -\mu \tag{46}$$

The Q matrix is therefore

$$Q = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix} \tag{47}$$

and since $\dot{A}(t) = A(t)Q$ it is possible to write the 2^n linear differential equations with constant coefficients. These are

$$\dot{a}_1(t) = -\lambda a_1(t) + \mu a_2(t) \tag{48}$$

$$\dot{a}_2(t) = \lambda a_1(t) - \mu a_2(t) \tag{49}$$

To solve this system of first order linear differential equations, we can make use of the Laplace transform[2/] and the relationship that $a_1(0) = 1$, $a_2(0) = 0$. Taking Laplace transforms of equations (48) and (49) and denoting the Laplace transform variable by s results in

$$sL(a_1) - a_1(0) + \lambda L(a_1) - \mu L(a_2) = 0 \tag{50}$$

$$sL(a_2) - a_2(0) - \lambda L(a_1) + \mu L(a_2) = 0 \tag{51}$$

2/ C. R. Wiley, Advanced Engineering Mathematics, McGraw-Hill; P. LeCorbeiller, Matrix Analysis of Electronic Network, Harvard University Press.

Substituting the values of $a_1(0)$ and $a_2(0)$ gives

$$sL(a_1) - 1 + \lambda L(a_1) - \mu L(a_2) = 0 \tag{52}$$

$$sL(a_2) - \lambda L(a_1) + \mu L(a_2) = 0 \tag{53}$$

or

$$(s + \lambda)L(a_1) - \mu L(a_2) = 1 \tag{54}$$

$$-\lambda L(a_1) + (s + \mu)L(a_2) = 0 \tag{55}$$

Making use of determinants we now solve for $L(a_1)$, and from the relationship $a_1(t) + a_2(t) = 1$ it is possible to obtain $a_2(t)$ once $a_1(t)$ has been determined.

$$L(a_1) = \frac{\begin{vmatrix} 1 & -\mu \\ 0 & (s+\mu) \end{vmatrix}}{\begin{vmatrix} (s+\lambda) & -\mu \\ -\lambda & (s+\mu) \end{vmatrix}} = \frac{s+\mu}{(s+\lambda)(s+\mu) - \mu\lambda} \tag{56}$$

$$L(a_1) = \frac{s + \mu}{s^2 + s\lambda + s\mu} = \frac{s + \mu}{s(s + \lambda + \mu)} \tag{57}$$

Expanding (57) in partial fractions results in

$$\frac{k_1}{s} + \frac{k_2}{s + \lambda + \mu} \tag{58}$$

$$k_1(s + \lambda + \mu) + k_2 s = s + \mu \tag{59}$$

$$k_1 s + (\lambda + \mu)k_1 + k_2 s = s + \mu \tag{60}$$

or

$$k_1 + k_2 = 1 \tag{61}$$

$$(\lambda + \mu)k_1 = \mu \tag{62}$$

therefore

$$k_1 = \frac{\mu}{\lambda + \mu} \quad \text{and} \quad k_2 = \frac{\lambda}{\lambda + \mu} \tag{63}$$

and

$$L(a_1) = \frac{\mu}{(\lambda + \mu)s} + \frac{\lambda}{(\lambda + \mu)(s + \lambda + \mu)} \tag{64}$$

therefore

$$a_1(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda e^{-(\lambda + \mu)t}}{\lambda + \mu} \tag{65}$$

Since

$$a_1(t) + a_2(t) = 1$$

$$a_2(t) = 1 - a_1(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda e^{-(\lambda + \mu)t}}{\lambda + \mu} \tag{66}$$

The steady state availabilities are easily obtained from equations (65) and (66) by letting $t \longrightarrow \infty$

$$a_1(\infty) = \frac{\mu}{\lambda + \mu} \tag{67}$$

$$a_2(\infty) = \frac{\lambda}{\lambda + \mu} \tag{68}$$

Case III: Sample Analysis of Two-Unit System

In this section the transition matrix Q will be determined and the system of linear differential equations set up. Figure 6 is the two unit system to be considered.

Unit No. 1	Unit No. 2
R_1 = probability of unit No. 1 working	R_2 = probability of unit No. 2 working
M_1 = probability of repairing unit No. 1 if not working	M_2 = probability of repairing unit No. 2 if not working

Two Unit System

Figure 6

The system has 2^n possible states and are tabulated in Table 7 below.

Table VII Tabulation of States of Two Unit System		
State	Unit No. 1	Unit No. 2
1	Working (0)	Working (0)
2	Working (0)	Failed (1)
3	Failed (1)	Working (0)
4	Failed (1)	Failed (1)

We now list the transition probabilities in Table 8 below.

Table VIII Transition Probabilities				
From \ To	1	2	3	4
1	p_{11}	p_{12}	p_{13}	p_{14}
2	p_{21}	p_{22}	p_{23}	p_{24}
3	p_{31}	p_{32}	p_{33}	p_{34}
4	p_{41}	p_{42}	p_{43}	p_{44}

where

p_{11} = probability of being in state one and remaining in state one

p_{32} = probability of being in state three and going to state two, etc.

NOTE: It will be shown that a double transition; i.e., the probability of going from state four to state one in a small increment of time (Δt) is impossible and therefore zero.

Substituting the probabilities given in [illegible] into [illegible] results in Table IX

Table IX				
From \ To	1	2	3	4
1	R_1R_2	$R_1(1-R_2)$	$(1-R_1)R_2$	$(1-R_1)(1-R_2)$
2	R_1M_2	$R_1(1-M_2)$	$(1-R_1)M_2$	$(1-R_1)(1-M_2)$
3	M_1R_2	$M_1(1-R_2)$	$(1-M_1)R_2$	$(1-M_1)(1-R_2)$
4	M_1M_2	$M_1(1-M_2)$	$(1-M_1)M_2$	$(1-M_1)(1-M_2)$

We now determine Q from the relationship

$$Q = \lim_{\Delta t \to 0} \frac{[p(\Delta t) - I)]}{\Delta t}$$

$$q_{11} = \lim_{\Delta t \to 0} \frac{(1-\lambda_1\Delta t)(1-\lambda_2\Delta t)-1}{\Delta t} = -(\lambda_1+\lambda_2) \qquad (69)$$

$$q_{12} = \lim_{\Delta t \to 0} \frac{(1-\lambda_1\Delta t)\lambda_2\Delta t}{\Delta t} = \lambda_2 \qquad (70)$$

$$q_{41} = \lim_{\Delta t \to 0} \frac{(\mu_1\Delta t)(\mu_2\Delta t)}{\Delta t} = 0 \text{ etc.} \qquad (71)$$

The complete Q matrix for this two unit system is given in equation (72).

$$\text{[illegible]} = \begin{pmatrix} -(\lambda_1+\lambda_2) & \lambda_2 & \lambda_1 & 0 \\ \mu_2 & -(\lambda_1+\mu_2) & 0 & \lambda_1 \\ \mu_1 & 0 & -(\mu_1+\lambda_2) & \lambda_2 \\ 0 & \mu_1 & \mu_2 & -(\mu_1+\mu_2) \end{pmatrix} \qquad (72)$$

The system of linear differential equations is as follows:

$$\left.\begin{array}{lllll} \dot{a}_1(t) = & -(\lambda_1+\lambda_2)a_1(t) & +\mu_2 a_2(t) & +\mu_1 a_3(t) & 0 \\ \dot{a}_2(t) = & \lambda_2 a_1(t) & -(\lambda_1+\mu_2)a_2(t) & 0 & +\mu_1 a_4(t) \\ \dot{a}_3(t) = & \lambda_1 a_1(t) & 0 & -(\mu_1+\lambda_2)a_3(t) & +\mu_2 a_4(t) \\ \dot{a}_4(t) = & 0 & \lambda_1 a_2(t) & +\lambda_2 a_3(t) & -(\mu_1+\mu_2)a_4(t) \end{array}\right\} \qquad (73)$$

This system of equations can be solved in a manner similar to that of the one unit system.

It is also possible to solve equation (73) by considering the elements of the availability vector to be functions of the single unit availability. For example, in a two unit system, as shown below

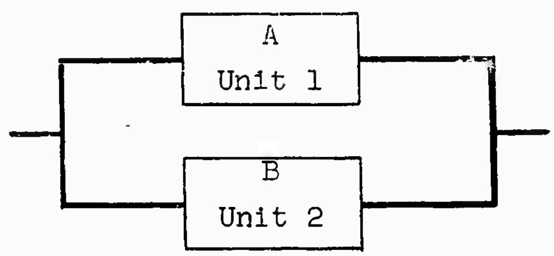


FIGURE 7

Has four states as follows:

Table X

Tabulation of System States
Two Unit System

Status	State
AB	1
$A\bar{B}$	2
$\bar{A}B$	3
$\overline{AB}$	4

a_{11} = unit A availability

$\bar{a}_{11}$ = unit A unavailability

a_{12} = unit B availability

$\bar{a}_{12}$ = unit B unavailability

Then for the two unit case the probability of being in any one of the four states is:

$$a_1 = a_{11}\, a_{12} \left\{\frac{\mu_1}{\lambda_1+\mu_1} + \frac{\lambda_1 e^{-(\lambda_1+\mu_1)t}}{\lambda_1+\mu_1}\right\}\left\{\frac{\mu_2}{\lambda_2+\mu_2} + \frac{\lambda_2 e^{-(\lambda_2+\mu_2)t}}{\lambda_2+\mu_2}\right\} \tag{74}$$

$$a_2 = a_{11}\, \bar{a}_{12} = \left\{\frac{\mu_1}{\lambda_1+\mu_1} + \frac{\lambda_1 e^{-(\lambda_1+\mu_1)t}}{\lambda_1+\mu_1}\right\}\left\{\frac{\lambda_2}{\lambda_2+\mu_2} - \frac{\lambda_2 e^{-(\lambda_2+\mu_2)t}}{\lambda_2+\mu_2}\right\} \tag{75}$$

$$a_3 = \bar{a}_{11}\, a_{12} = \left\{\frac{\lambda_1}{\lambda_1+\mu_1} - \frac{\lambda_1 e^{-(\lambda_1+\mu_1)t}}{\lambda_1+\mu_1}\right\}\left\{\frac{\mu_2}{\lambda_2+\mu_2} + \frac{\lambda_2 e^{-(\lambda_2+\mu_2)t}}{\lambda_2+\mu_2}\right\} \tag{76}$$

$$a_4 = \bar{a}_{11}\, \bar{a}_{12} = \left\{\frac{\lambda_1}{\lambda_1+\mu_1} - \frac{\lambda_1 e^{-(\lambda_1+\mu_1)t}}{\lambda_1+\mu_1}\right\}\left\{\frac{\lambda_2}{\lambda_2+\mu_2} - \frac{\lambda_2 e^{-(\lambda_2+\mu_2)t}}{\lambda_2+\mu_2}\right\} \tag{77}$$

This then is the solution for equation (73).

This procedure can be extended to encompass more than two units.

Conclusion and Summary

System performance measures availability, reliability, and design capability have been described. The product of these three parameters is a measure of system effectiveness. This measure will provide management with a useful tool for determining:

(1) The quantity and types of equipment required in a system.

(2) The required number of repair crews.

(3) The required number of spare parts.

(4) The system effectiveness predicted and measured.

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